Spin-orbit and impurity scattering in an integrable electron model: Exact results for dynamic correlations

A. A. Zvyagin[1,2] and H. Johannesson[3]

[1]*Max-Planck Institut für Physik Komplexer Systeme, Noethnitzer Strasse, 38, D-01187 Dresden, Germany*

[2]*B. I. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, Lenin Ave., 47, Kharkov 61103, Ukraine*

[3]*Department of Physics, University of Gothenburg, SE-412 96 Gothenburg, Sweden*

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We introduce an integrable model of spin-polarized interacting electrons subject to a spin-conserving spin-orbit interaction. Using the Bethe ansatz and conformal field theory, we calculate the exact large-time single-electron and density correlations and find that while the spin-orbit interaction enhances the single-electron Green's function, the density correlations get suppressed. Adding a localized impurity and coupling it to the electrons so that integrability is preserved, the dynamic correlations are found to change significantly after a quantum quench with the impurity interaction switched on suddenly. When the electrons are confined to a periodic structure, the correlations are indifferent to the location of the impurity and only carry an imprint of its intrinsic properties. We conjecture that this unusual feature originates from the integrability of the model.

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I. INTRODUCTION

In recent years, there has been a growing interest in materials and solid-state devices with strong spin-orbit interactions. Being a relativistic effect, a spin-orbit interaction (SOI) reveals itself as a velocity-dependent magnetic field acting on the spin of a particle moving in an electric field. This enables the polarization and manipulation of carrier spins by electric fields only—bypassing design complexities connected with local magnetic fields—and is at the heart of the current efforts to fuse spintronics with semiconductor technologies [1]. Spin polarization can be generated by an SOI in a variety of ways: impurity scattering (as in the anomalous [2] and spin Hall effects [3]), via an external electric bias ("current-induced spin polarization" [4]), or, topologically, through spin-momentum locking from strong atomic SOIs (as in topological insulators [5,6]). Once a spin-polarized current is produced, it may then be manipulated by exploiting the presence of other SOIs due to broken symmetries from interfaces, crystal structures, strain, or electric fields. The generic examples in semiconductor heterostructures are the Rashba and Dresselhaus SOIs [7].

In many proposals for spintronic devices, the interaction between electrons has to be taken into account, hence it is important to investigate the effects of SOIs together with electron-electron interactions. This is particularly so for low-dimensional structures where fluctuations are strongly enhanced due to nonanalyticities in the density of states. The additional presence of impurities and disorder leads to a complex problem, making nonperturbative theoretical results highly desirable.

In the present work, we make a first attempt at this task by studying an exactly solvable model of one-dimensional (1D) interacting electrons subject to spin-orbit *and* impurity scattering. To allow for an exact solution, we study a *minimal model* where the electrons are spin-polarized, and with the added SOI preserving the spin polarization. To simplify further, we consider a single impurity, and we devise its interaction with the itinerant electrons in such a way as to make the model integrable, amenable to a Bethe ansatz approach. While the resulting interaction becomes rather unwieldy—as expected from past work on integrable impurities [8]—it could nowadays conceivably be synthesized in a cold atomic gas confined to an optical nanotube [9]. Indeed, the study of synthetic SOIs in cold atomic gases, mimicking effects from semiconductor physics, is now coming of age [10], making this line of research quite timely.

The Bethe ansatz solvability of the model allows us to extract its finite-size spectrum, from which the scaling exponents for correlation functions can be obtained via conformal field theory [11]. Focusing on the large-time dynamical correlations, we find that while the spin-orbit interaction enhances the single-electron Green's function, the density correlations get suppressed. As expected, the presence of the integrable impurity does not influence the scaling exponents at equilibrium: Integrability implies that the impurity supports forward scattering only, with the sole effect that a scattered electron picks up a phase shift that can be absorbed in a twisted boundary condition on its wave function. Considering a local quantum quench—with the impurity-electron interaction suddenly switched on—one might anticipate that the large-time asymptotics maps onto the equilibrium impurity model and therefore is also insensitive to the presence of the impurity. However, this is not the case. When the electrons are confined to a ring, the scaling exponents for the large-time dynamic bulk correlation exponents *do* acquire a dependence on the impurity. Moreover, the quantum quench tends to boost electron density correlations, whereas the spin-orbit interaction does the opposite. This suggests that the very feature of integrability endows the ground state with a highly quantum-entangled structure where also "far-away" electrons feel the presence of the impurity. We conjecture that this feature reflects the way in which an integrable impurity embedded in a one-dimensional ring scatters electrons: All electrons are perfectly transmitted across the impurity site, with the quantum quench releasing a finite-momentum excitation that runs around the ring and influences correlations uniformly in the bulk at large times. This salient feature may enable the

