Measuring Luttinger Liquid Correlations from Charge Fluctuations in a Nanoscale Structure

Paata Kakashvili and Henrik Johannesson

Institute of Theoretical Physics, Chalmers University of Technology and Göteborg University, SE-412 96 Göteborg, Sweden

(Received 24 June 2003; published 28 October 2003)

We suggest an experiment to study Luttinger liquid behavior in a one-dimensional nanostructure, avoiding the usual complications associated with transport measurements. The proposed setup consists of a quantum box, biased by a gate voltage, and side coupled to a quantum wire by a point contact. Close to the degeneracy points of the Coulomb blockaded box, and in the presence of a magnetic field sufficiently strong to spin polarize the electrons, the setup can be described as a Luttinger liquid interacting with an effective Kondo impurity. Using exact nonperturbative techniques, we predict that the differential capacitance of the box will exhibit distinctive Luttinger liquid scaling with temperature and gate voltage.

DOI: 10.1103/PhysRevLett.91.186403 PACS numbers: 71.10.Pm, 73.21.–b, 73.23.Hk

It is theoretically well established that interacting electrons in one dimension (1D) do not form a Fermi liquid, but rather a composite—a *Luttinger liquid* [1]—where *all* low-lying excitations are collective, and separately carry charge and spin. Despite intense efforts, however, there are very few experiments that unambiguously point to Luttinger liquid behavior in a real 1D electron system. Quantum wires [2] and single-walled carbon nanotubes [3] are prime examples of systems where the electron dynamics is effectively one dimensional. Still, interpretations of relevant experimental data based on Luttinger liquid theory remain controversial. In most experiments until now, one has measured *transport* properties, and it has been notoriously difficult to assess the extent to which external sources, contacts, impurities, etc., influence the results.

In this Letter, we propose a *nontransport* experiment on a 1D nanoscale structure which avoids the problems mentioned above. The system is composed by a 1D quantum box side coupled to a single-mode quantum wire via a point contact (Fig. 1), and could be built from a gated GaAs semiconductor or cleaved edge overgrowth structure [2]. A magnetic field is applied such that the electrons become spin polarized. The charging of the box is then monitored as a function of an applied gate voltage or, alternatively, as a function of temperature at a fixed voltage bias. Using a simple model, we show that this setup can be analyzed in terms of Luttinger liquid theory. We find that the differential capacitance of the quantum box has a nonanalytic dependence on temperature and gate voltage, with a scaling exponent that encodes the electron correlations of the system. This fingerprint of Luttinger liquid behavior should be possible to identify by charge measurements using the recently developed single-electron transistor electrometer technique [4], given proper choice of parameters and design of the setup.

We take the quantum box to be sufficiently small to exhibit Coulomb blockade [5], but large enough for the electrons in the box to be modeled by a (confined) Luttinger liquid. More precisely, we study the limit $\delta E \ll k_B T_K \ll e^2/2C_\Sigma$, where δE is the average level spacing of the box close to the Fermi level, T_K is the temperature scale at which correlation effects set in (to be defined below), and $e^2/2C_\Sigma$ is the charging energy of the box (with C_Σ the full capacitance of the box). δE thus serves as a low-energy cutoff restricting the validity of our analysis [6].

The system can be modeled by a Hamiltonian

$$H = H_{el} + H_c + H_{\text{tun}}, \tag{1}$$

where

$$H_{el} = \sum_{k,\alpha} \epsilon_k a^\dagger_{k,\alpha} a_{k,\alpha} + \sum_{q,\alpha,\beta} \hat{U}_{\alpha\beta}(q) \rho_{q,\alpha} \rho_{-q,\beta}, \tag{2}$$

$$H_c = \frac{Q_1^2}{2C_\Sigma} + \zeta V Q_1, \tag{3}$$

$$H_{\text{tun}} = \frac{t}{\ell} \sum_{k,p} (a^\dagger_{k,0} a_{p,1} + \text{H.c.}). \tag{4}$$

Here $a_{k,\alpha}$ are the electron destruction operators in the wire ($\alpha = 0$) and the box ($\alpha = 1$), with the energy ϵ_k measured from the Fermi level ϵ_F. In the interaction term $\rho_{q,\alpha}$ are the Fourier components of the corresponding density operators in the wire and the box, and

