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Enhanced two-channel Kondo physics in a quantum box device

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Abstract – We propose a design for a one-dimensional quantum box device where the charge fluctuations are described by an anisotropic two-channel Kondo model. The device consists of a quantum box in the Coulomb blockade regime, weakly coupled to a quantum wire by a single-mode point contact. The electron correlations in the wire produce strong backscattering at the contact, significantly increasing the Kondo temperature as compared to the case of non-interacting electrons. By employing boundary conformal field theory techniques we show that the differential capacitance of the box exhibits manifest two-channel Kondo scaling with temperature and gate voltage, uncontaminated by the one-dimensional electron correlations. We discuss the prospect to experimentally access the Kondo regime with this type of device.

The study of the Kondo effect has been at the forefront of condensed matter research ever since its inception forty years ago [1]. While the simplest case studied by Kondo —a spin-(1/2) impurity coupled to a single band of conduction electrons— is by now well understood, a number of variations of the original problem continue to challenge the experimentalist as well as the theorist.

A particularly intriguing question is how to realize the (overscreened) *two-channel Kondo effect* in an experiment. Ideally, two-channel Kondo physics emerges when there are two competing channels in which the conduction electrons can screen a spin-(1/2) impurity. As a result, the impurity spin becomes overscreened below some characteristic temperature T_K, and various thermodynamic and transport properties show non-Fermi liquid (NFL) behavior [2–4]. Being the simplest example of NFL behavior driven by a localized degree of freedom, the model that encapsulates the effect —the two-channel Kondo model [5]— has attracted enormous interest. Many experimental realizations have been suggested over the years, including a proposal for a quantum dot device where a small spinful dot (the "impurity") is coupled to two screening channels defined by a pair of conducting leads and a larger dot [6]. A major challenge is how to control that the couplings to the two channels are identical —as required for the two-channel Kondo effect to appear— and also, how to make sure that there is no channel mixing from cotunneling of electrons (which would immediately destroy the effect). It was recently reported that the required level of control may in fact be achieved in the laboratory, and some experimental data in support of two-channel Kondo physics was presented [7].

A different approach is to search for realizations of two-channel Kondo physics in systems where the channel symmetry and independence are robustly protected by some conservation law. A case in point is a quantum box (a large semiconducting quantum dot or a metallic grain) weakly connected by a point contact to a conducting lead. As shown by Matveev [8], near a degeneracy point of the average charge of the box the charge fluctuations can be modeled by an anisotropic two-channel Kondo Hamiltonian. The two available charge states in the box (corresponding to $n-1$ and n electrons) take the role of the two spin states of the impurity, while the physical spin of the conduction electrons provides for the two independent channels. In the absence of a magnetic field, this guarantees that the channel symmetry is robust. Unfortunately, the Kondo temperature T_K of this device is very small, $T_K \sim E_C e^{-1/2t\nu}$, E_C being the single-electron charging energy of the box, and t the tunneling rate through the

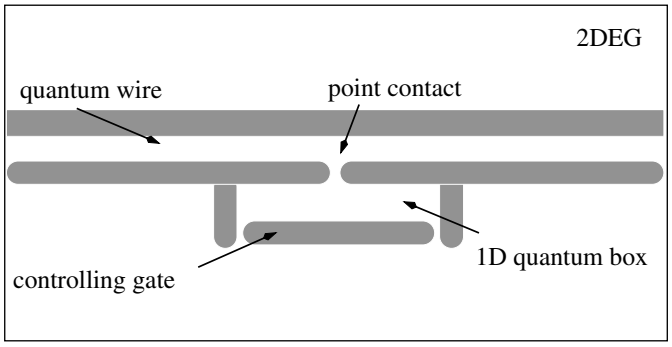

Fig. 1: Schematic picture of the proposed setup. A 1D quantum box side-coupled to a quantum wire via a point contact.

contact. Trying to increase T_K by increasing E_C requires that the box is made smaller, which in turn threatens to kill the effect since T_K must still be larger than the level spacing in the box. Whereas great progress has been achieved in high-precision charge measurements [9], and fingerprints of two-channel Kondo charge fluctuations may have been identified in the capacitance of a semiconducting quantum box connected to a lead [10], the difficulties to satisfy the conflicting constraints above make it unlikely that a fully developed effect can be observed in such a device [11].