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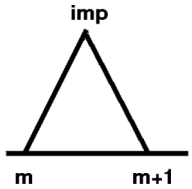

FIG. 1. Illustration of the coupling of the impurity to the host. The impurity is coupled to its neighboring sites with a hopping amplitude $f(\theta,\eta)\cosh(\theta)t$ and interaction $f(\theta,\eta)V$, with $f(\theta,\eta)$ and V defined in the text.

engineering of quantum states in one-dimensional structures with "functional quantum impurities" that do not corrupt electron transport and, moreover, could be used to promote electron correlations.

II. MODEL

We consider a 1D spin-polarized interacting electron system, with the SOI coming from an electric field perpendicular to both the spin polarization and the direction of electron propagation. Awaiting future cold atom realizations [10], such a setup may be materialized using a quantum wire patterned in a zinc-blende semiconductor quantum well where shear strain gradients emulate an internal electric field [12], and with the device put on top of a ferromagnetic insulator to provide for the spin polarization. We should stress, however, that we do not aspire to model a particular experiment. Instead, the main reason for the design of our model is to obtain a sufficiently simple but nontrivial theory that allows for an exact solution. Thus, we take as a Hamiltonian $\mathcal{H}_{\rm wire} = \sum_j h_{j,j+1}$, where

$$h_{j,j+1} = (t + i\alpha)c_j^\dagger c_{j+1} + \text{H.c.} + V n_j n_{j+1} - \mu n_j, \quad (1)$$

where $c_j^\dagger$ (c_j) is the creation (destruction) operator for an electron at the jth site, $n_j = c_j^\dagger c_j$, t is the hopping amplitude in the absence of an SOI, α is the SOI amplitude, and V is the interaction strength between electrons at neighboring sites (see Fig. 1). The hopping term in Eq. (1) can be rewritten [13] as $t'(e^{i2\pi\phi}c_j^\dagger c_{j+1} + \text{H.c.})$, where $t' = \sqrt{t^2+\alpha^2}$ and $\tan(2\pi\phi) = \alpha/t$, and one then recognizes $\mathcal{H}_{\rm wire}$ as a 1D analog of the Haldane-Hubbard model [14]. With the help of a gauge transformation, the phase factor can be removed completely from the theory for the case of an open chain, and transferred to twisted boundary conditions for a closed chain. We will consider the case $0 < V \leqslant t'$, where we can use the parametrization $\cos\eta = V/t'$. It is interesting to note that for this case the Hamiltonian can be mapped with the help of the Jordan-Wigner transformation onto that of an "easy-plane" antiferromagnetic spin-1/2 chain with Dzyaloshinskii-Moriya interaction, with V-μ playing the role of an external magnetic field [15].

Let us now introduce an impurity by adding a lattice site, labeled "imp" and located, say, between sites m and $m+1$ of the chain. To maintain the integrability of the theory, the coupling of the impurity site to the host has to be chosen judiciously. Using a template from Ref. [16] and adapting it to the present case, we are led to the following form of the impurity Hamiltonian:

$$\mathcal{H}_{\rm imp} = f(\theta,\eta)(h_{m,\rm imp} + h_{{\rm imp},m+1} - h_{m,m+1} - g(\theta,\eta)[h_{m,\rm imp}, h_{{\rm imp},m+1}]), \quad (2)$$