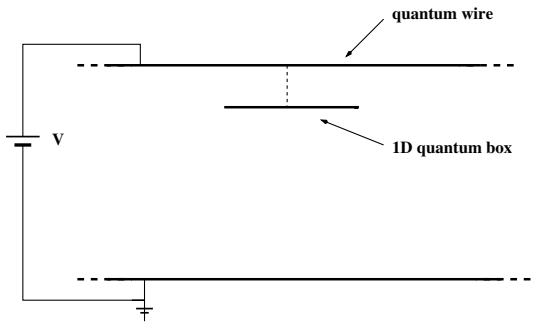

FIG. 1. Schematic picture of the proposed setup. A 1D quantum box side coupled to a quantum wire via a point contact. V is a gate voltage.

$\hat{U}_{\alpha\alpha}(q)[\hat{U}_{01}(q)]$ is the Fourier transform of the screened interaction potential *in* the wire and the box [*between* the wire and the box] (with the screening supplied by carriers in nearby gates). Since the wire and the box are defined on the same substrate, we shall take $\hat{U}_{00}(q) = \hat{U}_{11}(q)$, assuming that their transverse widths are the same. The charging energy of the box is described by H_c, with Q_1 measuring the surplus charge in the box with respect to the (zero bias) Fermi level, ζ being a dimensionless parameter which depends on the layout of the sample, and V the gate voltage. The last term, $H_{\rm tun}$, governs the tunneling between the wire and the box, with t the tunneling rate through the point contact. Note that all effects from the finite size of the 1D box (including Coulomb blockade) are carried by H_c, and that in H_{el} and $H_{\rm tun}$ the length ℓ of the box for simplicity is taken to be the same as that of the extended wire (here assumed to be sufficiently large for additional charging effects to be ignored). Also note that, while H_c encapsulates only the *mean-field* Coulomb interaction among electrons in the box, the electron-electron interaction in H_{el} is dynamic and influences the spectrum already for a fixed number of electrons in the wire and the box.

To make progress, we decompose the electron fields $\psi_\alpha(x) \sim \int dk\, e^{ikx} a_{k,\alpha}$ in left $[\psi_{L,\alpha}(x)]$ and right $[\psi_{R,\alpha}(x)]$ components (with x the coordinate along the wire), expanded about the two Fermi points $\pm k_F$ of the linearized spectrum. Keeping only the "local" piece $U_{\alpha\beta}(x) = \hat{U}_{\alpha\beta}(0)\delta(x)$ of the potential, and setting $\hat{U}_{01}(0) = \hat{U}_{00}(0) \equiv g$, H_{el} can be expressed on diagonal Sugawara form [7] as

$$H_{el} \approx \frac{1}{2\pi}\int dx\Big[\frac{v_c}{4}(:J_R J_R: + :J_L J_L:) + \frac{v_F}{3}(:\boldsymbol{J}_R\cdot\boldsymbol{J}_R: + :\boldsymbol{J}_L\cdot\boldsymbol{J}_L:)\Big], \quad (5)$$

where "$\approx$" is a reminder that (5) contains the local part of the interaction only. Here $v_c = v_F(1 + 4g/v_F)^{1/2}$, with v_F the Fermi velocity, and the normal ordering is taken with respect to the filled Dirac sea. The currents are defined by

$$J_{R/L} = \sinh\vartheta :\psi^\dagger_{R/L,\alpha}\psi_{R/L,\alpha}: + \cosh\vartheta :\psi^\dagger_{L/R,\alpha}\psi_{L/R,\alpha}:,$$

$$\boldsymbol{J}_{R/L} = \tfrac{1}{2}:\psi^\dagger_{R/L,\alpha}(x)\boldsymbol{\sigma}_{\alpha\alpha'}\psi_{R/L,\alpha'}(x):,$$

with $2\vartheta = \operatorname{arctanh}[2g/(v_F + 2g)]$, $\boldsymbol{\sigma}$ being the vector of Pauli matrices, and the indices $\alpha, \alpha' = 0, 1$ summed over.

One immediately recognizes H_{el} in (5) as a Luttinger liquid Hamiltonian, with dynamically separated charge and "pseudospin" currents $J_{L/R}$ and $\boldsymbol{J}_{L/R}$, respectively [8]. Taking into account the boundaries of the box [9], as well as the finite range of the screened Coulomb interaction [10], will add more structure to Eq. (5). Also, in a more realistic theory one expects that $\hat{U}_{01} < \hat{U}_{00}$, implying that the manifest SU(2) pseudospin symmetry of H_{el} in (5) gets broken. However, for transparency and ease of notation, we here choose to work with the simple theory where H_{el} is represented by (5), and return below to discuss the more general case.