In this letter we consider a novel design for a quantum box device with an enhanced Kondo temperature, allowing for a possible experimental entry to two-channel Kondo physics. Our scheme, inspired by that of Matveev [8], adds to several recent proposals for realizing the two-channel Kondo effect in a nanoscale structure [12–22]. With our effort we also wish to address an issue that has been notably absent from the discussion of this problem: the possible influence from *dynamic* electron interactions on a two-channel charge Kondo effect.

Our design, which is most easily implemented in a gated semiconductor heterostructure or cleaved edge overgrowth structure [23], is sketched in fig. 1: a one-dimensional (1D) quantum box is side-coupled to a quantum wire via a low-transmission single-mode point contact, putting the box in the Coulomb blockade regime. The box is biased by a gate voltage V, initially tuned to a value where the charging energy for $n-1$ and n electrons is the same *(degeneracy point)*: $V=-ne/2C_\Sigma$ with n an odd integer, e the electron charge, and C_Σ the capacitance of the box. The box should be made sufficiently large so that its level spacing Δ is much smaller than any other energy scale in the problem, allowing for the discrete levels to be represented by a quasi-continuum.

Introducing second-quantized operators $a_{k\mu\alpha}$ for electrons in the wire ($\alpha=0$) and the box ($\alpha=1$) with momentum k and spin $\mu=\uparrow,\downarrow$, we model the setup by the Hamiltonian

$$H = H_0 + H_I + H_C + H_T\,, \tag{1}$$

where

$$\begin{aligned}
H_0 &= \sum_{k,\mu,\alpha} \epsilon_k a^\dagger_{k\mu\alpha} a_{k\mu\alpha},\\
H_I &= \frac{1}{2}\sum_{\substack{\alpha,\alpha'\\ \mu,\mu'}} \sum_{k,k',q} V_{\alpha\alpha'}(q) a^\dagger_{k+q\,\mu\alpha} a^\dagger_{k'-q\,\mu'\alpha'} a_{k'\mu'\alpha'} a_{k\mu\alpha}\,,\\
H_C &= \frac{Q^2}{2C_\Sigma} + VQ\,,\\
H_T &= t\sum_{k,k',\mu} (a^\dagger_{k\mu 0} a_{k'\mu 1} + \text{H.c.}).
\end{aligned}$$

The term H_C encodes the charging energy of the box with Q the surplus charge with respect to the zero-bias Fermi level, while H_T governs the tunneling through the point contact, t being a constant tunneling matrix element. By construction we have split the electron interaction into two pieces: The *mean-field* capacitive part H_C that is effective only in the finite box (assuming that the charging energy of the wire can be neglected), and a part H_I which builds dynamic electron correlations into the model[1]. H_I is most easily specified in the relevant low-energy limit where all scattering processes are confined to the neighborhood of the Fermi points $\pm k_F$. Assuming that the electron density is incommensurate with the lattice of the underlying substrate, the allowed low-energy processes can be classified into dispersive, forward and backward scattering (with the latter taking place only inside the wire or box since there is no exchange of wire and box electrons away from the point contact). Passing to a continuum description and decomposing the electron fields $\Psi_{\mu\alpha}(x)\sim\int \mathrm{d}k \mathrm{e}^{ikx} a_{k\mu\alpha}$ in left- and right-moving parts, $\psi_{-\mu\alpha}(x)$ and $\psi_{+\mu\alpha}(x)$, respectively, these processes can be conveniently expressed using the currents

