Z. Phys. B 103, 225–229 (1997)

ZEITSCHRIFT
FÜR PHYSIK B

Two-channel Kondo effect in a Luttinger liquid

Mats Granath, Henrik Johannesson

Institute of Theoretical Physics, Chalmers University of Technology, S-412 96 Göteborg, Sweden

Abstract. We review exact results obtained for the overscreened multi-channel Kondo effect in a one-dimensional interacting electron system. A simple rederivation of our results for the case of two channels is presented.

PACS: 72.10.Fk; 72.15.Nj; 72.15.Qm; 75.20.Hr

1. Introduction

Since its inception in the mid-50s, Landau's Fermi-liquid theory has played a key role in the study of interacting many-fermion systems [1]. The theory assumes that the low-lying quasi-particle spectrum of a liquid of fermions can be mapped onto that of the free Fermi gas. This hypothesis and its ramifications has been enormously successful in the theory of ordinary metals, as well as for describing certain strongly correlated electron systems, such as the "conventional" heavy-fermion materials, or the (single-channel) Kondo effect in magnetically doped metallic alloys.

In recent years, however, experimentalists have found (or designed!) a number of systems which do not fit into the standard Fermi-liquid picture. These include the high-temperature superconductors, quasi one- and two-dimensional materials, d- and f-electron based metals close to a quantum critical point, and certain artificially designed nano-scale structures – to mention only the most prominent examples [2]. This situation presents a challenge to the theorist who must here abandon the comforting Fermi-liquid paradigm.

The best studied *model* problems showing non-Fermi liquid behavior are the *Luttinger liquid* [3, 4] and the *overscreened multi-channel Kondo effect* [5–7]. The notion of a "Luttinger liquid" refers to the universal low-energy behavior of interacting electrons in one dimension (1D). The perfect nesting due to the disjoint Fermi surface in 1D (consisting of the two Fermi points $\pm k_F$) produces a diverging particle-hole response at $2k_F$ for repulsive electron-electron interaction. This leads to a breakdown of Fermi liquid theory, with the quasi-particles decaying into spatially separated charge- and spin modes. The multi-channel Kondo effect, on the other hand, arises from the coupling of a local magnetic moment to several degenerate channels m of noninteracting electrons. When $m > 2S$ (with S the magnitude of the impurity spin) the m channels of electron spin bind with the impurity to form a new effective local moment of magnitude $m/2 - S$. The process repeats, with the participating electrons forming a strongly correlated composite exhibiting manifest non-Fermi liquid behavior.

Both problems have been studied extensively, and have shaped much of our current thinking on strongly correlated electron systems. In light of this it is natural to ask: What happens if a magnetic impurity is inserted into a Luttinger liquid with several copies of the conduction band? As for the multi-channel Kondo effect in a Fermi liquid we expect that the impurity also here will induce effective interactions among the electrons. But in a Luttinger liquid the electrons are already strongly correlated by their mutual interactions. Then, what happens? Does the interplay between "induced" and "direct" correlations lead to novel effects? Or do we recover the same Kondo physics as in a Fermi liquid? Aside from the possible experimental relevance of these questions to the study of artificial impurities in quantum wires [9, 10], their resolution is an interesting theoretical issue in its own right, and may lead to new insights into the breakdown of Fermi-liquid behavior.

We have recently attacked this problem using the techniques of *boundary conformal field theory* (BCFT) [11]. The method, as applied to quantum impurity problems, was pioneered by Affleck and Ludwig in a series of remarkable papers on the multi-channel Kondo effect in a Fermi liquid [7]. Its extension to the *single* channel Kondo problem in a Luttinger liquid was subsequently carried out in [12, 13], supporting earlier renormalization-group results conjectured by Furusaki and Nagaosa [14]. The heart of the method is to replace the impurity by a scale invariant boundary condition on the bulk theory. One then uses the tools of BCFT to extract the scaling dimensions of the boundary operators that get generated under renormalization to increasing length scales as $T \to 0$. The leading thermal and magnetic response added to the system by the presence of the impurity is driven by the *leading irrelevant boundary operator* and by computing its auto-correlation functions, the finite-temperature properties due to the presence of the boundary

(*alias* the impurity!) can be accessed via standard perturbative techniques, treating (Euclidean) time as an inverse temperature.