Having built in the Luttinger liquid correlations into the model via (5), we now explore how these influence the charging of the box. Let us first recall that in a "classical" picture the charge in a quantum box biased by a gate voltage V can change only when V is tuned to the discrete values $-ne/2\zeta C_\Sigma$ (with n an odd integer) for which the Coulomb blockade is lifted [5]. This leads to the celebrated "Coulomb staircase" with steps at the *degeneracy points* at which the charging energy for $(n/2) \pm 1/2$ electrons is the same. This simple picture is modified by quantum charge fluctuations, enhanced by the coupling of the box to the quantum wire.

To study the fluctuation effects, we probe the system with a gate voltage close to a degeneracy point, for example $\zeta V = -e/2C_\Sigma + u$, with $u \ll e/C_\Sigma$ (i.e., u is a small voltage bias away from the chosen degeneracy point). In the limit of small t, we can then truncate the Hilbert space to the $Q_1 = 0$ and $Q_1 = e$ states (since in this limit transitions to virtual states of higher energy are suppressed). Following an exact formulation of Matveev [11], the resulting two-level system $H_c + H_{\rm tun}$ in (1) can be mapped onto an anisotropic Kondo interaction

$$H_K = \frac{J_\perp}{2}\psi^\dagger_{\mu,\alpha}(0)\sigma^j_{\alpha\alpha'}\psi_{\mu',\alpha'}(0)S^j - hS^z, \quad (6)$$

where $J_\perp = 2t$ and $h = eu$, and where $\boldsymbol{S}$ is an additional "pseudospin" of magnitude $1/2$ that implements the constraint on the allowed states (with $\boldsymbol{S}$ localized at the position $x = 0$ of the point contact). Note that *all* indices in (6) ($\mu, \mu' = L, R$; $\alpha, \alpha' = 0, 1$; $j = x, y$) are summed over. It is here important to realize that the presence of backscattering terms in H_K is due to the fact that the quantum box is *side coupled* to the wire via the point contact. This is different from the case of an *end-coupled* box, which supports only forward Kondo scattering [11–13]. As it turns out, it is precisely the backscattering in (6) that imprints Luttinger liquid characteristics on the charging of the box, measured by the average $\langle Q_1\rangle$. Its dependence on the gate voltage is given by the *differential capacitance* $c(u, T) = -[1/(\zeta e^2)][\partial\langle Q_1\rangle/\partial V]$, which, via the Matveev mapping [11], gets modeled by an impurity susceptibility $\chi_{\rm imp}(h, T) = \partial\langle S^z\rangle/\partial h \equiv c(u, T)$, describing the response of the local pseudospin to a "magnetic field" $h \equiv eu$ at $x = 0$.

The original problem has thus been replaced by that of calculating the susceptibility of a (pseudo)spin-1/2 impurity coupled to a Luttinger liquid H_{el} [Eq. (5)] by an anisotropic Kondo interaction H_K [Eq. (6)]. The presence of backscattering in H_K still makes this a hard problem, however. A perturbative renormalization group (RG) analysis [14] reveals that the backscattering terms become relevant for interacting electrons, taking the theory to a nontrivial fixed point. Here we approach the problem via a nonperturbative route, exploiting boundary

conformal field theory (BCFT) [15] to trade the Kondo interaction H_K for a scale invariant boundary condition on the bulk theory H_{el} in (5). One can then use BCFT to extract the critical exponents that govern the scaling of $\chi_{\rm imp}$ (*alias* the differential capacitance) for small values of T and u (i.e., close to the fixed point).

The fixed point describing the *isotropic* spin-1/2 Kondo effect in a Luttinger liquid [14,16] has been shown to correspond to a particular selection rule for quantum numbers of the BCFT embedding $\mathrm{U}(1) \otimes \mathrm{U}(1) \otimes \mathrm{SU}(2)_2 \otimes \mathrm{Ising}$ [17]. Here the two U(1) factors represent the spectra of left- and right-moving charge excitations, while the $\mathrm{SU}(2)_2 \otimes \mathrm{Ising}$ block derives from a coset construction of the $\mathrm{SU}(2)_1 \otimes \mathrm{SU}(2)_1$ left- and right-moving pseudospin excitation spectra (with the indices labeling the *levels* of the corresponding Kac-Moody algebras [7]). Given this structure, it is straightforward to verify that the anisotropy in (6) introduces irrelevant operators only (in exact analogy to the Kondo effect for noninteracting electrons [18]). Thus, the fixed point for the present problem is the same as for the isotropic model, and we can exploit the BCFT scheme developed in Ref. [17].

