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Synthetic helical liquid in a quantum wire

George I. Japaridze,[1,2] Henrik Johannesson,[3] and Mariana Malard[4]
[1]*Andronikashvili Institute of Physics, Tamarashvili 6, 0177 Tbilisi, Georgia*
[2]*Ilia State University, Cholokasvili Avenue 3-5, 0162 Tbilisi, Georgia*
[3]*Department of Physics, University of Gothenburg, SE 412 96 Gothenburg, Sweden*
[4]*Faculdade UnB Planaltina, University of Brasilia, 73300-000 Planaltina-DF, Brazil*
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We show that the combination of a Dresselhaus interaction and a spatially periodic Rashba interaction leads to the formation of a helical liquid in a quantum wire when the electron-electron interaction is weakly screened. The effect is sustained by a helicity-dependent effective band gap which depends on the size of the Dresselhaus and Rashba spin-orbit couplings. We propose a design for a semiconductor device in which the helical liquid can be realized and probed experimentally.

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The concept of a *helical liquid*—a phase of matter where spin and momentum directions of electrons are locked to each other—underpins many of the fascinating features of the recently discovered topological insulators [1]. In the case of an ideal two-dimensional (2D) topological insulator, electron states at its edges propagate in opposite directions with opposite spins, forming a one-dimensional (1D) helical liquid (HL) [2,3]. Given the right conditions [4,5], the spin-filtered modes of the HL may serve as ballistic conduction channels [6], holding promise for novel electronics/spintronics applications.

The HL is expected to exhibit several unusual properties, such as charge fractionalization near a ferromagnetic domain wall [7], interaction-dependent response to pinching the sample into a point contact [8], and enhanced superconducting correlations when two HLs are coupled together [9]. A particularly tantalizing scenario is the appearance of Majorana zero modes in an HL in proximity to a superconductor and a ferromagnet [10]. However, testing these various predictions in experiments on the HgTe/CdTe quantum well structures in which the HL phase has been observed is a formidable challenge: The softness and reactivity of HgTe/CdTe makes it difficult to handle [11], and moreover, charge puddles formed due to fluctuations in the donor density may introduce a helical edge resistance [12]. Alternative realizations of the HL are therefore in high demand. The prospect of using the dissipationless current of an HL in future chip designs adds to the importance of this endeavor [13].

One suggestion is to use a nanowire made of a "strong topological insulator" material [1]. When pierced with a magnetic flux quantum, the electrons in the wire are predicted to form an interacting HL [14]. In another scheme—appearing in attempts to engineer Kitaev's toy model [15] for p-wave pairing [16]—electrons in a quantum wire form an HL when subject to a Rashba spin-orbit coupling combined with a transverse magnetic field [17]. These, like most other proposals for HLs in quantum wires [18], specifically rely on the presence of a magnetic field.

In this Rapid Communication we show that an HL can be produced and controlled in a quantum wire using electric fields only. The advantages of employing electric fields rather than magnetic fields are manifold. Most importantly, an electric field does not corrupt the feature that counterpropagating helical modes carry antiparallel spins. Also, an electric field can easily be generated and applied locally, and eliminates many of the design complexities that come with the use of magnetic fields [19]. Our proposed device (see Fig. 1) exploits an unexpected effect that appears when interacting electrons are subject to a Dresselhaus spin-orbit interaction combined with a *spatially periodic* Rashba interaction: When the electron density is tuned to a certain value, determined by the wavelength of the Rashba modulation, a band gap tied to the helicity of the electrons opens. This gives rise to an HL. Notably, the required setup for realizing this HL is built around standard nanoscale semiconductor technology, and is very different from the recently proposed all-electric setup in Ref. [20] using carbon nanotubes. In what follows we derive an effective model that captures the surprising effect from the interplay between the Dresselhaus and the modulated Rashba interaction. We analyze the model and explain how the HL materializes, and also discuss the practicality and robustness of this type of a synthetic HL.