where $[\ldots,\ldots]$ denotes a commutator, $f(\theta,\eta) \equiv \sin^2\eta/[\sinh^2\theta + \sin^2\eta]$, $g(\theta,\eta) \equiv i\tanh\theta/\sin\eta$, and where $h_{m,\rm imp}$ and $h_{{\rm imp},m+1}$ have the same structure as in Eq. (1) but with $t \to t_{\rm imp} \equiv t\cosh\theta$. The real parameter θ defines the coupling of the impurity to the host. The case $\theta = 0$ simply corresponds to the addition of a lattice site with no other modification, while for $\theta \to \infty$ the impurity site decouples from the host. Note that for any $\theta \neq 0$, the hopping and interaction between the neighboring sites m and $m+1$ also get modified by $\mathcal{H}_{\rm imp}$. It is worth pointing out that the structure of the impurity Hamiltonian becomes much simpler for the case of an open chain with the impurity situated at its edge: For that case, we have $\mathcal{H}_{\rm imp} = f(\theta,\eta)h_{M,\rm imp}$ (where $M = L/a$ labels the last site in the chain, with L the length of the chain and a the lattice spacing). Also note that the commutator term in Eq. (2), while necessary for integrability, is irrelevant from the point of view of the renormalization group and can be neglected in the long-wavelength limit [17]. It can be checked that the gauge transformation, which removes the phase shift $2\pi\phi$ from the Hamiltonian for open boundary conditions and transfers it to twisted boundary conditions for the closed chain, can be applied also when the impurity interaction in (2) is included.

III. PERIODIC CHAIN: FINITE-SIZE SPECTRUM FROM THE BETHE ANSATZ

While the impurity Hamiltonian in Eq. (2) breaks lattice translational invariance, single-particle backscattering (reflection) is not possible. This is a necessity for the applicability of the BA method to which we now turn. For the case of a periodic chain, with the SOI encoded by twisted boundary conditions, we obtain the BA equations (cf. the corresponding equations for the homogeneous chain without SOI [18])

$$e_1(\lambda_\alpha + \theta)e_1^M(\lambda_\alpha)(-1)^{-\frac{M}{2}-N}e^{i2\pi\phi} = -\prod_{\beta=1,\beta\neq\alpha}^{N} e_2(\lambda_\alpha - \lambda_\beta), \quad (3)$$

which determine the quantum numbers $\{\lambda_\alpha\}_{\alpha=1}^N$ (with N the number of electrons) that parametrize the eigenfunctions and eigenvalues,

$$E = E_0 - \sum_{\alpha=1}^{N}\left(V - \mu - t'\frac{\sin^2\eta}{\cosh\lambda_\alpha - \cos\eta}\right), \quad (4)$$

of the total Hamiltonian $\mathcal{H} = \mathcal{H}_{\rm wire} + \mathcal{H}_{\rm imp}$. Here $\alpha = 1,\ldots,N$, and $e_n(x) = \sinh[(x+in\eta)/2]/\sinh[(x-in\eta)/2]$, with $E_0 = MV/4$. In the noninteracting limit $\theta = V = 0$, the quantum numbers $\{\lambda_\alpha\}_{\alpha=1}^N$ become ordinary rapidities connected to the crystal momenta, and one recovers the expected result for noninteracting spinless fermions with an SOI. Less trivial is the property that the BA equations in (4) are blind to the position of the impurity [19]. This feature, signaling that the impurity is nonreflecting, appears also in the related problem of "mobile" integrable impurities [20,21].

As we shall see, it has dramatic consequences for correlation functions and observables.

The parameter θ, determining the coupling of the impurity to the host, introduces a low-energy scale $T_\theta \sim t\exp(-\pi|\theta|)$, analogous to a Kondo temperature [22]. It defines a crossover between a low-energy regime where the impurity site is strongly coupled to the host, and a high-energy regime with the site being "asymptotically free." Importantly, the scale T_θ characterizes how the impurity influences the zero-frequency response of the system to an applied electric field: At low temperatures, $T \ll T_\theta$, one finds that the impurity contribution $\kappa_{\rm imp}$ to the charge stiffness behaves as $\kappa_{\rm imp} \sim 1/T_\theta$, while at high temperatures, $T \gg T_\theta$, $\kappa_{\rm imp} \sim 1/T\cosh^2(\mu/2T)$, in both cases with corrections $\sim 1/\ln(T/T_\theta)$ for $V = t$. It is important to point out that the appearance of the energy scale T_θ hinges on the presence of the interaction $\sim V$ in Eq. (2). This is different from the archetypal Anderson single-impurity model, in which the charge sector does not feature a crossover scale [22].

