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Exact results on the two-channel Anderson impurity model: single-electron Green's function and resistivity

Henrik Johannesson[a,*,1], Natan Andrei[b], Carlos J. Bolech[c]

[a]*Institute of Theoretical Physics, Chalmers University of Technology SE 41296 Göteborg, Sweden and Göteborg University, Sweden*
[b]*Department of Physics, Rutgers University, USA*
[c]*DPMC, Université de Genève, Switzerland*

Abstract

We exploit our recent boundary conformal field theory description (Johannesson et al., Phys. Rev. B 68 (2003) 075112) of the two-channel Anderson impurity model to construct its exact space- and time-dependent single-electron Green's function. The universal zero-temperature resistivity and leading temperature-dependent term are derived. We discuss possible implications for a quadrupolar-magnetic mixed-valent scenario for the UBe_{13} compound.

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The possibility of non-Fermi liquid (NFL) behavior in certain cerium- and uranium-based alloys has been a topic of intense discussion and research ever since the first discovery of the anomalous transport and thermodynamics in these materials [1]. Recently, the two-channel Anderson impurity model was proposed to account for the NFL physics of UBe_{13} [2]. In this model, a low-lying quadrupolar $5f^2$ doublet mixes with a magnetic $5f^3$ doublet via the hybridization between the local f orbital and the conduction band:

$$H = H_0 + \varepsilon_f f^\dagger_\sigma f_\sigma + \varepsilon_q b^\dagger_\alpha b_\alpha + V(\psi^\dagger_{\alpha\sigma}(0) b^\dagger_{\bar{\alpha}} f_\sigma + f^\dagger_\sigma b_{\bar{\alpha}} \psi_{\alpha\sigma}(0)). \quad (1)$$

The free electron Hamiltonian is denoted here by H_0, with the conduction electrons represented by radial (1D) fields $\psi^\dagger_{\alpha\sigma}$ carrying spin ($\sigma = \uparrow, \downarrow$) and quadrupolar ($\alpha = \pm$) quantum numbers. The electrons hybridize with the local f levels via the matrix element V. The quadrupolar [magnetic]

*Corresponding author. Tel.: +46 31 7723185; fax: +46 31 7723204.
E-mail address: johannesson@fy.chalmers.se (H. Johannesson).
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doublet of energy ε_q [ε_s] is created by a boson [fermion] operator $b^\dagger_\alpha$ [$f^\dagger_\sigma$]. Strong Coulomb repulsion implies single occupancy of the localized levels: $f^\dagger_\sigma f_\sigma + b^\dagger_\alpha b_\alpha = 1$. For more details, see Ref. [3].

We have applied our recent boundary conformal field theory (BCFT) description of the model [3] to construct its exact space- and time-dependent single-electron Green's function in the limit of low temperatures. Within the BCFT formalism, a quantum impurity gets replaced by a scale-invariant boundary condition on the free electron fields, inserted at the location of the impurity [4]. The specific boundary condition that emulates the presence of the f levels in Eq. (1) was identified in Ref. [3]. The single-electron Green's function picks up a dependence on this boundary condition (alias the impurity) via the one-particle S-matrix.

$$S_{(1)}(\omega_F) = e^{2i\delta_F} C_{(1)}(\omega_F). \tag{2}$$

Here $C_{(1)}(\omega_F)$ is the amplitude for a single electron to scatter elastically off the impurity at the Fermi level ω_F, and $\delta_F = \pi n_c/4$ is the corresponding single-electron scattering phase shift, with n_c the impurity charge valence. Carrying out the same kind of analysis as in Ref. [5] for the two-channel Kondo model (integer valence limit, $n_c = 1$, of the present model), we find that $C_{(1)}(\omega_F) = 0$, independent of the value of n_c. In other words, the outgoing scattering state has no remaining single-electron component after interaction with the impurity. This extreme non-Fermi liquid behavior is the same as for the two-channel Kondo model and *is not modified as one moves into the mixed valence regime where* $n_c \neq 0, 1$.