Carrying out this program for a multi-channel (antiferromagnetic) Kondo interaction in a Luttinger liquid [15], we find that the change of the low-temperature specific heat due to the impurity-electron (screening cloud) composite acquires a new leading term when $m > 2S$ [16], scaling in temperature with a critical exponent $\alpha_m = 1/m(1/K_{\rho,m} - 1)$. Here $K_{\rho,m} = (1 + 2(2m-1)g/v_F)^{-\frac{1}{2}} \leq 1$ is a channel-dependent Luttinger liquid charge parameter, measuring the strength g of the repulsive electron-electron interaction, with v_F the Fermi velocity. (Notably, by putting $m = 1$ we recover the anomalous exponent for the exactly screened single-channel case, first proposed by Furusaki and Nagaosa [14].) In contrast, the leading impurity magnetic response for any m remains the *same* as for noninteracting electrons. In what follows we shall rederive these results for the special case of *two channels*, $m = 2$, and with an impurity spin $S = 1/2$. For this particular case the analysis can be routed via the so called *coset construction* [17] in conformal field theory, thus making certain aspects of the theory more transparent.

2. The model

We describe the electrons in the bulk by a two-band spinful Tomonaga-Luttinger type model:

$$\begin{aligned} H_{el} = \frac{1}{2\pi}\int dx \Big\{ & v_F \Big[:\psi^\dagger_{L,i\sigma}(x) i\frac{d}{dx}\psi_{L,i\sigma}(x): \\ & - :\psi^\dagger_{R,i\sigma}(x) i\frac{d}{dx}\psi_{R,i\sigma}(x):\Big] \\ & + \frac{g}{2} :\psi^\dagger_{r,i\sigma}(x)\psi_{r,i\sigma}(x)\psi^\dagger_{s,j\mu}(x)\psi_{s,j\mu}(x): \\ & + g :\psi^\dagger_{L,i\sigma}(x)\psi_{R,i\sigma}(x)\psi^\dagger_{R,j\mu}(x)\psi_{L,j\mu}(x):\Big\}. \end{aligned} \tag{1}$$

Here $\psi_{L/R,i\sigma}(x)$ are the left/right moving components of the electron field $\Psi_{i\sigma}(x)$, expanded about the Fermi points $\pm k_F$: $\Psi_{i\sigma}(x) = e^{-ik_F x}\psi_{L,i\sigma}(x) + e^{ik_F x}\psi_{R,i\sigma}(x)$. Summation over repeated indices for chirality $r, s = R, L$, spin $\sigma = \uparrow, \downarrow$, and band *(flavor)* $i, j = 1, 2$ is implied. The first term in (1) is that of free relativistic electrons in 1D, while the second and third terms describe forward and backward electron-electron scattering respectively. The normal ordering is defined w.r.t. the filled Dirac sea.

The model in (1) can be obtained from a two-band Hubbard chain

$$\begin{aligned} H_{el} = & -t\sum_{n,i\sigma}(c^\dagger_{n,i\sigma}c_{n+1,i\sigma} + c^\dagger_{n+1,i\sigma}c_{n,i\sigma}) \\ & + U\sum_{n,ij,\mu\sigma} n_{n,i\sigma}n_{n,j\mu}\,, \end{aligned} \tag{2}$$

by performing a continuum limit $c_{n,i\sigma} \to \sqrt{\frac{a}{2\pi}}\Psi_{i\sigma}(na)$ (with a the lattice spacing), linearizing the spectrum, and then decomposing $\Psi_{i\sigma}(na)$ in chiral components. The parameters v_F and g in (1) thus become connected to the lattice by $v_F = 2at\sin(ak_F)$ and $g = Ua/\pi$ respectively. The procedure is standard and gives a well-defined theory for small energies and momenta in the limit of weak on-site repulsion U.

We couple the electrons to a localized spin-1/2 impurity $\boldsymbol{S}$ at the origin, using an antiferromagnetic ($\lambda > 0$) spin-exchange interaction. In the continuum, with chirally decomposed electron fields, this takes the form

$$H_K = \lambda :(\psi^\dagger_{L,i\sigma}(0) + \psi^\dagger_{R,i\sigma}(0))\frac{1}{2}\boldsymbol{\sigma}_{\sigma\mu}(\psi_{L,i\mu}(0) + \psi_{R,i\mu}(0))\cdot\boldsymbol{S}: \tag{3}$$

Note that in contrast to the effective 1D Kondo model describing free 3D electrons in several degenerate orbital channels [5–7], the interaction in (3) mixes left- and right-moving fields. As we shall see, this introduces a new twist into the problem.