The SOI shows up twofold in the BA equations (3), as a renormalization of the hopping t due to the SOI amplitude α and in the phase factor $\exp(i2\pi\phi)$. Their influence on persistent currents and correlation functions is most easily obtained via the finite-size corrections to the energy [23]. The derivation of the finite-size corrections ΔE for the homogeneous model in Eq. (1) follows standard routes. To leading order in $1/L$,

$$\Delta E = \frac{2\pi v}{L}\Delta\,, \tag{5}$$

with $\Delta = [2Z]^{-2}(\Delta N)^2 + Z^2[D-\phi]^2 + n^+ + n^-$, and v is the velocity of low-lying excitations at the Fermi points. Here Z is the "dressed charge" [23], connected to the ground-state charge stiffness $\kappa(\mu)$ by $Z^2 = \pi v\kappa(\mu)$ and taking values from $\sqrt{\pi/2(\pi-\eta)}$ to 1 as μ increases from V to $t' + V$ (where the number of electrons becomes zero). The quantum numbers, ΔN, $D\,(= \Delta N/2 \bmod 1)$, and $n^\pm$, keep track of particle excitations, excitations from one Fermi point to the other (from umklapp), and particle-hole excitations, respectively.

Let us now see how the result in Eq. (5) gets modified when adding the impurity. An analysis similar to that for the homogeneous model yields the same expression for ΔE as in Eq. (5), but with $\Delta \to \Delta_{\rm imp}$, where

$$\Delta_{\rm imp} = [2Z]^{-2}[\Delta N - n_{\rm imp}]^2 + Z^2[D-\phi-d_{\rm imp}]^2, \tag{6}$$

where $n_{\rm imp} = \int_{-\Lambda}^{\Lambda} d\lambda\rho(\lambda)$ is the valence of the impurity site, and

$$d_{\rm imp} = \frac{1}{2}\left(\int_{-\infty}^{-\Lambda} d\lambda\rho(\lambda) - \int_{\Lambda}^{\infty} d\lambda\rho(\lambda)\right). \tag{7}$$

Here $\rho(\lambda)$ satisfies the integral equation

$$\rho(\lambda) = a_1(\lambda-\theta) - \int_{-\Lambda}^{\Lambda} d\lambda' a_2(\lambda-\lambda')\rho(\lambda'), \tag{8}$$

where $a_n(x) \equiv 2\partial_x\{\tan^{-1}[\cot(n\eta/2)\tanh(x/2)]\}$, and the integration limits $\pm\Lambda$ play the role of Fermi points. Note that the values of $n_{\rm imp}$ and $d_{\rm imp}$ are defined mod 1.

IV. CORRELATION FUNCTIONS FOR THE PERIODIC CHAIN

Given the results in Eqs. (5) and (6), one can now calculate the persistent current [24] (Aharonov-Bohm-Casher effect [25]) by differentiating the finite-size correction to the ground-state energy with respect to the external flux (which can be introduced similar to ϕ). Here, we focus instead on how to obtain the asymptotics of correlation functions. The method for this is well known, and uses conformal field theory (CFT) [26] to take advantage of the conformal symmetry underlying the model. Introducing the conformal dimensions $\Delta^\pm$, a correlation function for an operator O in the ground state of the closed homogeneous chain can be written as

$$\langle O(x,t)O(0,0)\rangle \sim \frac{e^{-2i(D-\phi)k_F x}}{(x-ivt)^{2\Delta^+}(x+ivt)^{2\Delta^-}}, \tag{9}$$

where $k_F = \pi N/2L$ is the Fermi wave number, and with the distance $x = ja$ satisfying $a \ll x \ll L$, with j an integer. For small nonzero temperatures T one has to replace $(x \mp ivt)$ by $v\sinh[\pi T(x\mp ivt)/v]/\pi T$ in Eq. (9). By Cardy's formula [11], the conformal dimensions $\Delta^\pm$ are related to Δ in Eq. (5) by $\Delta = \Delta^+ + \Delta^-$ with ϕ absorbed in a twisted boundary condition on the operator O [20], as manifest in Eq. (9). One thus obtains for the homogeneous model without impurity

$$\Delta^\pm = \frac{1}{2}\left[ZD \pm \frac{\Delta N}{2Z}\right]^2 + n^\pm. \tag{10}$$

For the density-density correlation function, the choice of quantum numbers is $\Delta N = 0$, with D a nonzero integer [23]. It follows that the long-time dynamical density correlations are given by

$$\langle n(x,t)n(x,0)\rangle = n_c + \text{const}\times t^{-\gamma_1} + \cdots\,, \tag{11}$$

where $\gamma_D \equiv 2(ZD)^2$, and n_c is a constant. For the single-electron Green's function, we must instead choose $\Delta N = 1$ with D a half-odd integer [23]. We thus obtain

$$\langle c(x,t)c^\dagger(x,0)\rangle = \text{const}\times t^{-\nu_{1/2,1}} + \cdots\,, \tag{12}$$

where $\nu_{D,\Delta N} = 2(ZD)^2 + (\Delta N)^2/2Z^2$. As revealed by Eqs. (11) and (12), the dependence of the dressed charge Z on the renormalized coupling $t' = \sqrt{t^2+\alpha^2}$ makes the SOI suppress large-time density-density correlations while the single-electron Green's function instead gets enhanced.