Knowing the fixed point allows us to identify the *leading* boundary operators that drive the finite-T scaling of $\chi_{\rm imp}$. Note that, in contrast to the isotropic case in Ref. [17], operators that break (pseudo)spin-rotational invariance are now allowed [by the anisotropy of H_K in (6)]. A systematic search [19] yields two leading operators $\mathcal{O}^{(1)} = T_s \otimes \mathbb{1}_{\rm Ising} \otimes \mathbb{1}_c$ and $\mathcal{O}^{(2)} = J^z \otimes \epsilon \otimes \mathcal{O}_c$, with scaling dimensions $\Delta^{(1)} = 2$ and $\Delta^{(2)} = 3/2 + 1/2K_c$, respectively. K_c is the usual Luttinger liquid "charge parameter" with perturbative expression $K_c = (1 + 4g/v_F)^{-1/2}$ (here allowed to take values in the interval $1/2 \le K_c \le 1$), T_s is the $\mathrm{SU}(2)_2$ energy-momentum tensor, $\mathbb{1}$ is the identity operator in the indexed sector, J^z is the z component of the $\mathrm{SU}(2)_2$ pseudospin current, ϵ is the Ising energy density, and $\mathcal{O}_c$ is a symmetrized product of U(1) vertex operators (for details, see Ref. [17]).

Given the operators $\mathcal{O}^{(1)}$ and $\mathcal{O}^{(2)}$, the scaling behavior of $\chi_{\rm imp}(T, h = 0)$ can be calculated via an expansion in their conjugate scaling fields λ_1 and λ_2. Passing to a Lagrangian formalism, we write the partition function as a path integral, treating the (inverse) temperature as an imaginary time. To simplify the calculation, we also replace the local field h in the definition of $\chi_{\rm imp}$ by a uniform field coupling to the pseudospins of all electrons. This will change the amplitude of the impurity susceptibility [20] but, since we shall be interested in the scaling exponents only, this change is immaterial. Using a linked cluster expansion, we can then write

$$\chi_{\rm imp}(T, 0) = \lambda_1 I[\mathcal{O}_3^{(1)}] + \frac{1}{2}\sum_{i=1,2} \lambda_i^2 I[\mathcal{O}_3^{(i)}, \mathcal{O}_4^{(i)}] + \cdots, \tag{7}$$

where

$$I[\mathcal{O}_3, \ldots, \mathcal{O}_j] \equiv \int_{-\infty}^{\infty} \frac{dx_1 dx_2}{4\pi^2 \beta} \times \int_{-\beta/2}^{\beta/2} d\tau_1 \cdots d\tau_j \langle J_1^z J_2^z \mathcal{O}_3 \cdots \mathcal{O}_j \rangle_c,$$

with $\langle \cdots \rangle_c$ a connected n-point function, and $J_k^z \equiv J^z(\tau_k, x_k)$, $k = 1, 2$. Given the boundary operators $\mathcal{O}_j^{(1,2)} \equiv \mathcal{O}^{(1,2)}(\tau_j)$, $j = 3, 4$, that enter (7), we use the appropriate operator product expansions (OPEs) [7] to collapse the integrands to products of two-point functions. This allows us to easily calculate the integrals and we obtain [using $c(T, u = 0) = \chi_{\rm imp}(T, h = 0)$]

$$c(T, u = 0) = A + B[K_c] T^{1/K_c} + CT^2 + \cdots, \tag{8}$$

with A, $B[K_c]$, and C constants (where $B[K_c] = \mathrm{const}\{1/K_c - 1\}$), and where "$\cdots$" indicates subleading corrections. The short-range electron-electron interaction, encoded by the parameter K_c, is thus seen to induce a *nonanalytic term in the differential capacitance*, scaling as T^{1/K_c}, while vanishing in the noninteracting limit ($K_c = 1$).