$$\begin{aligned}
J_\pm &= \mathrm{sh}\vartheta :\psi^\dagger_{\pm\mu\alpha}\psi_{\pm\mu\alpha}: + \mathrm{ch}\vartheta :\psi^\dagger_{\mp\mu\alpha}\psi_{\mp\mu\alpha}:\,,\\
\boldsymbol{J}^{[\mu]}_\pm &= \frac{1}{2} :\psi^\dagger_{\pm\mu\alpha}\boldsymbol{\sigma}_{\mu\mu'}\psi_{\pm\mu'\alpha}:\,,\\
\boldsymbol{J}^{[\alpha]}_\pm &= \frac{1}{2} :\psi^\dagger_{\pm\mu\alpha}\boldsymbol{\sigma}_{\alpha\alpha'}\psi_{\pm\mu\alpha'}:\,.
\end{aligned} \tag{2}$$

The normal ordering is taken w.r.t. the filled Dirac sea (obtained after linearizing the spectrum around $\pm k_F$), $\boldsymbol{\sigma}$ is the vector of Pauli matrices, and the indices $\alpha,\alpha',\sigma,\sigma'$ are summed over. The parameter ϑ is given by $2\vartheta=\mathrm{arctanh}(3g/(v_F+3g))$, with v_F the Fermi velocity and with g the strength of the screened Coulomb interaction, here approximated by its dominating component at zero-momentum transfer. For simplicity we take this interaction to be the same *between* wire and box ($\sim V_{01}(q)$) as *in* the wire and the box ($\sim V_{00}(q)=V_{11}(q)$): $V_{01}(0)=V_{00}(0)\equiv g$. In a real device one expects that $V_{01}(0)<V_{00}(0)$. However, as will be seen below, the value of the ratio $V_{01}(0)/V_{00}(0)$ at most influences subleading corrections to the charge fluctuations in the box, and for the purpose of extracting

[1] For simplicity, in H_0 and H_I we take the length of the box to be equal to that of the wire, putting all finite-size effects into H_C.

the leading behavior we may put it to unity, yielding a more transparent formalism. Given the currents in (2) we can now cast the low-energy limit of $H_0 + H_I$ in the form [24]

$$H_0 + H_I = \frac{1}{2\pi} \sum_{\substack{\ell=\pm \\ \eta=\alpha,\mu}} \int \mathrm{d}x \left(\frac{v_c}{8} :J_\ell(x) J_\ell(x): + \frac{v_{[\eta]}}{4} :\boldsymbol{J}_\ell^{[\eta]}(x) \cdot \boldsymbol{J}_\ell^{[\eta]}(x): \right) , \quad (3)$$

with $v_c = v_F(1 + 6g/v_F)^{1/2}$, $v_{[\mu]} = v_{[\alpha]} = v_F - g$. We have here removed two marginally irrelevant interactions, including an unphysical exchange process between wire and dot electrons away from the point contact[2]. The $U(1)$ charge, $SU(2)_2$ spin and $SU(2)_2$ *pseudospin* currents J_ℓ, $\boldsymbol{J}_\ell^{[\mu]}$ and $\boldsymbol{J}_\ell^{[\alpha]}$, respectively, are decoupled in (3), and one recognizes $H_0 + H_I$ as the Hamiltonian for a spinful Luttinger liquid (written in "Sugawara form" [24]) with an extra pseudospin channel. Having thus coded the dynamic part of the theory we turn to its effect on the charge fluctuations in the box.

In the vicinity of the degeneracy point and with $k_B T \ll e^2/2C_\Sigma$, only the $Q = 0$ and $Q = e$ states are accessible and higher charge states can be removed from the theory. Following Matveev [8], the resulting two-level system $H_C + H_T$ in (1) can then be mapped onto an anisotropic two-channel Kondo interaction H_K:

$$H_K = \frac{J_\perp}{2} \psi^\dagger_{\ell\mu\alpha}(0) \boldsymbol{\sigma}^j_{\alpha\alpha'} \psi_{\ell'\mu\alpha'}(0) S^j - h S^z . \quad (4)$$