Adding the impurity, now considering the entire Hamiltonian $\mathcal{H} = \mathcal{H}_{\rm wire} + \mathcal{H}_{\rm imp}$, the theory is no longer invariant under the full conformal group as the presence of the impurity breaks translational invariance. However, exploiting a *boundary CFT* approach [27], we can still extract information about correlation functions using the following trick [28]: We fold the system in half at the impurity position $x = 0$ [taken between sites m and $m+1$ in Eq. (2)], and we represent left- (right-) moving electrons at $x < 0$ by an auxiliary channel of electrons moving right (left) at $x > 0$. Via this construction, the impurity gets traded for a boundary condition at $x = 0$ that is left invariant under a restricted set of conformal transformations and where a forward scattering process [the only process allowed by $\mathcal{H}_{\rm imp}$ in Eq. (2)] corresponds to having an electron come in through one channel and then be reflected back through the other. As shown in the Appendix, the sum

of the boundary scaling dimensions in the auxiliary problem for $x > 0$ precisely defines the bulk scaling dimensions of the original problem, and one finds that these are identical to those of the homogeneous chain without the impurity. As a consequence, the long-time density and single-electron correlations in the presence of the impurity differ from those in Eqs. (11) and (12) only by the shift $\phi \to \phi + d_{\rm imp}$. This result signals the distinctive feature of an *integrable* quantum impurity embedded in a one-dimensional system: All particles impinging on the impurity are perfectly transmitted across the impurity site, with the scattering phase shift $d_{\rm imp}$ absorbable into a twisted boundary condition.

Correlation effects become different when considering the dynamic response after a quantum quench at $t = 0$, set off by suddenly switching on the impurity-electron interaction in Eq. (2). As detailed in the Appendix, the impurity-renormalized boundary condition now implies that $\Delta N \to \Delta N - n_{\rm imp}$ and $D \to D - d_{\rm imp}$ in Eq. (10). Thus, not only the amplitudes but also the exponents γ_D and $\nu_{D,\Delta N}$ get modified by the presence of the impurity. Remarkably, the large-time correlations are translationally invariant, insensitive to the particular location of the impurity. We conjecture that also this property reflects the integrability of the system: By the quantum quench, energy is transferred to the system via the impurity-electron interaction, and the perfectly transmitting impurity releases a finite-momentum excitation that runs around the ring and influences the correlations uniformly in the bulk. This picture is suggestive considering the structure of the BA equations, Eq. (3), which makes it possible to associate nonzero momentum with the impurity. In this way, it effectively comes to play the role of a wave spreading over the ring, illustrating a kind of particle-wave duality.

To elucidate the phenomenon, it may be useful to make an analogy with recent work on interacting 1D spinless fermions with nonlinear dispersion relations [21]. Formally, the nonlinear corrections to the low-energy spectrum can be related here to the presence of a fictitious impurity with properties very similar to the one introduced in our model. In short, the difference between our impurity and the fictitious one is in the definition of the parameter θ. For our case, θ is determined by the impurity-host coupling, whereas for the fictitious impurity θ is instead the rapidity of a high-energy excitation. With this observation, it also becomes easy to generalize our results for the correlation functions to include the nonzero curvature of the dispersion relation. We simply use the additivity of the $1/L$ corrections, and we add $n^f_{\rm imp}(\Lambda_h)$ and $d^f_{\rm imp}(\Lambda_h)$ (with f denoting "fictitious") to $n_{\rm imp}(\theta)$ and $d_{\rm imp}(\theta)$, where Λ_h defines the rapidity of the high-energy excitation. For $\Lambda_h \sim \Lambda$, $n^f_{\rm imp}(\Lambda)$ and $d^f_{\rm imp}(\Lambda)$ (both determined mod 1) can be expressed in terms of the dressed charge Z [21]. In related, earlier work, Tsukamoto *et al.* [20] argued that the sudden insertion of a mobile impurity into an interacting 1D electron system produces nontrivial bulk correlation functions at large times when backscattering is suppressed, thus presaging our exact results via the analogy above. Interestingly, the new correlations produced by the quench are interpreted as being due to an orthogonality catastrophe [29] similar to that in the x-ray edge singularity for systems with a suddenly created localized core hole: The screening effects due to the electrons lead to an "infrared catastrophe," yielding a nontrivial asymptotic behavior of correlation functions in the long-time regime.