Our result in (8) predicts a distinct signal of Luttinger liquid correlations in the proposed setup. For what temperatures should one expect to see it? Taking the 1D quantum box to have a length $\ell' \sim 1$ μm and choosing parameters assuming an experiment using a GaAs heterostructure [2], the energy spacing δE close to the Fermi level corresponds to roughly 0.5 K. The temperature that sets the upper limit for the validity of our theory is the effective Kondo temperature T_K, with expression $T_K = E_C^* \exp(-1/2t\nu)$ in the limit $g\ell < 2t$ [14]. Here $E_C^* = E_C[1 - 4(t\nu)^2 + \cdots]$ is the renormalized charging energy [21], and ν is the density of states at the Fermi level. With $t \sim 0.2/\nu$ and $E_C \sim e^2/2C_\Sigma$, where $C_\Sigma \sim 30$ aF in a typical device, we obtain $T_K \sim 2$ K. With these estimates, our prediction in (8) applies for temperatures in the interval $0.5\ \mathrm{K} < T < 2\ \mathrm{K}$.

Considering the narrowness of the estimated temperature interval, it may experimentally be easier to study the scaling of the capacitance with gate voltage at a fixed temperature. Approximating the window $0.5\ \mathrm{K} < T < 2\ \mathrm{K}$ by the $T \to 0$ limit, the scaling can be obtained via a *Wegner expansion* [22] of the effective ("Kondo language") impurity free energy. Close to the critical point $T = 0, h = 0$, we thus write

$$F_{\rm imp} = \mathrm{const} + Tf\left(\frac{h}{T^\Delta}\right) + g' T^{1-\Delta'} f'\left(\frac{h}{T^\Delta}\right) + \cdots. \tag{9}$$

Here f is a scaling function, $\Delta = 1/2$ is the boundary dimension acquired by the *local* magnetic field h, and f' is the gradient of f with respect to the leading irrelevant scaling field g'. The corresponding operator $\epsilon \otimes \mathcal{O}_c$ is generated from the OPE of J^z with $J^z \otimes \epsilon \otimes \mathcal{O}_c$, and g' is thus proportional to h and carries RG eigenvalue $\Delta' = -(1/K_c - 1)/2 < 0$. In the limit $s \to \infty$, $f(s) \sim s^{1/\Delta}$. Thus, when $T \to 0$, the second term in Eq. (9) gives

an analytic contribution $\sim h^2$. Inspection of the third term in (10) reveals that it can contribute a *finite* correction $\delta F_{\rm imp}$ only via a term $\sim s^{(1-\Delta')/\Delta}$ in the expansion of f', implying that $\delta F_{\rm imp} \sim h^{1+(1-\Delta')/\Delta} = h^{2+1/K_c}$. Contributions from higher order terms in Eq. (9) are of $O[h^4]$. Summarizing, we obtain

$$c(T = 0, u) = D + E[K_c]u^{1/K_c} + Fu^2 + \cdots. \qquad (10)$$

Here D, $E[K_c]$, and F are constants, with $E[K_c] \to 0$ as $K_c \to 1$.

Before concluding, we must address the question how the boundaries of the box, as well as the finite range and the anisotropy of the screened Coulomb interaction, influence the physics. Although these features must be accounted for in a faithful modeling of an experimental sample, they will not *qualitatively* change the charge fluctuation effects derived in Eqs. (8) and (10): As for the boundary effects from the quantum box, these will suppress the spectral weight at the Fermi level, at low energies reducing the effective value of K_c [9]. The finite range R of the screened Coulomb interaction further depresses K_c by a factor $[\ln(R/d)]^{-1/2}$, where d is the (common) transverse width of the wire and the box [10] (with $3 < R/d < 15$ in typical experiments on gated GaAs heterostructures [2]). Both effects are moderate, though, and as long as the renormalized K_c is larger than $1/2$ the nonanalytic terms in (8) and (10) will remain the leading ones. Turning to the expected anisotropy $\hat{U}_{01}(0) \equiv g' < \hat{U}_{00}(0) = g$, this will generate an exactly marginal term proportional to $(g - g')J_L^z J_R^z$, in addition to shift the velocities in (5). While the boundary operators identified above will still be present {with K_c renormalized upwards, with a new perturbative expression $K_c = (1 + 2[g + g']/2)^{-1/2}$}, it is conceivable that the spin sector may now contribute additional boundary operators with noninteger dimensions. However, if there results an exponent *smaller* than $1/K_c$, this implies only that the nonanalytic scaling of the capacitance gets enhanced. Conversely, the $1/K_c$ scaling remains the leading one. In either case, the picture that we have uncovered by using an SU(2) invariant description in (5) will remain valid.

