Loschmidt Echo Revivals: Critical and Noncritical

R. Jafari[1,2,3] and Henrik Johannesson[1,2]
[1]*Beijing Computational Science Research Center, Beijing 100094, China*
[2]*Department of Physics, University of Gothenburg, SE 412 96 Gothenburg, Sweden*
[3]*Department of Physics, Institute for Advanced Studies in Basic Sciences (IASBS), Zanjan 45137-66731, Iran*

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A quantum phase transition is generally thought to imprint distinctive characteristics on the nonequilibrium dynamics of a closed quantum system. Specifically, the Loschmidt echo after a sudden quench to a quantum critical point—measuring the time dependence of the overlap between initial and time-evolved states—is expected to exhibit an accelerated relaxation followed by periodic revivals. We here introduce a new exactly solvable model, the extended Su-Schrieffer-Heeger model, the Loschmidt echo of which provides a counterexample. A parallell analysis of the quench dynamics of the three-site spin-interacting XY model allows us to pinpoint the conditions under which a periodic Loschmidt revival actually appears.

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Taking a quantum system out of equilibrium can be done in many ways, such as injecting energy through an external reservoir or applying a driving field. The simplest paradigm is maybe that of a quantum quench, where a closed system is pushed out of equilibrium by a sudden change in the Hamiltonian which controls its time evolution. Studies of quantum quenches have spawned a large body of results on equilibration and thermalization [1] (and its breakdown in integrable systems [2]), on entanglement dynamics [3], and more [4,5]. In this context, an important task is to identify nonequilibrium dynamical signatures of a quantum phase transition (QPT). The problem comes in a variety of shapes, ranging from the Kibble-Zurek mechanism for defect production [6] to the time evolution of correlations in strongly correlated out-of-equilibrium systems at a QPT [7]. A basic variant is to ask the question: If a Hamiltonian is suddenly quenched to a quantum critical point (or its vicinity), is there any special characteristic of the subsequent dynamics?

To address this question one may invoke the Loschmidt echo (LE) [8], which measures the overlap between the initial (prequench) and time-evolved (postquench) state. Applied to a quantum critical quench—i.e., with the quench parameter pulled to a quantum critical point—finite-size case studies reveal that the time dependence of the LE of several models exhibits a periodic pattern, a revival structure, formed by brief detachments from its mean value [9–16], implying revivals also for expectation values of local observables [17,18]. The amplitudes of these revivals may decay with time; however, their presence appears to be independent of the initial state and the size of the quench [13]. Indeed, the distinctive structure of revivals of the LE after a quench has been conjectured to be a faithful witness of quantum criticality [9,10].

In this Letter we challenge the notion that quantum criticality and LE revival structures are intrinsically linked. We do this by way of example, introducing a new exactly solvable model, the extended Su-Schrieffer-Heeger (ESSH) model, which exhibits several distinct quantum phases with associated QPTs. The ESSH model serves as a representative of a large class of quasifree 1D Fermi systems, and contains as special cases the original SSH model [19], the Creutz model [20], and the Kitaev chain [21] and its dimerized version [22]. Moreover, via a Jordan-Wigner transformation [23], and with suitably chosen parameters, the ESSH model embodies several generic spin chain models, including the 1D quantum compass model [24]. Important for the present work, the quench dynamics of the ESSH model highlights the conditions under which the LE may show a revival structure. Informed by this, and by results extracted from another exactly solvable model, the three-site spin-interacting (TSSI) XY model [25,26], we come to the conclusion that quantum criticality is neither a sufficient nor a necessary condition for the LE to exhibit an observable revival structure. Instead, what matters is that the quasiparticle modes which control the LE are massless *and* have a group velocity $v_g \gg L/t$, where L is the length of the system and t is the observation time. Only if these modes coincide with the quantum critical modes is a revival structure tied to a QPT. These conditions, which are general, bring new light on the important issue of how to read a LE after a quantum quench.