To make progress it is useful to first replace the spin and flavor indices on the chiral fields, i and σ, by a single index Λ, running from 1 to 4, and then cast the bulk Hamiltonian $\mathscr{H}_{el}$ on the form

$$\begin{aligned} \mathscr{H}_{el} = \frac{1}{2\pi}\int dx \Big\{ & \frac{v_F + 3g}{8} :J_r(x)J_r(x): \\ & + \frac{v_F - g}{5} :J^{\mathscr{A}}_r(x)J^{\mathscr{A}}_r(x): \\ & + \frac{3}{4}gJ_L(x)J_R(x) - 2gJ^{\mathscr{A}}_L(x)J^{\mathscr{A}}_R(x)\Big\}, \end{aligned} \tag{4}$$

with currents

$$J_r(x) = :\psi^\dagger_{r,\Lambda}(x)\psi_{r,\Lambda}(x): \tag{5}$$

$$J^{\mathscr{A}}_r(x) = :\psi^\dagger_{r,\Lambda}(x)T^{\mathscr{A}}_{\Lambda\Gamma}\psi_{r,\Gamma}(x):, \tag{6}$$

obeying the $U(1)$ and $SU(4)_1$ Kac-Moody algebras, respectively (with $T^{\mathscr{A}}_{\Lambda\Gamma}$, $A = 1, .., 15$ the generators of $SU(4)$). The charge sector can be diagonalized by means of the canonical transformation $J_{L/R}(x) = \cosh(\theta)j_{L/R} - \sinh(\theta)j_{R/L}$, with the parameter θ given by $\tanh(2\theta) = \frac{3g}{v_F+3g}$. Furthermore, the non-diagonal term $J^{\mathscr{A}}_L J^{\mathscr{A}}_R$ is (marginally) irrelevant [18] and can hence be dropped. Doing this, we can then separate the spin- and flavor degrees of freedom via the conformal embedding $SU(2)_2 \times SU(2)_2 \to SU(4)_1$. At the level of currents this can be expressed as

$$\begin{aligned} \frac{1}{5} :J^{\mathscr{A}}_{L/R}(x)J^{\mathscr{A}}_{L/R}(x): \doteq & \frac{1}{4} :\boldsymbol{J}^s_{L/R}(x)\cdot\boldsymbol{J}^s_{L/R}(x): \\ & + \frac{1}{4} :\boldsymbol{J}^f_{L/R}(x)\cdot\boldsymbol{J}^f_{L/R}(x):, \end{aligned} \tag{7}$$

with the $SU(2)_2$ spin- and flavor currents

$$\boldsymbol{J}^s_r(x) = :\psi^\dagger_{r,i\sigma}(x)\frac{1}{2}\boldsymbol{\sigma}_{\sigma\mu}\psi_{r,i\mu}(x): \tag{8}$$

$$\boldsymbol{J}^f_r(x) = :\psi^\dagger_{r,i\sigma}\frac{1}{2}\boldsymbol{\sigma}_{ij}\psi_{r,j\sigma}: \quad , \tag{9}$$

$\boldsymbol{\sigma}$ being the Pauli matrices. Collecting these results, we can finally put the *renormalized* bulk Hamiltonian on diagonal Sugawara form, with manifestly decoupled charge- *(c)*, spin- *(s)*, and flavor *(f)* degrees of freedom:

$$\begin{aligned} H^*_{el} = \frac{1}{2\pi}\int_{-\ell}^{\ell} dx \Big\{ & \frac{v_c}{8} :j^i_L(x)j^i_L(x): + \frac{v_s}{4} :\boldsymbol{J}^{si}_L(x)\cdot\boldsymbol{J}^{si}_L(x): \\ & + \frac{v_f}{4} :\boldsymbol{J}^{fi}_L(x)\cdot\boldsymbol{J}^{fi}_L(x):\Big\}, \end{aligned} \tag{10}$$