We consider a setup with a single-channel quantum wire formed in a gated 2D quantum well supported by a semiconductor heterostructure. The electrons in the well are subject to two types of spin-orbit interactions, the *Dresselhaus* and *Rashba* interactions [21]. For a heterostructure grown along [001], with the electrons confined to the xy plane, the leading term in the Dresselhaus interaction takes the form $H_{\rm D} = \beta(k_x\sigma_x - k_y\sigma_y)$ with β a material-specific parameter. The Rashba interaction is given by $H_{\rm R} = \alpha(k_x\sigma_y - k_y\sigma_x)$, where α depends on several distinct features of the heterostructure [22,23], including the applied gate electric field. The latter feature allows for a gate control of the Rashba coupling α [24]. It is important to mention that large fluctuations of α [22] may drive the HL to an insulating state through an Anderson-type transition [25]. We shall return to this issue below.

Taking the x axis along the wire, adding to $H_{\rm D}$ and $H_{\rm R}$ the kinetic energy of the electrons as well as the chemical potential, one obtains—using a tight-binding formulation—the Hamiltonian $H_0 + H_{\rm DR}$, where

$$H_0 = -t\sum_{n,\alpha} c^\dagger_{n,\alpha}c_{n+1,\alpha} + \frac{\mu}{2}\sum_{n,\alpha} c^\dagger_{n,\alpha}c_{n,\alpha} + \text{H.c.}, \quad (1)$$

$$H_{\rm DR} = -i\sum_{n,\alpha,\beta} c^\dagger_{n,\alpha}\left[\gamma_{\rm D}\,\sigma^x_{\alpha\beta} + \gamma_{\rm R}\,\sigma^y_{\alpha\beta}\right]c_{n+1,\beta} + \text{H.c.}, \quad (2)$$

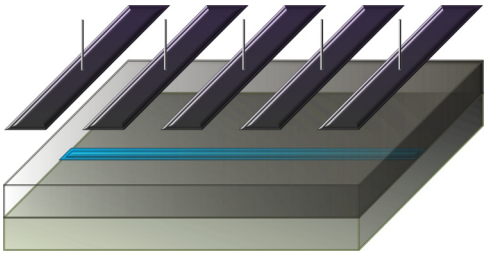

FIG. 1. (Color online) Device supporting a 1D synthetic helical liquid: Electrons in a single-channel quantum wire (blue) formed in a heterostructure supporting a Dresselhaus interaction are subject to a modulated Rashba field from a periodic sequence of charged top gates (dark gray).

with $H_{\rm DR}$ the second-quantized projection of $H_{\rm D}+H_{\rm R}$ along the wire. Here $c^\dagger_{n,\alpha}$ ($c_{n,\alpha}$) is the creation (annihilation) operator for an electron with spin $\alpha=\uparrow,\downarrow$ on site n (with spin along the growth direction $\hat z$), t is the electron hopping amplitude, and μ a chemical potential controllable by a back gate. The signs and magnitudes of $\gamma_{\rm D}\equiv\beta a^{-1}$ and $\gamma_{\rm R}\equiv\alpha a^{-1}$ (a being the lattice spacing) depend on the material as well as on the particular design of the heterostructure.

We now envision that we place a sequence of equally charged nanoscale electrodes on top of the heterostructure (cf. Fig. 1). As a result, the Rashba coupling will pick up a modulated contribution due to the modulation of the electric field from the electrodes. Taking their separation to be the same as their extension along the wire (cf. Fig. 1), we model the Rashba modulation by a simple harmonic,

$$H_{\rm R}^{\rm mod}=-i\gamma'_{\rm R}\sum_{n,\alpha,\beta}\cos(Qna)c^\dagger_{n,\alpha}\sigma^y_{\alpha\beta}c_{n+1,\beta}\ +\ \text{H.c.},\qquad(3)$$

with $\gamma'_{\rm R}$ the amplitude and Q its wave number. Besides the modulation of the Rashba interaction, also the chemical potential gets modulated by the external gates:

$$H_{\rm cp}^{\rm mod}=\frac{\mu'}{2}\sum_{n,\alpha}\cos(Qna)c^\dagger_{n,\alpha}c_{n,\alpha}\ +\ \text{H.c.}\qquad(4)$$

As follows from the analysis in Ref. [26], this term has no effect at low energies unless the electron density is tuned to satisfy the commensurability condition $|Q-2k_F|\ll O(1/a)\bmod 2\pi$, with k_F the Fermi wave number: At all other densities, including those for which an HL emerges, $H_{\rm cp}^{\rm mod}$ in Eq. (4) is rapidly oscillating and gives no contribution in the low-energy continuum limit. Hence, we shall neglect it here.