In summary, we predict, under conditions specified above, that the differential capacitance of a quantum box side coupled to a quantum wire exhibits a nonanalytic scaling in temperature and gate voltage, with *the same scaling exponent in both cases*. We have traced the effect to the strong electron correlations inherent in one-dimensional systems, and we expect that high-precision charge measurements [4] should be able to detect it. An experimental verification may shed new light on the elusive Luttinger liquid behavior of electrons in one dimension.

We thank S. Eggert, J. Kinaret, and X. Wang for helpful discussions. This work was supported by the Swedish Research Council under Grant No. 621-2002-4947.

[1] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
[2] A. R. Goñi *et al.*, Phys. Rev. Lett. **70**, 1151 (1993); S. Tarucha, T. Honda, and T. Saku, Solid State Commun. **94**, 413 (1995); A. Yacoby *et al.*, Phys. Rev. Lett. **77**, 4612 (1996); M. Rother *et al.*, Physica (Amsterdam) **6E**, 551 (2000); O. M. Auslaender *et al.*, Science **295**, 825 (2002).
[3] M. Bockrath *et al.*, Science **275**, 1922 (1997); S. J. Tans *et al.*, Nature (London) **386**, 474 (1997).
[4] R. J. Schoelkopf *et al.*, Science **280**, 1238 (1998); W. Lu *et al.*, Nature (London) **423**, 422 (2003).
[5] D. V. Averin and K. K. Likharev, *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier Science, New York, 1991).
[6] The case of a small quantum dot (with level spacing larger than the temperature) is also experimentally relevant but requires a different modeling from the one presented here for a large box.
[7] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, England, 1998).
[8] Note that, given $\hat{U}_{00}(0) = \hat{U}_{11}(0) = \hat{U}_{01}(0) = g$, the absence of renormalization of the pseudospin velocity in (5) reflects the fact that there is no exchange of electrons between wire and box away from the point contact.
[9] A. E. Mattsson, S. Eggert, and H. Johannesson, Phys. Rev. B **56**, 15 615 (1997); F. Anfuso and S. Eggert, cond-mat/0302625.
[10] H. J. Schulz, Phys. Rev. Lett. **71**, 1864 (1993); W. Häusler, L. Kecke, and A. H. MacDonald, Phys. Rev. B **65**, 085104 (2002).
[11] K. A. Matveev, Sov. Phys. JETP **72**, 892 (1991).
[12] E. Lebanon, A. Schiller, and V. Zevin, Phys. Rev. B **64**, 245338 (2001); K. Le Hur and G. Seelig, Phys. Rev. B **65**, 165338 (2002).
[13] A. Furusaki and K. A. Matveev, Phys. Rev. Lett. **88**, 226404 (2002); E. B. Kolomeisky, R. M. Konik, and X. Qi, Phys. Rev. B **66**, 075318 (2002); E. H. Kim, Y. B. Kim, and C. Kallin, cond-mat/0205054.
[14] A. Furusaki and N. Nagaosa, Phys. Rev. Lett. **72**, 892 (1994).
[15] I. Affleck and A. W. W. Ludwig, Nucl. Phys. **B360**, 641 (1991).
[16] R. Egger and A. Komnik, Phys. Rev. B **57**, 10620 (1998).
[17] P. Fröjdh and H. Johannesson, Phys. Rev. Lett. **75**, 300 (1995); Phys. Rev. B **53**, 3211 (1996).
[18] I. Affleck, A. W. W. Ludwig, H.-B. Pang, and D. L. Cox, Phys. Rev. B **45**, 7918 (1992); P. Schlottmann, Phys. Rev. Lett. **84**, 1559 (2000).
[19] P. Kakashvili and H. Johannesson (unpublished).
[20] G. Zaránd, T. Costi, A. Jerez, and N. Andrei, Phys. Rev. B **65**, 134416 (2002).
[21] X. Wang, R. Egger, and H. Grabert, Europhys. Lett. **38**, 545 (1997).
[22] F. J. Wegner, Phys. Rev. B **5**, 4529 (1972).