Loschmidt echo.—A quantum quench is a sudden change in the Hamiltonian $H(\theta_1)$ of a quantum system, with θ_1 denoting the value(s) of the parameter(s) that will be quenched. The system is initially prepared in an eigenstate $|\Psi_m(\theta_1)\rangle$ to the Hamiltonian $H(\theta_1)$. The quench is carried out at time $t=0$, when θ_1 is suddenly switched to θ_2. The system then evolves with the quench Hamiltonian $H(\theta_2)$ according to $|\Psi_m(\theta_1,\theta_2,t)\rangle = \exp[-iH(\theta_2)t]|\Psi_m(\theta_1)\rangle$. In this case the LE [8], here denoted by $\mathcal{L}(\theta_1,\theta_2,t)$, reduces to a dynamical version of the ground-state fidelity (return probability),

$$\mathcal{L}(\theta_1,\theta_2,t) = |\langle\Psi_m(\theta_1)|\exp[-iH(\theta_2)t]|\Psi_m(\theta_1)\rangle|^2, \quad (1)$$

measuring the distance between the time-evolved state $|\Psi_m(\theta_1,\theta_2,t)\rangle$ and the initial state $|\Psi_m(\theta_1)\rangle$.

The LE typically decays in a short time $T_{\rm rel}$ (relaxation time), from unity to some mean value around which it then fluctuates [27]. *Revivals* are also visible in the LE as pronounced deviations from the average value [13]. For quenches to a quantum critical point in a finite system there is an expectation that the LE relaxation is accelerated [9–11,15,28–30] and that the revivals are periodic [9,10,13,14]. Conversely, such behavior has been proposed as a signature of quantum criticality [9,10]. However, the matter turns out to be more complex. To see how, we next introduce the ESSH model and exhibit its quench dynamics.

Extended Su-Schrieffer-Heeger model.—We define the Hamiltonian of the ESSH model by

$$H = \sum_{n=1}^{N}[-(wc_n^{A\dagger}c_n^B + \tau c_{n+1}^{A\dagger}c_n^B + \Delta e^{-i\theta}c_n^{A\dagger}c_n^{B\dagger} + \Lambda e^{i\theta}c_{n+1}^{A\dagger}c_n^{B\dagger}) + \frac{\mu}{2}(c_n^{A\dagger}c_n^A + c_n^{B\dagger}c_n^B)] + \text{H.c.}, \quad (2)$$

where A and B are sublattice indices labeling fermion creation and annihilation operators $c_n^{A/B\dagger}$ and $c_n^{A/B}$, w and τ are hopping amplitudes, Δ and Λ are superconducting pairing gaps, $\pm\theta$ are the phases of the pairing terms, and μ is a chemical potential. Choosing $\mu = 0$ and introducing the Nambu spinor $\Gamma^\dagger = (c_k^{A\dagger}, c_k^{B\dagger}, c_{-k}^A, c_{-k}^B)$, the Fourier transformed Hamiltonian can be expressed in Bogoliubov–de Gennes form [31], $H = \sum_{k\geq 0}\Gamma^\dagger H(k)\Gamma$, with

$$H(k) = \begin{pmatrix} 0 & p_k & 0 & q_k \\ p_k^* & 0 & -q_{-k} & 0 \\ 0 & -q_{-k}^* & 0 & -p_{-k}^* \\ q_k^* & 0 & -p_{-k} & 0 \end{pmatrix}, \quad (3)$$

where $p_k = -(w + \tau e^{-ika})$ and $q_k = -(\Delta e^{-i\theta} - \Lambda e^{i(\theta-ka)})$. Here, $k = 2m\pi/L$, $m = 0,\ldots,N/2$, given periodic boundary conditions, and $L = Na$, with a the lattice spacing, taken as unity in arbitrary units.

By diagonalizing $H(k)$ one obtains the quasiparticle Hamiltonian $H = \sum_{\alpha=1}^4\sum_k \varepsilon_k^\alpha \gamma_k^{\alpha\dagger}\gamma_k^\alpha$, with $\gamma_k^{\alpha\dagger}$ and γ_k^α linear combinations of the elements in the Nambu spinor, and with corresponding energy bands $\varepsilon_k^1 = -\varepsilon_k^4 = -\sqrt{a_k+\sqrt{a_k^2-b_k}}$ and $\varepsilon_k^2 = -\varepsilon_k^3 = -\sqrt{a_k-\sqrt{a_k^2-b_k}}$, where $a_k = |q_k|^2 + |p_k|^2 + |q_{-k}|^2 + |p_{-k}|^2$ and $b_k = 4(p_k^*p_{-k} - q_k^*q_{-k})(p_kp_{-k}^* - q_kq_{-k}^*)$. The ground state $|\Psi_0\rangle$ is obtained by filling up the negative-energy quasiparticle states, $|\Psi_0\rangle = \prod_k\gamma_k^{2\dagger}\gamma_k^{1\dagger}|V\rangle$, where $|V\rangle$ is the Bogoliubov vacuum annihilated by the γ_k's (see Supplemental Material [32]).