Given the full Hamiltonian $H=H_0+H_{\rm DR}+H_{\rm R}^{\rm mod}$, we pass to a basis which diagonalizes $H_0+H_{\rm DR}$ in spin space,

$$\begin{pmatrix}d_{n,+}\\ d_{n,-}\end{pmatrix}\equiv\frac{1}{\sqrt2}\begin{pmatrix}-ie^{-i\theta}c_{n,\uparrow}+e^{i\theta}c_{n,\downarrow}\\ e^{-i\theta}c_{n,\uparrow}-ie^{i\theta}c_{n,\downarrow}\end{pmatrix},\qquad(5)$$

with $2\theta=\arctan\gamma_{\rm D}/\gamma_{\rm R}$. The index $\tau=\pm$ of the operator $d_{n,\tau}$ labels the new quantized spin projections along the direction of the combined Dresselhaus ($\propto\gamma_{\rm D}\hat x$) and uniform Rashba ($\propto\gamma_{\rm R}\hat y$) fields. Putting $\gamma'_{\rm R}=0$ in Eq. (3) and using (5), the system is found to exhibit four Fermi points $\pm k_F+\tau q_0$, where $q_0a=\arctan\sqrt{(\tilde t/t)^2-1}$ with $\tilde t=\sqrt{t^2+\gamma_{\rm R}^2+\gamma_{\rm D}^2}$, and where $k_F=\pi\nu/a$ with $\nu=N_e/2N$, N_e (N) being the number of electrons (lattice sites). The corresponding Fermi energy ϵ_F is given by $\epsilon_F=-2\tilde t\cos(k_Fa)+\mu$.

To analyze what happens when $\gamma'_{\rm R}$ is switched on, we focus on the physically relevant limit of low energies, linearize the spectrum around the Fermi points, and take the continuum limit $na\to x$. By decomposing $d_{n,\tau}$ into right- and left-moving fields $R_\tau(x)$ and $L_\tau(x)$,

$$d_{n,\tau}\to\sqrt{a}[e^{i(k_F+\tau q_0)x}R_\tau(x)+e^{i(-k_F+\tau q_0)x}L_\tau(x)],$$

and choosing $|Q-2(k_F+\tau q_0)|\ll O(1/a)\bmod 2\pi$ one thus obtains an effective theory with two independent branches, $H\to\sum_{i=1,2}\int dx\,\mathcal{H}_i$. Let us first consider $Q=2(k_F+q_0)$ for which $\mathcal{H}_1$ applies to the Fermi points $\pm k_F\mp q_0$, and $\mathcal{H}_2$ to $\pm k_F\pm q_0$ and come back to the general case below. Omitting all rapidly oscillating terms that vanish upon integration, one finds

$$\mathcal{H}_1=-iv_F(:R_-^\dagger\partial_xR_-:-:L_+^\dagger\partial_xL_+:),\qquad(6)$$

$$\begin{aligned}\mathcal{H}_2=&-iv_F(:R_+^\dagger\partial_xR_+:-:L_-^\dagger\partial_xL_-:)\\&+i\lambda\,(R_+^\dagger\partial_xL_-+L_-^\dagger\partial_xR_+),\end{aligned}\qquad(7)$$

where $v_F=2a\tilde t\,\sin(\pi\nu)$, $\lambda=a\gamma'_{\rm R}\gamma_{\rm D}(\gamma_{\rm R}^2+\gamma_{\rm D}^2)^{-1/2}$, $:\cdots:$ denotes normal ordering, and where we have absorbed the constant phase $e^{i(k_F+q_0)a}$ into $R_+(x)$.