V. OPEN CHAIN

Turning to the case of an open chain, with a local potential h attached to its edges, the stratagem from above can be repeated step by step. We find the following for the finite-size corrections:

$$\Delta E = \frac{\pi v}{L}\Delta_b, \tag{13}$$

with

$$\Delta_b = [2Z^2]^{-1}[\Delta N + \Theta(h,\theta)]^2 + n. \tag{14}$$

By putting the impurity at one of the edges, choosing m in Eq. (2) as the corresponding boundary site, Δ_b in Eq. (14) takes the role of boundary scaling dimensions governing the large-time correlation functions in the neighborhood of the impurity. Here

$$\Theta(h,\theta) = -\frac{1}{2}\int_{-\Lambda}^{\Lambda} d\lambda \rho(\lambda), \tag{15}$$

with $\rho(\lambda)$ the solution of the integral equation,

$$\rho(\lambda) = \frac{1}{2}\left(\sum_{j=0,\pm 1} a_1(\lambda + j\theta) + a_2(\lambda)\right) + a_{\mu_h}(\lambda) - \int_{-\Lambda}^{\Lambda} d\lambda' a_2(\lambda - \lambda')\rho(\lambda'). \tag{16}$$

The inhomogeneous term a_{μ_h} is determined as a_n above with the formal substitution $n \to \mu_h$, with $\mu_h = \ln[g_-(\eta,h/t)/g_+(\eta,h/t)]^{1/2}$ an *effective* boundary potential determined by h, and with $g_\pm(\eta,h/t) \equiv \sinh[\ln\sqrt{\cos\eta \pm (2h/t)} \pm i\eta/2]$. Since there is now only a single Fermi point, $D \to 0$, $n^\pm \to n$, as manifest in Eqs. (13) and (14). The nonappearance of the phase ϕ reflects the trivial topology of the open chain, with the spin-conserving SOI only renormalizing the hopping amplitude t. Given our results for the bulk correlations in the periodic chain, we conjecture that the boundary correlations governed by Δ_b are insensitive to a displacement of the integrable impurity away from the boundary. Unfortunately, proof of this is not easily constructed within a boundary CFT formalism.

VI. DISCUSSION

In summary, using a combined Bethe ansatz and conformal field theory approach, we have obtained the exact asymptotic behavior of correlation functions in an integrable model of spin-polarized interacting electrons with a spin-conserving spin-orbit interaction. When the electrons are confined to a ring, the spin-orbit interaction tends to enhance the large-time single-electron correlations while the density-density correlations get suppressed. After a sudden insertion of an integrable quantum impurity, with the impurity-electron interaction switched on abruptly, the scaling of the dynamic correlations picks up a nontrivial dependence on the presence of the impurity. The way the scaling dimensions depend on the spin-orbit coupling and the impurity phase shifts reveals that

the quench enhances the long-time correlations, thus reducing the suppressing effect of the spin-orbit interaction on the density-density correlations. At large times, the phenomenon plays out with the same strength anywhere on the ring, independent of the distance to the impurity. We conjecture that this reflects the integrability of the impurity-electron interaction, which acts to produce a delocalized finite-momentum excitation after the quench, with electrons suffering only forward scattering off the impurity. Conceivably, the effect could be exploited in a future device for boosting electron correlations via a local quantum quench. The rapid progress in "on-demand" design of interactions in fermionic cold-atom systems holds promise for an experimental test.

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APPENDIX A: BOUNDARY CFT FOR A PURELY TRANSMITTING IMPURITY

In this appendix, we show how to reformulate the problem of an integrable—purely transmitting—quantum impurity embedded in the bulk of a one-dimensional spinless fermion system so that Cardy's boundary conformal field theory (CFT) applies [30].