with velocities $v_c = v_F\sqrt{1+6g/v_F}$ and $v_s = v_f = v_F - g$ respectively. We have here confined the system to a finite interval $x \in [-\ell, \ell]$ and replaced the right-moving currents by a second species of left-moving currents: $j^2_{L/R}(x) \equiv j_{R/L}(-x)$ for $x > 0$ (with $j^1_{L/R}(x) \equiv j_{L/R}(x)$). This is equivalent to folding the interval in half to $[0, \ell]$, imposing a boundary condition

$$j_L^{1/2}(0) = j_R^{2/1}(0), \tag{11}$$

and then using (11) to analytically continue the charge currents back to the full interval (and repeating the procedure for spin- and flavor currents). The Sugawara structure in (10) implies that H^*_{el} is invariant under independent $U(1)$ and $SU(2)_2$ transformations on the two species of charge- and spin/flavor currents, mirroring the *chiral* symmetry in charge-, spin-, and flavor sectors of the renormalized bulk theory. Also note that the decoupling of the three sectors in (10) leads to three dynamically independent bulk theories, each conformally invariant with a single effective velocity.

3. Kondo interaction: symmetries and boundary operators

Adopting the hypothesis that any quantum impurity can be traded for a scale invariant boundary condition on the applicable bulk theory [19], we expect that the effect of the Kondo interaction in (3) is to "renormalize" (11) into a new, nontrivial boundary condition on $\mathscr{H}^*_{el}$. In BCFT, a boundary condition is equivalent to a selection rule for quantum numbers of a conformal embedding of the symmetries of the critical bulk Hamiltonian [11]. In our case, the conformal embedding is $\times_{i=1,2}[U(1)\times SU(2)_2 \times SU(2)_2]^i$ with $U(1)$ quantum numbers q^i and $SU(2)$ spin- and flavor quantum numbers j^i and I^i respectively. These label the *conformal towers* [20] in charge-, spin- and flavor sectors into which the eigenstates of $\mathscr{H}^*_{el}$ organize, and take values according to

$$q^{\frac{1}{2}} = \frac{1}{2}C^{\frac{1}{2}}e^{\theta} \pm \frac{1}{2}D^{\frac{1}{2}}e^{-\theta} \quad j^i, I^i \in \{0, \frac{1}{2}, 1\} \tag{12}$$

with $C^i, D^i \in Z$. For each boundary condition there is a *selection rule* which determines the allowed combinations of quantum numbers from (12). The eigenstates of $\mathscr{H}^*_{el}$ (with a particular boundary condition imposed) are in 1-1 correspondence with the *boundary operators* $\mathscr{O}_n$ of the same theory boosted to Euclidean space-time (with the time-axis as boundary), their scaling dimensions Δ_n being connected to the finite-size energy levels E_n by $E_n = E_0 + \pi v \Delta_n/\ell$ [11]. It follows that the selection rule completely specifies the boundary operator content, and hence the boundary critical behavior. The trivial boundary condition (11) is an artifact of our construction, and the selection rule associated with it must therefore be such as to reproduce the *bulk* scaling dimensions of $\mathscr{H}^*_{el}$. It is less obvious how to identify the selection rule for the nontrivial boundary condition representing (3). This is different from the ordinary two-channel Kondo problem where the selection rule can easily be obtained via a canonical transformation on the spin current [7]: In our case the Kondo interaction in (3) mixes left- and right moving electrons and can therefore not be expressed in terms of a spin current built from the chiral fields $\psi_{L/R}(x)$. This obstructs the use of the standard approach. As it turns out, however, we do not need the full selection rule to obtain the impurity critical behavior: It is sufficient to identify the *leading irrelevant boundary operator* (LIBO) contained in the spectrum, as this is the operator that governs the response of the impurity-electron composite. This opens up a short cut for attacking the problem: We consider *all* selection rules for combining quantum numbers in (12) (thus exhausting all conceivable boundary fixed points), for each selecting the corresponding LIBO. We then extract the *possible* impurity critical behaviors by selecting those LIBOs that (i) respect the symmetries of $\mathscr{H}^*_{el} + \mathscr{H}_K$ and (ii) correctly reproduce known results in the noninteracting limit $g \to 0$. Remarkably, via the supplementary condition that (iii) the LIBO should emulate the symmetry breaking induced by the Kondo interaction, we are able to pinpoint a unique solution to the problem.