One easily verifies that the gap to the first excited state vanishes for all momenta k when $\theta = \pi/2$, $w = \Delta$, and $\tau = \Lambda$. The ground state here acquires a degeneracy of $2^{N/2}$ [enlarged to $2\times 2^{N/2}$ at the isotropic point (IP) $\Delta = \Lambda$] [32]. It follows that the line $\theta = \pi/2$ in parameter space is critical for any ratio Δ/Λ. Its interpretation is most easily phrased in spin language by connecting the ESSH model to the general quantum compass model [24,33] via a Jordan-Wigner transformation [23]. The critical line $\theta = \pi/2$ is then seen to define a (nontopological) QPT between two distinct phases with large short-range spin correlations in the x and y direction, respectively. As expected [34], this QPT is signaled by a sharp decay of the ground-state fidelity $F(\theta,\theta+\delta\theta) = |\langle\Psi_0(\theta)|\Psi_0(\theta')\rangle|$, cf. Fig. (S2) in Supplemental Material [32].

Loschmidt echo in the ESSH model.—By a rather lengthy calculation, one can obtain the complete set of eigenstates of the model, yielding an exact expression for the LE [32] When the system is initialized in the ground state $|\Psi_0(\theta_1)\rangle$ and quenched to the critical line, i.e., with $\theta_2 = \theta_c = \pi/2$, one obtains

$$\mathcal{L}(\theta_1,\theta_c,t) = \prod_{0\leq k\leq\pi}\left|1 - A_k\sin^2[\varepsilon_k^1(\theta_c)t] - B_k\sin^2\left[\frac{\varepsilon_k^1(\theta_c)t}{2}\right]\right|, \quad (4)$$

where A_k and B_k measure overlaps between k modes of the initial ground state, $|\psi_{0,k}(\theta_1)\rangle$, and eigenstates $|\psi_{m,k}(\theta_c)\rangle$ of $H(\theta_c)$; cf. Fig. 1 and Ref. [32]. The energies $\varepsilon_k^1(\theta_c)$ are those of the quasiparticles in the lowest filled band in the ground state of the critical quench Hamiltonian.

In Fig. 2 we have plotted $\mathcal{L}(\theta_1,\theta_2,t)$ versus Δ and time t for quenches to the critical line $\theta_2 = \theta_c = \pi/2$ starting

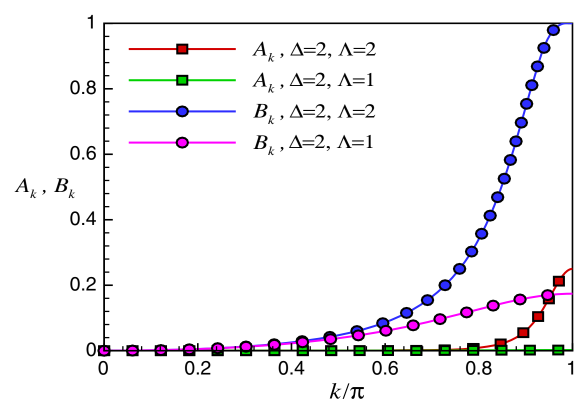

FIG. 1. The amplitudes A_k and B_k in Eq. (4) plotted versus k at the isotropic point $w = \Delta = \tau = \Lambda = 1$ and away from the isotropic point $w = \Delta = 2$, $\tau = \Lambda = 1$.