Given the Hamiltonian in Eq. (1), we begin by taking a continuum limit, representing the lattice fermion operators c_n by

$$c_n \to \sqrt{a}[e^{ik_F x}\psi_R(x) + e^{-ik_F x}\psi_L(x)], \quad x = na, \tag{A1}$$

where ψ_L and ψ_R are chiral fields defined in the neighborhood of the Fermi points k_F and $-k_F$, respectively, satisfying

$$\{\psi_\lambda(x), \psi^\dagger_{\lambda'}(y)\} = \delta_{\lambda,\lambda'}\delta(x-y), \quad \lambda,\lambda' = L,R. \tag{A2}$$

Linearizing the spectrum around the Fermi points, the continuum limit of Eq. (1) can then be expressed in current algebra form,

$$H = \frac{v}{2}\int dx \left[\sum_{\alpha=L,R} :J_\alpha(x)J_\alpha(x): + gJ_L(x)J_R(x)\right] \tag{A3}$$

with U(1) currents

$$J_\lambda(x) =: \psi^\dagger_\lambda(x)\psi_\lambda(x): , \tag{A4}$$

and where v and g are parametrized by v_F and V [31]. The normal ordering $: \cdots :$ is carried out with respect to the filled Dirac sea. The Hamiltonian in Eq. (A3) mixes left and right currents but can be diagonalized by the Bogoliubov transformation

$$J_{L/R} = \cosh\theta j_{L/R}(x) - \sinh\theta j_{R/L}(x) \tag{A5}$$

with $2\theta = \operatorname{arctanh}[g/(v_F + g)]$. One thus obtains

$$H = \frac{v}{2}\int dx[: j_L(x)j_L(x): + : j_R(x)j_R(x):], \tag{A6}$$

with the new currents satisfying the U(1) Kac-Moody algebra,

$$[j_{L/R}(x), j_{L/R}(y)] = \pm i\delta'(x-y). \tag{A7}$$

We now boost the currents into the complex plane $\{z = \tau + ix\}$ (with τ a Euclidean time) and identify the impurity site in Eq. (2) with the time axis $x = 0$. Whereas the impurity-electron interaction in Eq. (2) is not easily expressible in terms of the currents, the current algebra formulation is still helpful for understanding how this interaction can be handled within the boundary CFT formalism. In this approach—first used for a quantum impurity problem in Ref. [32]—the interaction in Eq. (2) is traded for a conformally invariant boundary condition at $x = 0$ [27]. As follows from the integrability of the model, in the present case the impurity is perfectly transmitting. This simplifies the problem. However, there is a catch: In the boundary CFT formalism, no momentum or charge is allowed to pass through the boundary. To be able to use boundary CFT, we therefore have to reformulate the problem in such a way that our perfectly transmitting impurity gets represented by a perfectly reflecting boundary. The "trick" how to do this involves the introduction of an auxiliary channel of fermions, where pure transmission through the impurity site gets represented by pure reflection from one channel into the other [28]. Upon analytic continuation, one is left with two channels of left-moving (or right-moving) currents, both respecting translational invariance. The imprint of the impurity (which has now superficially disappeared from the problem) is seen in the new spectrum of scaling dimensions. These dimensions can be read off from the exact finite-size Bethe ansatz spectrum, thus providing access to the asymptotic correlation functions.

To see how this blueprint plays out in mathematical terms, we first impose periodic boundary conditions on the transformed currents,

$$j_\lambda(\tau,0_+) = j_\lambda(\tau,0_-), \tag{A8}$$

thinking of the time axis $x = 0$ as a boundary with periodic boundary conditions when there is no impurity present. Next, we restrict the system to the interval $-L \leqslant x \leqslant L$ (taking $L \to \infty$ at the end), fold it in half, double the currents, and identify $x = -L$ and $x = L$. The new currents, defined in the semi-infinite complex plane $x \geqslant 0$, are connected to the old ones by

$$j_{1L}(x) \equiv j_L(x), \qquad j_{1R}(x) \equiv j_R(x), \tag{A9}$$

$$j_{2L}(x) \equiv j_R(-x), \quad j_{2R}(x) \equiv j_L(-x), \tag{A10}$$

where we have suppressed the common time argument. As a consequence, the periodic boundary condition in Eq. (A8) takes the form

$$j_{1L}(0) = j_{2R}(0), \;\; j_{2L}(0) = j_{1R}(0). \tag{A11}$$

By this procedure, the Hamiltonian in Eq. (A6) is now defined for $x \geqslant 0$ only. Using Eq. (A9) and the boundary condition in Eq. (A11), however, we can analytically continue the left-moving currents to $x < 0$, with