Here $\boldsymbol{S}$ is a pseudospin-(1/2) operator that implements the constraint on the allowed charge states in the box. The coupling $J_\perp$ and the field h are given by $J_\perp = 2t$ and $h = eu$, respectively, with u a small voltage bias away from the degeneracy point. Note that *all* indices in (4), $(\ell, \ell' = \pm; \alpha, \alpha' = 0, 1; \mu, \mu' = \uparrow, \downarrow, j = x, y)$, are summed over. To complete the mapping to a Kondo pseudospin formulation one makes use of the fact that $\langle Q(u) \rangle = e[1/2 - \langle S^z \rangle(h, J_\perp)]$, implying that the differential capacitance $c(u, T)$ of the box gets modeled by an impurity pseudospin susceptibility $\chi_{imp}(h, T) \equiv -(1/e^2)\partial \langle Q \rangle / \partial u = c(u, T)$. Having translated the original problem into Kondo language, the task has thus become that of calculating the susceptibility of a pesudospin-(1/2) impurity (eq. (4)) coupled to a two-channel Luttinger liquid $H_0 + H_I$ (eq. (3)), with the two channels provided by the physical spin of the electrons.

It is important to realize that the backscattering of electrons off the impurity in (4) is due to the side-coupling of the box to the wire, see fig. 1. As it turns out, it is precisely this novel feature that yields the desired properties of our setup. As shown by Le Hur, the amplitude for two-channel electron impurity backscattering renormalizes to a strong-coupling fixed point as the temperature is lowered [25]. This is in exact analogy with the single-channel Kondo problem in a Luttinger liquid, where this effect was first noted [26]. The flow to strong coupling produces a crossover from an exponentially suppressed Kondo temperature $T_K \sim E_c \mathrm{e}^{-1/2t\nu}$ for $g \ll J_\perp$ to a power law $T_K \sim E_C (2t\nu)^{2/(1-K_c)}$ for $g \gg J_\perp$, with K_c the Luttinger liquid charge parameter. Exploiting the crossover formula from ref. [26], with input parameters chosen for a GaAs-based device [27] and taking $t\nu \lesssim 0.2$, one finds that while T_K in the non-interacting limit ($K_c = 1$) is comparable to or below the level spacing in the box, T_K becomes almost an order of magnitude larger as K_c approaches 0.6 (which is easily reached in a low-density quantum wire [28]). Although the temperature window that opens is probably too narrow for a full-fledged two-channel Kondo scaling to develop, it should at least allow for a controlled experimental study of its transient behavior. Using a *metallic* quantum wire/box with its much larger effective electron mass would stretch the window by another order of magnitude (provided that the electron density is suppressed by proper gating of the device). We here point to the recent observation that electron shell effects can stabilize arbitrarily long metallic quantum wires, making the fabrication of a metallic device a viable and realistic prospect [29].

Suppose that the limitations set by current semiconductor-based technology can indeed be overcome, allowing for the capacitance to be measured in the critical region $T \ll T_K$ using a metallic device. Would a logarithmic scaling with temperature and gate voltage emerge —as predicted for non-interacting electrons [8]— or will the strong dynamic electron correlations in the wire and the box cause a new type of behavior? In the case of spin polarized electrons it has been shown that 1D correlations strongly influence charge fluctuations [30], and one may anticipate a similar outcome also in the present case. To find out, we have employed the tools of boundary conformal field theory (BCFT) [31], building on an earlier approach to the isotropic two-channel Kondo effect in a Luttinger liquid [32]. The required analysis closely follows that presented in ref. [32], and we here only sketch the key ideas together with the results.