While the nondiagonal term in Eq. (7) is renormalization-group (RG) irrelevant in the absence of e-e interactions, it may turn relevant and open a gap at the Fermi points $\pm k_F\pm q_0$ when the e-e interaction

$$H_{e\text{-}e}=\sum_{n,n';\alpha,\beta}V(n-n')c^\dagger_{n,\alpha}c^\dagger_{n',\beta}c_{n',\beta}c_{n,\alpha},\qquad(8)$$

is included. Its low-energy limit can be extracted by following the procedure from above, and we obtain $H_{e\text{-}e}\to\int dx\,\mathcal{H}_{e\text{-}e}$, where

$$\begin{aligned}\mathcal{H}_{e\text{-}e}=&\,g_1:R_\tau^\dagger L_\tau L_{\tau'}^\dagger R_{\tau'}:+g_2:R_\tau^\dagger R_\tau L_{\tau'}^\dagger L_{\tau'}:\\&+\frac{g_2}{2}(:L_\tau^\dagger L_\tau L_{\tau'}^\dagger L_{\tau'}:+L\to R),\end{aligned}\qquad(9)$$

with $\tau,\tau'=\pm$ summed over, and where $g_1\sim\tilde V(k\sim2k_F)$ and $g_2\sim\tilde V(k\sim0)$, $\tilde V(k)$ being the Fourier transform of the screened Coulomb potential $V(n-n')$ in Eq. (8). The backscattering $\sim g_1$ is weak in a semiconductor structure and renormalizes to zero at low energies also in the presence of spin-orbit interactions [27]. In effect we are thus left with only the dispersive and forward scattering channels $\sim g_2$ in Eq. (9), to be added to $\mathcal{H}_1$ and $\mathcal{H}_2$ from Eqs. (6) and (7). Passing to a bosonized formalism [28], the resulting full Hamiltonian density can be written as $\mathcal{H}=\mathcal{H}^{(1)}+\mathcal{H}^{(2)}+\mathcal{H}^{(12)}$ with

$$\mathcal{H}^{(i)}=\mathcal{H}_0^{(i)}+\frac{\lambda\,\delta_{i2}}{\sqrt{\pi K}a}\cos(\sqrt{4\pi K}\phi_2)\partial_x\theta_2,\quad i=1,2,\qquad(10)$$

$$\mathcal{H}^{(12)}=\frac{g_2K}{\pi}\partial_x\phi_1\partial_x\phi_2,\qquad(11)$$

where $K\approx(1+g_2/\pi v_F)^{-1/2}$. Here $\mathcal{H}_0^{(i)}=u[(\partial_x\theta_i)^2+(\partial_x\phi_i)^2]$ is a free boson theory with $u\approx v_F/2K$, and with θ_i the dual field to ϕ_i. The indices "1" and "2" tagged to the fields label the two branches originating from Eqs. (6) and (7).

We should point out that our fields ϕ_i $(i = 1,2)$ are rotated with respect to the conventional bosonic fields $\phi_\tau^{R,L}$ $(\tau = \pm)$ [29] representing the original fermion fields R_τ and L_τ, $\phi_i = \phi_\pm^R + \phi_\mp^L$, with upper (lower) sign attached to $i = 1$ $(i = 2)$. This nonstandard spin-mixing basis $\{\phi_i\}$ is suitable for revealing how the nondiagonal term in Eq. (7) combines with the *e-e* interaction in Eq. (9) to gap out the states near $\pm k_F \pm q_0$: The term in Eq. (7) transforms into the sine-Gordon-like potential in Eq. (10) [30], controlled by *e-e* interactions through the Luttinger liquid K parameter. As we shall see, the theory expressed in the form of Eqs. (10) and (11) can be efficiently handled by using an adiabaticity argument.