$$j_{1L}(-x) = j_{2R}(x), \quad j_{2L}(-x) = j_{1R}(x), \tag{A12}$$

and then write the Hamiltonian in the full complex plane in terms of left-moving currents only (after having taken $L \to \infty$):

$$H = \frac{v}{2} \sum_{i=1,2} \int dx : j_L(x) j_L(x) : . \tag{A13}$$

We now bring in the impurity-electron interaction, Eq. (2). Introducing the notation $\{\Delta_L\}$ for the subset of chiral ("left-moving," say) scaling dimensions that make up the *boundary scaling dimensions* for a given boundary condition, Cardy's finite-size boundary formula is expressed as $E = E_0 + \pi v \Delta_L / l$ [11]. Adapting it to our case with two copies of left-moving channels, indexed by $i = 1,2$, we have

$$E = E_0 + \frac{\pi v}{l}(\Delta_1 + \Delta_2). \tag{A14}$$

This formula connects the energy spectrum of the theory on the strip $\{w = u + iv\}$, $0 \leqslant v \leqslant l$, $-\infty < u < \infty$, to the sum of boundary scaling dimensions $\Delta_1 + \Delta_2 = 2\Delta_L$ in the semi-infinite plane $\{z = \exp(\pi w/l)\}$ associated with the boundary condition at $x = 0$ which emulates the impurity interaction [33]. It is crucial here to realize that the images of this boundary condition at the two edges of the strip effectively correspond to the insertion of *two* copies of the impurity. While in our case we are not able to pinpoint the appropriate boundary condition *per se*, having obtained the exact finite-size spectrum from the Bethe ansatz solution, we can nonetheless identify the spectrum of scaling dimensions using Eq. (A14): The subset of chiral scaling dimensions $\{\Delta_L\}$ that corresponds to the new boundary condition is simply selected via inspection of the finite-size Bethe ansatz spectrum after insertion of two auxiliary impurities in each channel, one at each edge of the strip. It follows that the quantum numbers ΔN and D in Eq. (10) get renormalized *twice*, with $\Delta N \to \Delta N - n_{\rm imp}(v = 0) = \Delta N'$ and $D \to D - d_{\rm imp}(v = 0) = D'$ from the $v = 0$ edge, and $\Delta N' \to \Delta N' + n_{\rm imp}(v = l) = \Delta N$ and $D' \to D' + d_{\rm imp}(v = l) = D$ from the $x = l$ edge. The opposite signs of the charge valences $n_{\rm imp}$ and level shifts $d_{\rm imp}$ at the two edges here originate from the opposite signs of the phase shifts at $v = 0$ and $v = l$ (corresponding to $\tau < 0$ and $\tau > 0$, respectively, in the semi-infinite plane). In contrast, when the impurity interacts with the fermions only when $\tau \geqslant 0$, as after a quantum quench at $\tau = 0$, only the boundary condition at the corresponding edge of the strip, $v = 0$, gets renormalized. As a result, the dynamic correlation exponents pick up a nontrivial contribution from the impurity, with $\Delta N \to \Delta N - n_{\rm imp}$ and $D \to D - d_{\rm imp}$. As was made explicit in our analysis above, the second channel of left-moving currents in Eq. (A13) simulates the right-moving currents in (A6). Therefore, *bulk scaling dimensions* $\{\Delta\}$ appear in Eq. (A14), disguised as sums of chiral scaling dimensions labeled by the channel index: $\Delta = \Delta_1 + \Delta_2$. It is important to emphasize that this conclusion is certain to be valid only for an *integrable* impurity, since only for this case are we ensured that the impurity is purely transmitting in the basis of the $j_{L/R}(x)$ currents that diagonalize the bulk interactions, thus maintaining the decoupling of the two channels.

As a concluding remark in this appendix, it is important to realize that it is precisely the absence of backscattering from the integrable impurity that causes *all* large-time dynamic correlation functions to be governed by the *same* scaling dimensions—independent of the distance from the impurity. Hence there is no crossover from bulk to boundary critical behavior as one approaches the impurity site. The breaking of translational invariance due to the impurity shows up only in a shift of the phase of the full space-time correlation function in Eq. (9). Clearly, as emphasized throughout this work, this feature is not generic but crucially hinges on the design of the impurity interaction, having made it integrable and therefore purely transmitting.

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