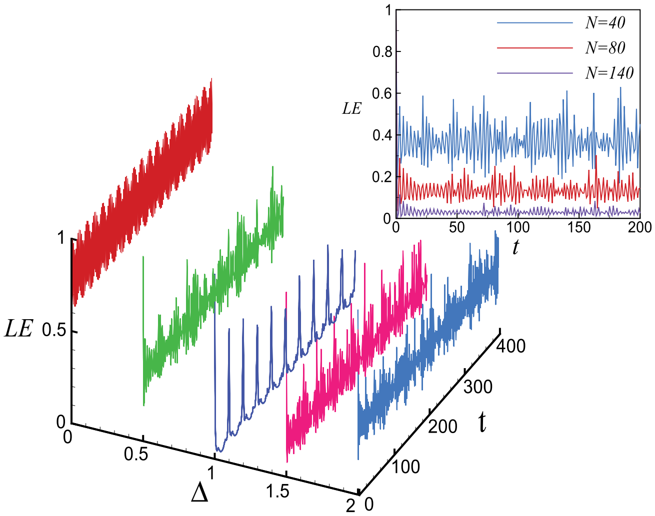

FIG. 2. The LE versus Δ and time t for quenches to the critical line $\theta_2 = \theta_c = \pi/2$ starting from $\theta_1 = 0.45\pi$, for $w = \Delta$, $\tau = \Lambda = 1$, and $N = 40$. Inset: The LE versus time t for quenches to the critical line $\theta_c = \pi/2$ starting from $\theta_1 = 0.45\pi$, for different system sizes N and with $w = \Delta = 2$, $\tau = \Lambda = 1$.

from $\theta_1 = 0.45\pi$, for $w = \Delta$, $\tau = \Lambda = 1$, and $N = 40$. One clearly sees a rapid decay of the LE, with periodic revivals in time when quenching to the IP, $\Delta = 1$. This is in agreement with several studies of LEs at quantum criticality [9–16,27–29,35]. However, departing from the IP, taking $\Delta \neq \Lambda$, but remaining at the critical line $\theta_c = \pi/2$, a surprising result occurs: The periodic revivals get wiped out for sufficiently large anisotropies, with the LE oscillating randomly around its mean value.

To find out why the LE exhibits a revival structure at or very close to the IP, but not farther away from the IP, let us begin by pinpointing the revival periods at the IP, manifest in Fig. 3(a). Plotting T_{rev} versus N, cf. Fig. 3(b), unveils a linear scaling,

$$T_{\mathrm{rev}} = \frac{Na}{K}, \tag{5}$$

where K has dimension of velocity with value $K = 4.00 \pm 0.03$. A numerical spectral analysis suggests that $K \approx v_{\max}$, where $v_{\max} = \max[\partial_k \epsilon_k^1(\theta_c)]$, cf. inset, Fig. 3(b). This result is anticipated from a study of the spin-1/2 XY model [13], where the LE revival period is also governed by the maximum quasiparticle group velocity produced by the critical quench Hamiltonian. However, Eq. (5), with $K \approx v_{\max}$, fails to account for the disappearance of periodic revivals away from the IP. Why is that?

The answer lies in Eq. (4). First, note that a revival requires that all k modes in Eq. (4) contribute sizably to the LE, in turn requiring that the oscillating terms are small. An analysis shows that the oscillation amplitudes A_k and B_k are indeed small except for B_k when approaching the BZ boundary (at which B_k takes its maximum), cf. Fig. 1. It follows that the corresponding modes can contribute constructively to the LE only at time instances at which their oscillation terms get suppressed. Thus, we expect that the most pronounced revivals happen when the vanishing of the term proportional to $B_{k=\pi}$ is concurrent with the near vanishing of B_k terms with k close to π. To obtain the revival period at the IP we thus make the ansatz $\epsilon_{k_0}^1(\theta_c)t/2 = m\pi$, with m an integer and with k_0 the mode with the largest group velocity in the vicinity of the BZ boundary. A Taylor expansion to first order, $\epsilon_{k_0 - p\delta k}^1(\theta_c) \approx \epsilon_{k_0}^1(\theta_c) - \partial_k \epsilon_k^1(\theta_c)|_{k_0} p\delta k$ shows that B_k terms of neighboring k modes are strongly suppressed whenever t is a multiple of $Na/v_{\max}$ with $v_{\max} = \partial_k \epsilon_k^1(\theta_c)|_{k_0}$ and (as before) $a = 1$. Here, $p \ll N$ are integers and $\delta k = 2\pi/N$. This estimate of the revival period agrees with the numerical result in Eq. (5).