Starting with condition (i) we note that the Kondo interaction (3) breaks the $U(1)_4 \times U(1)_4$ (charge), $SU(2)_2 \times SU(2)_2$ (spin) and $SU(2)_2 \times SU(2)_2$ (flavor) symmetries of $\mathscr{H}^*_{el}$ down to the corresponding diagonal subgroups. For the charge sector this means that we may allow operators with non-zero charges q_1 and q_2, provided that $q_1 = -q_2$ as required by conservation of total charge. As follows from the analysis in [12], this implies that any $U(1)$ operators with dimensions

$$\Delta_c = \frac{1}{4}n^2 e^{\pm 2\theta} + \mathscr{N}, \qquad n, \mathscr{N} \in N \tag{13}$$

are allowed. The procedure for factoring out the diagonal subgroups in spin- and flavor sectors is more involved. Working at the level of conformal towers it corresponds to decomposing products of two spin/flavor towers $(j^1)_2/(I^1)_2$ and $(j^2)_2/(I^2)_2$ of $SU(2)_2$ into (possibly sums of) products of towers $(j)_4/(I)_4$ of $SU(2)_4$ and the *coset* $SU(2)_2 \times SU(2)_2/SU(2)_4$. The coset is generated by the $N = 1$ *superconformal algebra* (SCA) of central charge $c = 1$, which is an $N = 1$ supersymmetric extension of the $c = 1$ Virasoro algebra [17]. The algebra is divided into two sectors: the Ramond (R) and the Neveu-Schwartz (NS) algebras with primary dimensions $\{\frac{1}{24}, \frac{1}{16}, \frac{3}{8}, \frac{1}{16}\}$ and $\{0, \frac{1}{16}, \frac{1}{6}, 1\}$ respectively. In addition, the grade of a generic state is integer in the R sector, whereas it is half-integer in the NS sector. Conservation of total spin and flavor, respected by $\mathscr{H}_K$, requires that any operator in the spin/flavor $SU(2)_4$ sector transforms as a singlet. The two first singlet states are the vacuum $(j = I = 0)_4$ and the first Kac-Moody descendant in the $(j = I = 1)_4$ tower (with contracted vector indices). Primary states of the spin $SU(2)_4$ sector have conformal dimension $j(j+1)/6$ with $j \in \{0, 1/2, ...2\}$ (and equivalently for the flavor sector), and we thus obtain:

$$\Delta^{s/f}_{SU(2)_4} = 0, \frac{4}{3}, ... \tag{14}$$

with "..." denoting higher dimensions of singlet operators, of no relevance to us here. On the other hand, there is no restriction on the coset sectors since the spin- and flavor symmetries are broken for each species. Thus, excluding the states of zero norm *(null descendants)*, any SCA operators with dimensions

$$\Delta^{s/f}_{SCA} = \{0, \frac{1}{16}, \frac{1}{6}, 1\} + \frac{1}{2}\mathscr{N} \quad \{0, \frac{1}{24}, \frac{3}{8}, 1\} + \mathscr{N}, \qquad (15)$$

with $\mathscr{N} \in N$ may appear.

The scaling dimensions of the full (composite) boundary operators, consistent with the symmetry constraint, are accordingly given by

$$\Delta = \Delta_c + \sum_{j=s,f} (\Delta^j_{SU(2)_4} + \Delta^j_{SCA}) \qquad (16)$$

where the combination of values from (13), (14) and (15) depends on the particular selection rule considered. Note that any selection rule forces $\Delta \geq 1$, as the $g = 0$ boundary fixed point is stable with g a marginal perturbation.

4. Impurity critical behavior

Each selection rule defines a scaling Hamiltonian

$$\mathscr{H}_{scaling} = \mathscr{H}^*_{el} + \mu_1 \mathscr{O}_1(0) + \mu_2 \mathscr{O}_2(0) + ... \qquad (17)$$

with $\{\mathscr{O}_n\}$ boundary operators with dimensions chosen from (16). Considering *all* combinations from (13), (14) and (15), for each case keeping the leading terms in (17), the *possible* impurity critical behaviors can be calculated perturbatively in the scaling fields μ_j, using standard techniques [7]. As we expect all response functions – including those of the impurity-electron composite – to be smooth functions of the marginal bulk coupling g, we must here invoke also the constraint in (ii). In particular, the $g = 0$ scaling for the change of the low-temperature specific heat and magnetic susceptibility due to the impurity,