To make progress we pass to a Lagrangian formalism by Legendre transforming Eqs. (10) and (11). Using that $\Pi_i = \sqrt{K}\partial_x\theta_i$ serves as conjugate momentum to $\phi_i/\sqrt{K}$, Π_i can be integrated out from the partition function Z, with the result

$$Z \sim \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 e^{-(S^{(1)}+S^{(2)}+S^{(12)})}, \tag{12}$$

with Euclidean actions

$$S^{(i)} = S_0^{(i)} - \delta_{i2}\frac{m_0}{\pi a}\int d\tau\, dx\, \cos(\sqrt{16\pi K}\phi_2), \quad i = 1,2, \tag{13}$$

$$S^{(12)} = \frac{g_2 K}{\pi}\int d\tau dx \partial_x\phi_1 \partial_x\phi_2. \tag{14}$$

Here $S_0^{(i)} = (1/2)\int d\tau\, dx[(1/v)(\partial_\tau\phi_i)^2 + v(\partial_x\phi_i)^2]$ is a free action with $v = 2u$, and $m_0 = \lambda^2/4Kva$.

Having brought the theory on the form of Eqs. (13) and (14), valid for a Rashba modulation with $Q = 2(k_F + q_0)$, we first consider the auxiliary problem where the amplitude g_2 in Eq. (14) is replaced by a tunable parameter, call it g_2'. Putting $g_2' = 0$ and refermionizing $S^{(1)}$ we then obtain a helical Dirac action for the first branch (with Fermi points $\pm k_F \mp q_0$), with the second branch (with Fermi points $\pm k_F \pm q_0$) described by a sine-Gordon action, $S^{(2)}$. The cosine term in $S^{(2)}$ becomes RG relevant for $K < 1/2$, driving this branch to a stable fixed point with massive soliton-antisoliton excitations [30]. The energy to create a soliton-antisoliton pair defines an insulating gap Δ, and one finds from the exact solution of the sine-Gordon model [31] that

$$\Delta = c(K)\Lambda\left(\frac{m_0}{\Lambda}\right)^{1/(2-4K)}, \quad K < \frac{1}{2}, \tag{15}$$

where $\Lambda = v/a$ is an energy cutoff, and $c(K)$ is expressible in terms of products of Gamma functions. The opening of a gap implies that the field ϕ_2 gets pinned at one of the minima of the cosine term. Thus, in the neighborhood of the fixed point its gradient is suppressed with the effect that the action $S^{(12)}$ remains vanishingly small also after g_2' has been restored to its true value, $g_2' \to g_2$. In particular, it follows that $S^{(12)}$ cannot close the gap. Note that this "argument by adiabaticity" is perfectly well controlled as the approach to a stable fixed point rules out any nonanalyticities in the spectrum. In summary, when $K < 1/2$, a Rashba modulation $Q = 2(k_F + q_0)$ opens a gap in the second branch which becomes insulating, *leaving behind a conducting helical electron liquid* in the first branch [see Fig. 2(a)].

