# CORRIGENDUM

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# Corrigendum: A generalization of the quantum Rabi model: exact solution and spectral structure (2017 *J. Phys. A: Math. Theor.* 50 294004)

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(Some figures may appear in colour only in the online journal)

In our original paper, we solved exactly the quantum Rabi–Stark model, a generalization of the quantum Rabi model (for a recent elementary discussion of the quantum Rabi model, see [1]), and discussed its spectral structure. Doing this required a generalization of Braak's method [2] developed for the exact solution of the quantum Rabi model. This method crucially uses the  $\mathbb{Z}_2$  symmetry of the models. Working in the Bargmann space representation, the ensuing coupled first-order singular ordinary differential equations are solved by a modified power series ansatz, the so-called Frobenius method. The coefficients in this power series are then given, for the quantum Rabi–Stark model, by coupled three-term recurrence relations. Once these recurrence relations are solved numerically, the energy spectra is obtained as the zeros of the so-called *G* functions.

The recurrence relations for the power series coefficients contain twelve coefficient functions, equations (33)–(44) in our original paper, which depend on the model parameters (i.e. the oscillator frequency ( $\omega \equiv 1$ ), the qubit splitting ( $\Delta$ ), the Rabi (g), and Stark ( $\gamma$ ) coupling) and the energy E.

Two of these coefficients,  $\bar{C}_2$  in equation (39) and  $\bar{C}_1$  in equation (40) were miscalculated and should be replaced by

$$\bar{C}_2 = (1 - \gamma^2)w + g = g\left(\sqrt{1 - \gamma^2} + 1\right),$$
(1)

$$\bar{C}_1 = E - 2gw - g^2 + \gamma\Delta \tag{2}$$

where  $w = g/\sqrt{1-\gamma^2}$ .

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**Figure 1.** Energy spectra of the quantum Rabi–Stark model for  $\gamma = 0.5$  ( $\Delta = 0.4$ ). The blue (red) lines correspond to odd (even) parity levels calculated by exact numerical diagonalization (with truncation number N = 400). The blue (red) rings correspond to odd (even) parity levels calculated by iterating the coupled recurrence relations.



**Figure 2.** Energy spectra of the quantum Rabi–Stark model for  $\gamma = 0.95$  ( $\Delta = 0.4$ ). Truncation number and colour and symbol code are the same as in figure 1.

Using the corrected coefficients (1) and (2), one obtains the energy spectra in figures 1 and 2 (replacing figures 4 and 6, respectively, in our original paper). Also shown in figures 1 and 2 are comparisons with energy spectra obtained by exact numerical diagonalization with a truncation number of N = 400. The numerically exact diagonalization of the quantum Rabi–Stark model

is completely independent of the procedure carried out in our original paper. However, the fit of the spectra obtained by these two independent methods is excellent.

While the energy spectra in figures 5 and 6 of our original paper are numerically incorrect and should not be used for extracting quantitative information, the general conclusions drawn in the paper remain unchanged and valid. This includes the scenario for the lifting of degeneracies in the G functions illustrated in figure 2 of our original paper.

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