$$C_{imp} \sim T \ln(\frac{T_K}{T}) + ..., \quad \chi_{imp} \sim \ln(\frac{T_K}{T}) + ... \quad T \to 0 \quad (18)$$

should be recovered in the limit $g \to 0$. (Here T_K is a Kondo temperature and "..."' denotes subleading contributions in T.) The result in (18) can be obtained by making a canonical transformation to a Weyl basis of definite-parity ($P = \pm$) fields $\psi_{\pm,i\sigma}(x) = 1/\sqrt{2}\,(\psi_{L,i\sigma}(x) \pm \psi_{R,i\sigma}(-x))$ [21]. In this basis $\mathscr{H}_{el}(g = 0) + \mathscr{H}_K$ becomes identical to the Hamiltonian representing 3D noninteracting electrons in four channels ($P = \pm, i = 1, 2$), coupled to a local spin in the positive parity channels only. At low temperatures it is known to renormalize to the overscreened *two-channel* Kondo fixed point with response functions as in (18) [22].

Performing the analysis for the impurity specific heat C_{imp}, we find that there are only three candidates for critical scaling satisfying the constraints in (i) and (ii):

$$C_{imp} = c_1(1/K_\rho - 1)^2 T^\alpha + c_2 T \ln(\frac{1}{T}) + \mathscr{O}(T), \quad T \to 0 \qquad (19)$$

where (a) $c_{1,2} \neq 0$, $\alpha = 1/2(K_\rho^{-1} - 1)$, (b) $c_{1,2} \neq 0$, $\alpha = 2(K_\rho^{-1} - 1)$, *or* (c) $c_1 = 0, c_2 \neq 0$. Here $K_\rho \equiv K_{\rho,2} = \sqrt{v_F/(v_F + 6g)}$ is the two-channel Luttinger liquid charge parameter, and $c_{1,2}$ are amplitudes of second order in the scaling fields. The leading term in case (a) is generated by the composite operator $\mathscr{O}_1 \sim : e^{i(\sqrt{\pi}/2mK_\rho)\,\phi^1_L} : \times : e^{i(\sqrt{\pi}/2mK_\rho)\,\phi^2_L} : \times \varphi^s \times \varphi^f$ of dimension $\Delta_{\mathscr{O}_1} = 1/4e^{2\theta} + 3/4$, where ϕ^i_L is a chiral boson of species i, and $\varphi^{s/f}$ is a dimension $\Delta_{\varphi^{s/f}} = 3/8$ Ramond field in the spin/flavor coset sector. This is the *only* candidate LIBO that breaks the chiral symmetry in charge-, spin- *and* flavor sectors, as it contains nontrivial operator factors from all sectors. In the derivation of (a) - (c) we used a truncated scaling Hamiltonian, containing only the first few irrelevant boundary operators. If this is to faithfully represent the effect of the Kondo interaction, including its breaking of chiral symmetry, we must require it to contain operators that break this symmetry as well. With this condition we identify (a) as the unique solution for C_{imp}. Note that the leading term in (19) vanishes in the noninteracting limit $K_\rho \to 1$ and we recover the expected result in (18) for $g = 0$.

Turning to the impurity susceptibility χ_{imp}, its leading scaling behavior is produced by the lowest-dimensional irrelevant boundary operator that contains a singlet spin $SU(2)_4$ factor of non-zero dimension. By our symmetry analysis this is identified as the LIBO for the noninteracting problem [7], which in our scheme is obtained by combining the first descendant in the spin-1 $SU(2)_4$ conformal tower with a $\Delta = 1/6$ Neveu-Schwartz field, thus yielding a composite operator of dimension $\Delta = 3/2$. (Note that the operator $\mathscr{O}_1$ only contains the identity in the $SU(2)_4$ sector, and hence does not contribute to χ_{imp}.) Thus, the leading behavior of χ_{imp} is insensitive to the electron-electron interaction, and is still given by (18). It is important to emphasize that this result remains valid also in the presence of the (marginally) irrelevant spin terms which have been removed from the renormalized bulk Hamiltonian (10). It is easy to verify that any mixing between irrelevant bulk- and boundary operators would produce subleading contributions to χ_{imp} compared to that coming from the leading irrelevant boundary operator alone [23]. Hence we can safely conclude that the *leading* impurity magnetic response is blind to the electron-electron interaction.