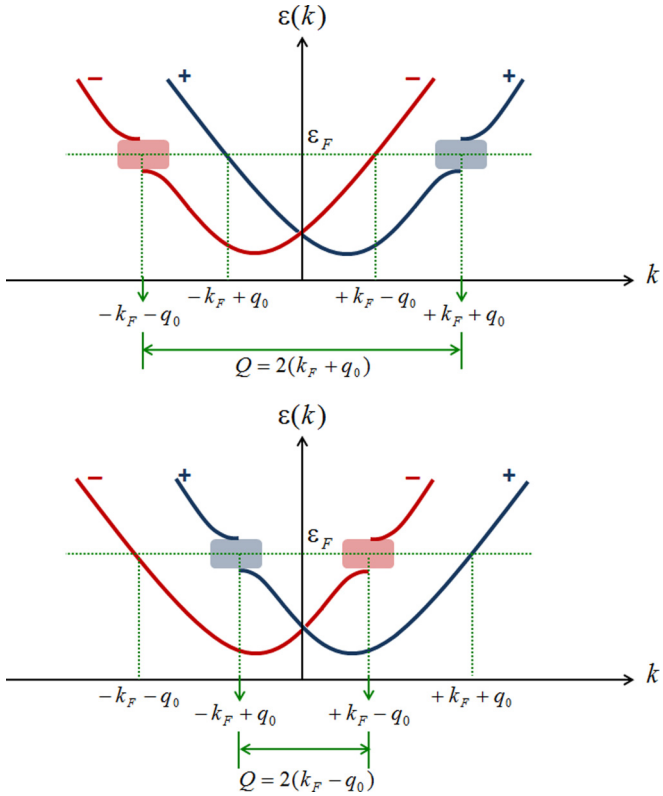

FIG. 2. (Color online) Schematic plot of the dispersion relations for the two types of helical liquid phases, with (a) $Q = 2(k_F + q_0)$ and (b) $Q = 2(k_F - q_0)$.

The analysis above is readily adapted to the case with $Q = 2(k_F - q_0)$, and one finds that the gap now opens in the first branch. Note that our results remain valid in the presence of the weakened commensurability condition $|Q - 2(k_F + \tau q_0)| \ll O(1/a) \bmod 2\pi$, $\tau = \pm$, as this condition still allows us to throw away the rapidly oscillating terms in the low-energy limit of $H_R^{\rm mod}$.

Our interpretation of the dynamically generated gap Δ as an effective band gap—as in Fig. 2–draws on a result by Schulz [32] where a bosonized theory similar to that defined by our Eqs. (10) and (11) is refermionized into a noninteracting two-band model, with the bands separated by a gap corresponding to the dynamic gap of the bosonized theory. This picture—while heuristic only—helps to conceptualize the role of the commensurability conditions for the emergence of the synthetic HL.

The fact that *e-e* interactions can open a gap in an HL is well known from the literature [4,5,16]. In particular, Xu and Moore [5] noted that if a dynamically generated gap opens in one of two coexisting Kramers pairs (*alias* "branches" 1 and 2 in our model), this gives rise to a stable HL in the other pair. Their observation pertains to the case where the scattering within each branch is governed by distinct strengths of the *e-e* interaction: A gap may then open in the branch with the stronger interaction. For this reason the Xu-Moore observation does not apply to the realistic case of of a single quantum wire with the same interaction strength in the two spin-split branches. This is where our proposal injects a different element

into the picture: By properly combining a modulated Rashba spin-orbit interaction with a Dresselhaus interaction we find that the gap-opening mechanism from *e-e* interactions can indeed be triggered in such a way as to open a gap in one of the branches only, leaving behind a stable HL in the other. This HL is of a different type compared to the ones hitherto probed experimentally: It owes its existence neither to being "holographic" [33] (like the edge states of an HgTe quantum well [6]) nor to being "quasihelical" [18] (as is the case for magnetic-field-assisted HLs [34]). The time-reversal analog of the notorious fermion-doubling problem [35] is instead circumvented by the fact that the gapped branch breaks time-reversal symmetry *spontaneously* by developing a spin-density wave [36]. As there is no need to apply a magnetic field to realize the synthetic HL, it escapes the complications from time-reversal symmetry breaking that mar a quasihelical liquid [18]. By this, it becomes an attractive candidate for renewed Majorana fermion searches [37].

Having established a proof of concept that a synthetic HL can be sustained in a quantum wire by application of electric fields only, is our proposal also "deliverable" in the laboratory? The query can be broken down into three specific questions: (i) Is it feasible to realize a regime with sufficiently strong *e-e* interactions (as required by the condition $K < 1/2$)? (ii) Can the size of the gap Δ be made sufficiently large to block thermal excitations at experimentally relevant temperatures? (iii) Is the synthetic HL robust against disorder?