Turning to the anisotropic case $\Delta \neq \Lambda$ and repeating the analysis from above immediately reveals why the revival structure now gets lost. First, as exemplified in Fig. 1, the B_k amplitudes are here small for *all* k modes. Thus, the

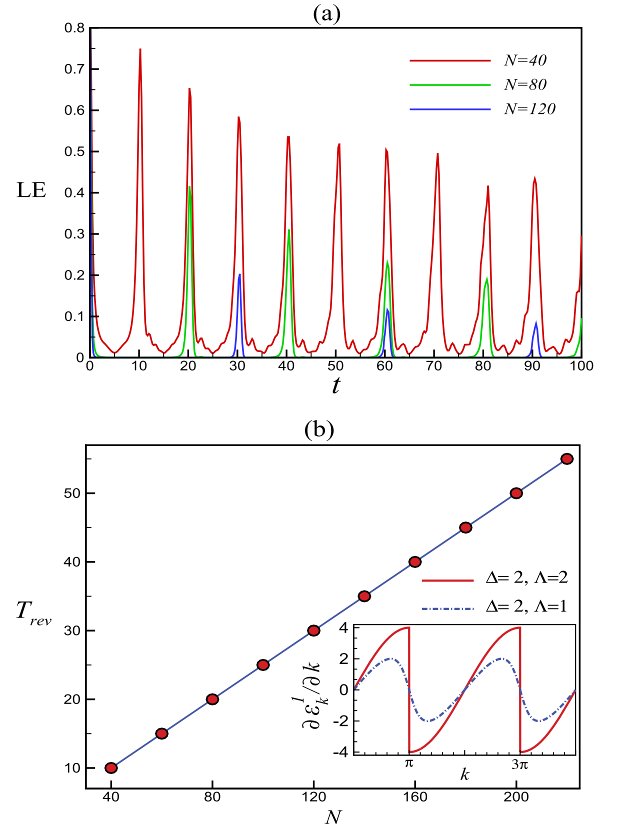

FIG. 3. (a) LE versus time t, with initial pairing phase $\theta_1 = 0.45\pi$ and quenching to the critical line $\theta_c = \pi/2$, for various system sizes N at the IP $w = \Delta = 2$, $\tau = \Lambda = 2$. (b) Scaling of the revival period T_{rev} with system size N for a quench to the critical line at the IP. Inset: The derivative of the ground-state energy modes ϵ_k^1 (group velocity) at the critical line $\theta = \pi/2$ for isotropic (red line) and anisotropic (blue hatched line) cases.

simultaneous suppression of the dominant (but still small) oscillation terms is not expected to have a significant effect on the LE. Moreover, as seen in the inset of Fig. 3(b), the group velocities $v_k = \partial_k \epsilon_k^1(\theta_c)$ away from the IP are quite small throughout the k range where B_k is nonvanishing. As a consequence, with $T_{\text{rev}} \approx L/v_{k=\pi}$ (as before obtained by expanding the quasiparticle energies close to $k=\pi$, where the B_k amplitudes are largest), one would have to wait an exceedingly long time to see any trace of a weak revival structure, if at all present.

To understand the origin of the different behaviors of the LE at the IP and away from the IP, recall from Eq. (4) that the revivals are controlled by quasiparticles in the lowest energy band, ε_k^1. This is so, since the second filled quasiparticle band in the ground state, ε_k^2, collapses to zero and becomes dispersionless at the critical line $\theta_c = \pi/2$ [32]. Away from the IP, the ε_k^1 band remains gapped for all k also at the critical line, thus holding back quasiparticle excitations from that band. This is different from the critical line at the IP where the gap closes at the BZ boundary [32]. Since the oscillation amplitudes can be interpreted as measuring the probabilities of quasiparticle excitations, k modes at or near the gap-closing points are indeed expected to yield much larger amplitudes. As follows from our result for the revival period, if these modes also give rise to a group velocity $v_g \gg L/t$, with t the observation time, a revival structure will ensue. Note that here v_g is the group velocity of quasiparticles at which the oscillation amplitudes peak. While v_g happens to be at a global maximum in the ESSH model at the IP, this property is not expected to be generic.

Loschmidt echo in the three-site spin-interacting XY model.—Having established that quantum criticality is not a sufficient condition for a revival structure in a LE, what about the converse? Can a LE exhibit a revival structure without the presence of a QPT?