5. Summary

To summarize, we have shown that the low-T specific heat of a Luttinger liquid with two degenerate electron bands coupled to a localized $S = 1/2$ impurity gets shifted by the impurity by an interaction-dependent term, scaling with a critical exponent $\alpha = 1/2(K_\rho^{-1} - 1)$. In contrast, the leading change of the magnetic susceptibility due to the impurity is insensitive to the electron-electron interaction. These results are exact, given the central assumption that the effect of the impurity is described by a renormalized boundary condition on the bulk Hamiltonian. As reported in [15], an extended analysis can be carried out for any number of electron bands m and magnitude S of the impurity spin. Details of this analysis, covering also transport properties of the system, will be published elsewhere [23].

It is a pleasure to thank I. Affleck, X. Wang, A. A. Zvyagin and, in particular, P. Fröjdh for discussions and useful suggestions. H. J. acknowledges support from the Swedish Natural Science Research Council.

References

1. L. D. Landau, Sov. Phys. - JETP **3**, 920 (1956)
2. For a more detailed catalogue, see e.g. the Proceedings from the ITP Conference on Non-Fermi Liquid Behaviour in Metals, edited by P. Coleman, B. Maple and A. Millis, J. Phys.: Condens. Matter **8**, 9675 (1996)
3. F. D. M. Haldane, J. Phys. C: Solid State Phys. **14**, 2585 (1981)
4. For a review, see J. Voit, Rep. Prog. Phys. **58**, 977 (1995)
5. P. Nozières and A. Blandin, J. Physique **41**, 193 (1980)
6. N. Andrei and C. Destri, Phys. Rev. Lett. **52**, 364 (1984); P. B. Wiegmann and A. M. Tsvelik, Z. Phys. B **54**, 201 (1985)
7. I. Affleck and A. W. W. Ludwig, Nucl. Phys. **B352**, 849 (1991); **B360**, 641 (1991); Phys. Rev. Lett. **67**, 3160 (1991)
8. For a review, see P. Schlottmann and P. D. Sacramento, Adv. Phys. **42**, 641 (1993)
9. G. Kirczenow et al., Phys. Rev. Lett. **72**, 2069 (1994)
10. Y. Meir, N. S. Wingreen, and P. A. Lee, Phys. Rev. Lett. **70**, 2601 (1993)
11. J. L. Cardy, Nucl. Phys. **B324**, 581 (1989)
12. P. Fröjdh and H. Johannesson, Phys. Rev. Lett. **75**, 300 (1995); Phys. Rev. B **53**, 3211 (1996)
13. P. Durganandini, Phys. Rev. B **53**, R8832 (1996)
14. A. Furusaki and N. Nagaosa, Phys. Rev. Lett. **72**, 892 (1994)
15. M. Granath and H. Johannesson, to be published
16. When $m \leq 2S$ the BCFT approach as employed here can only produce a list of possible impurity critical behaviors. For details, see [15]
17. P. Goddard, A. Kent, and D. Olive, Comm. Math. Phys. **103**, 105 (1986)
18. For a detailed analysis of the analogous $SU(2)_1$ case, see I. Affleck et al., J. Phys. A: Math. Gen. **22**, 511 (1989)
19. For a review, see A. W. W. Ludwig, in Quantum Field Theory and Condensed Matter Physics, edited by S. Randjbar-Daemi and L. Yu (World Scientific, Singapore 1994), and I. Affleck, Conformal Field Theory Approach to the Kondo Effect, `cond-mat/9512099`
20. For a review, see P. Ginsparg, in Fields, Strings and Critical Phenomena, edited by E. Brézin and J. Zinn-Justin (North-Holland, Amsterdam, 1990)
21. Note that the Weyl basis cannot be used when $g \neq 0$ since the interacting part of the bulk Hamiltonian $\mathscr{H}_{el}$ becomes nonlocal in this basis
22. I. Affleck, A. W. W. Ludwig, H.-B. Pang, and D. L. Cox, Phys. Rev. B **45**, 7918 (1992)
23. M. Granath and H. Johannesson, in preparation