Using the BCFT machinery to extract the effective scaling Hamiltonian at low temperatures, a perturbative analysis combined with the result for the one-particle S-matrix, produces an exact analytic expression for the leading terms of the single-electron Green's function. For a dilute distribution of uncorrelated impurities we obtain the standard form

$$G(\omega_n, k) = \frac{1}{i\omega_n - \varepsilon_k - \Sigma(\omega_n)}, \tag{3}$$

where the Matsubara self-energy $\Sigma(\omega_n)$ splits into a universal zero-temperature part which is independent of frequency ω_n *and* impurity valence n_c, and a finite-T part which contains two n_c-dependent "scaling fields" λ_q and λ_s which measure the participation of the quadrupolar and spin degrees of freedom in the scattering of the conduction electrons off the impurity [6].

Having obtained the self-energy $\Sigma(\omega_n)$ in Eq. (3) we can calculate the resistivity ρ of the model by analytically continuing $\Sigma(\omega_n) \to \Sigma^R(\varepsilon_k)$, with Σ^R the self-energy of the retarded Green's function. As follows from the analysis in Ref. [7] for this class of problems, vertex corrections to the resistivity involve s-wave correlations which vanish identically. The resistivity is thus determined by the quasi-particle life time

$$\tau(\varepsilon_k) = -\tfrac{1}{2}(\mathrm{Im}\,\Sigma^R(\varepsilon_k))^{-1}, \tag{4}$$

via the simple Kubo formula

$$\rho^{-1}(T) = \frac{4e^2}{3m_e} \int \frac{d^3k}{(2\pi)^3} \left[-\frac{dn_F(\varepsilon_k)}{d\varepsilon_k} \right] k^2 \tau(\varepsilon_k). \tag{5}$$

Here e and m_e are the electron charge and mass, respectively, with $n_F(\varepsilon_k)$ the Fermi distribution function. From Eqs. (4) and (5) we obtain

$$\rho(T) = \frac{3n_i}{4\pi(e g_F v_F)^2} (1 - A(n_c)\sqrt{T} + \ldots), \tag{6}$$

with "..." denoting subleading temperature corrections. The $T = 0$ resistivity is universal and is the same as for the two-channel Kondo model [5]. The leading finite-T term also exhibits the same $\sqrt{T}$ scaling as the two-channel Kondo model, but now with an amplitude $A(n_c)$ which depends on the impurity valence n_c. This amplitude can be determined numerically by fitting the BCFT scaling fields $\lambda_{q,s}$ to the impurity-free energy obtained from the exact *Bethe Ansatz* solution of the model [8]. This work is currently underway [6].

As we mentioned in the introduction, the two-channel single-impurity Anderson model has been proposed as a description of the NFL physics of the UBe_{13} alloy [2]. However, our exact result for the resistivity does not support this conjecture: The experimentally observed $T^{\frac{3}{2}}$-behavior of the low-temperature resistivity for this material [9] is in conflict with the $T^{\frac{1}{2}}$-scaling in Eq. (6). This may not come as a surprise: The thermodynamics of the model found in Ref. [8] also does not seem to agree

with available experimental data on UBe_{13}. Moreover, experimental results for the third-order susceptibility are difficult to explain with the present model [11]. Taken together, these results suggest that additional effects (near-degenerate impurity multiplets [8], excited crystalline electric field states [10], etc.) may have to be taken into account in order to explain the anomalous behavior of this compound.

References

[1] For a review, see D.L. Cox, A. Zawadowski, Adv. Phys. 47 (1998) 599.
[2] A. Schiller, F.B. Anders, D.L. Cox, Phys. Rev. Lett. 81 (1998) 3235.
[3] H. Johannesson, N. Andrei, C.J. Bolech, Phys. Rev. B 68 (2003) 075112.
[4] I. Affleck, A.W.W. Ludwig, Nucl. Phys. B 360 (1991) 641.
[5] I. Affleck, A.W.W. Ludwig, Phys. Rev. B 48 (1993) 7297.
[6] H. Johannesson, N. Andrei, C.J. Bolech, to appear.
[7] N.E. Bickers, D.L. Cox, J.W. Wilkins, Phys. Rev. B 36 (1987) 2036.
[8] C.J. Bolech, N. Andrei, Phys. Rev. Lett. 88 (2002) 237206.
[9] F. Steglich, et al., Z. Phys. B 103 (1997) 235.
[10] M. Koga, D.L. Cox, Phys. Rev. Lett. 82 (1999) 2575.
[11] A.P. Ramirez, P. Chandra, P. Coleman, Z. Fisk, J.L. Smith, H.R. Ott, Phys. Rev. Lett. 73 (1994) 3018.