The answer is yes. A case in point is the LE of a quench to the $h_s = 0$ line in the $J_3 - h$ parameter space of the three-site spin-interacting XY model [25,26],

$$H_{\text{TSSI}} = -\frac{J}{2}\sum_{j=1}^{N}(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y) - h_s\sum_{j=1}^{N}(-1)^j\sigma_j^z - \frac{J_3}{4}\sum_{j=1}^{N}(\sigma_j^x\sigma_{j+2}^x + \sigma_j^y\sigma_{j+2}^y)\sigma_{j+1}^z, \tag{6}$$

where σ^x, σ^y, and σ^z are the usual Pauli matrices. In Ref. [36] it was noted that the decay rate of the LE shows an accelerated decay in such a quench, independent of whether the quench is critical ($J_3 = 0$) or noncritical ($J_3 \neq 0$). In contrast, the LEs of quenches to the critical lines $h_s = \pm J_3/2$, which define a QPT between an antiferromagnetic and type-I spin-liquid phase, display neither enhanced decays nor revival structures.

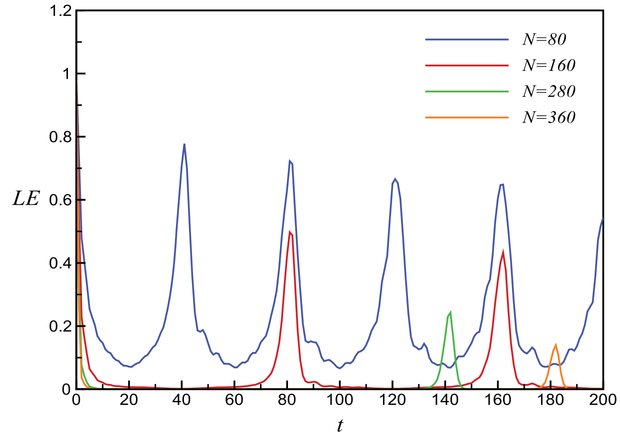

FIG. 4. The LE of the TSSI XY model versus time t at the noncritical point where $J_3 = 4$ and $h_s = 0$.

Guided by our results for the ESSH model, we resolve this conundrum by numerically confirming that the absence of a revival structure for a quench from the antiferromagnetic phase to the $h_s = \pm J_3/2$ critical lines of the TSSI XY model is linked to consistently small oscillation amplitudes in the mode decomposition of the LE. Analogous to the ESSH model away from the IP, this can be attributed to the fact that the quasiparticles which control the LE remain fully gapped as one approaches the QPT. On the contrary, the revival structures which *do* appear in the TSSI LEs are associated with large oscillation terms in the mode decomposition of the LE, with amplitudes that peak at wave numbers where nearby quasiparticles have a sizable group velocity. This, in turn, emulates the scenario for the ESSH model at the IP, but now for quenches to special parameter values which do *not* define a critical point of a QPT. One should here note that while a QPT may favor large LE oscillation amplitudes [37] (however, as transpires from our analysis, only if these are controlled by the quasiparticles which become massless at the QPT), large amplitudes can incidentally appear also *within* a quantum phase if this phase supports massless excitations. Provided that these excitations have sizable group velocities, an observable revival structure may then emerge, as evidenced when quenching to the *noncritical* ($J_3 \neq 0, h_s = 0$) line within the type-I spin-liquid phase of the TSSI XY model, cf. Fig. 4.

Summary.—We have shown that the presence of a quantum phase transition is neither a sufficient nor a necessary condition for observing a revival structure in the Loschmidt echo after a quantum quench. Periodic revivals are preconditioned by a LE controlled by massless quasiparticle modes with a group velocity $v_g \gg L/t$, where L is the length of the system and t is the observation time. This property may or may not be present at a quantum critical point. The suppression of a critical revival structure is strikingly illustrated away from the isotropic quantum critical point in the extended Su-Schrieffer-Heeger model, introduced in this Letter. Here, the revivals are found to be controlled by quasiparticle states which remain gapped at the anisotropic quantum phase transition, implying small oscillation amplitudes in the mode decomposition of the LE. Our findings may call for a revisit of earlier results on revival structures and quantum criticality, and should encourage efforts to identify more reliable nonequilibrium markers of quantum criticality.

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