To answer these questions, we take as a case study a quantum wire patterned in an InAs quantum well (QW) [24,38]. Starting with (i), a detailed analysis yields that

$$\tilde{V}(k \sim 0) \approx \frac{e^2}{\pi \epsilon_0 \epsilon_r} \ln\left(\frac{2d}{\eta}\right) + O\left(\frac{\eta^2}{d^2}\right) \qquad (16)$$

with η the half width of the wire, and where ϵ_r is the averaged relative permittivity of the dopant and capping layers between the QW and a metallic back gate at a distance d from the wire [39]. The commonly used $In_{1-x}Al_xAs$ capping layer has $\epsilon_r \approx 12$ when $x = 0.25$, with roughly the same value when doped with Si. With $\eta \approx 5$ nm and $v_F \approx 6 \times 10^5$ m/s [40], taking $d > 1$ μm and using that $g_2 = 4\tilde{V}(k \sim 0)/\pi\hbar$ [28], one verifies that $K \approx (1 + g_2/\pi v_F)^{-1/2} < 1/2$. Thus, the desired "strong-coupling" regime is attainable without difficulty.

Turning to (ii), we need to attach a number to the gap Δ in Eq. (15). Reading off data from Ref. [24], applicable when the InAs QW is separated from the top gates by a solid PEO/$LiClO_4$ electrolyte, the Rashba coupling $\hbar\alpha$ is found to change from 0.4×10^{-11} eV m to 2.8×10^{-11} eV m when tuning a top gate from 0.3 to 0.8 V. With $a \approx 5$ Å [40], we may thus take $\hbar\gamma_R = 8$ meV and $\hbar\gamma_R' = 60$ meV, assuming that (the spacers between) the top gates in Fig. 1 are biased at (0.3 V) 0.8 V. As for the Dresselhaus coupling, experimental data for InAs QWs come with large uncertainties. We here take $\hbar\gamma_D = 5$ meV, guided by the prediction that $1.6 < \alpha/\beta < 2.3$ in conventionally gated structures [38]. Inserting $\lambda = a\gamma_R'\gamma_D(\gamma_R^2 + \gamma_D^2)^{-1/2}$ into Eq. (15), and choosing, say, $K = 1/4$ with $c(1/4) = 1$ [31] we obtain $\Delta \approx 0.3$ meV (with smaller values of K producing a larger gap). While this value of Δ is much smaller than the bulk gap in an HgTe QW [6], it is still large enough—with safe margins—to protect the synthetic HL at sub-Kelvin temperatures. This allows one to probe it by standard quantum transport experiments. It is here interesting to note that a recent proposal for an "all-electric" topological insulator in an InAs double well arrives at an inverted band gap of roughly the same size as our interaction-assisted gap [41].

Finally, let us address (iii). As shown in Refs. [4] and [5], a 1D helical liquid may undergo a localization transition due to disorder-generated correlated two-particle backscattering. A case in point is when a Rashba interaction is present [25], with a fluctuating component $\alpha_{rand}(x)$ from the random ion distribution in nearby doping layers [22]. Fortuitously, the localization length ξ_{rand} for an InAs wire, making the usual assumption that $\sqrt{\langle\alpha_{rand}^2(x)\rangle} \approx \langle\alpha(x)\rangle$ [22], turns out to be much larger than the renormalization scale $\xi = \hbar v/\Delta$ at which the helicity gap develops [43]. Moreover, estimates of the elastic mean free path ℓ_e for InAs quantum wires [42] show that $\xi < \ell_e < \xi_{rand}$ when $1/5 < K < 1/2$ and $\alpha_{rand}(x) < 4 \times 10^{-11}$ eV m. It follows that the synthetic HL is well protected within these parameter intervals.

In summary, we have unveiled a scheme for producing an interacting helical electron liquid in a quantum wire using electric fields only, exploiting an interplay between a Dresselhaus and a spatially periodic Rashba spin-orbit interaction. This synthetic helical liquid is of a different type than existing varieties, being neither "holographic" [6] nor "quasihelical" [34]. While a number of nontrivial design criteria have to be satisfied for its realization in the laboratory, none of them are beyond present-day capabilities. Indeed, considering the principal simplicity and robustness of the required setup, the synthetic helical liquid could become a workhorse for exploring many of the intriguing phenomena associated with helical electrons in one dimension.

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