Let us first recall that the fixed point of the *isotropic* two-channel Kondo effect in a Luttinger liquid corresponds to a particular selection rule for quantum numbers of the BCFT embedding $\otimes_{i=1,2}[U(1) \otimes SU(2)_2 \otimes SU(2)_2]^i$ implied by the Hamiltonian in (3) [32]. Here the $U(1)$ factor represents charge, while the $SU(2)_2$ factors represent spin and pseudospin, with $i = 1, 2$ corresponding to left and right moving fields. The Kondo interaction couples left and right movers, and therefore the symmetries above are broken down to their diagonal subgroups. For the charge sector this implies that any $U(1)$ operator with dimension

$$\Delta_c = \frac{1}{4} n^2 e^{\pm 2\theta} + N; \qquad n, N \in \mathbb{N}, \quad (5)$$

[2] The excluded interactions can be written in terms of $SU(4)_1$ Kac-Moody currents as $H_{excl} = -2g J_L^{\mathcal{A}} J_R^{\mathcal{A}}$ $(\mathcal{A} = 1, \ldots, 15)$, where H_{excl} is marginally irrelevant for $g > 0$.

is allowed [32]. Factorizing the diagonal subgroups in the spin and pseudospin sectors amounts to a coset construction at the level of conformal towers. The two $SU(2)_2$ towers in the spin and pseudospin sectors are decomposed into $SU(2)_4$ and a coset which is generated by the $N=1$ superconformal algebra (SCA) of central charge $c=1$: $SU(2)_2 \otimes SU(2)_2 = SU(2)_4 \otimes \text{SCA}$. Primary states of the spin (pseudospin) $SU(2)_4$ sectors have conformal dimensions $j(j+1)/6$ with $j \in \{0, 1/2, 1, 3/2, 2\}$. The SCA in turn is divided into two sectors: the Ramond (R) and Neveu-Schwartz (NS) algebras with primary dimensions $\{1/24, 1/16, 3/8, 9/16\}$ and $\{0, 1/16, 1/6, 1\}$, respectively. In addition, the grade of a generic state is integer in the R sector, whereas it is half-integer in the NS sector. The effect of the Kondo coupling is encoded by the *leading irrelevant boundary operators* [4] (which are products of operators from charge, spin, pseudospin and SCA sectors) with possible scaling dimensions

$$\Delta = \Delta_c + \sum_{j=s,\,p} (\Delta^j_{SU(2)_4} + \Delta^j_{\text{SCA}}), \tag{6}$$

where

$$\Delta^{s/p}_{SU(2)_4} = \frac{j(j+1)}{6} + N, \qquad j = 0, \frac{1}{2}, 1, \frac{3}{2}, 2; \tag{7}$$

$$\Delta^{s/p}_{\text{SCA}} = \left\{0, \frac{1}{16}, \frac{1}{6}, 1\right\} + \frac{N}{2}, \quad \left\{\frac{1}{24}, \frac{1}{16}, \frac{3}{8}, \frac{9}{16}\right\} + N, \tag{8}$$

with $N \in \mathbb{N}$. Valid boundary operators i) must respect all symmetries of the theory and ii) must not violate the known critical scaling of observables in the non-interacting limit $g \to 0$. The criterion i) restricts the choice of operators in each conformal sector, while the criterion ii) defines the rules for combination of operators in different sectors to obtain the full boundary operators.

Given the above construction and criteria it is in principle straightforward to pinpoint the effect from the exchange anisotropy (broken pseudospin rotational symmetry) and the magnetic field (broken time-reversal symmetry) in (4). In contrast to the $SU(2)_4$ invariant case above where only operators that transform as singlets are allowed, more operators now appear in the pseudospin sector. By inspection we find that the only relevant operator produced is the $j=1$ primary field ϕ^z, with conformal dimension $\Delta_{\phi^z} = 1/3$. This operator is present only if pseudospin rotational *and* time-reversal symmetries are simultaneously broken. In the limit of vanishing magnetic field the anisotropy is irrelevant, implying that the magnetic field is a relevant perturbation, as for the two-channel Kondo model for non-interacting electrons [33].

There are two more operators appearing because of the broken pseudospin symmetry: the first descendant J^z of the $j=0$ identity operator, and the $j=2$ primary field ϕ^{zz}, both being exactly marginal of dimension $\Delta = 1$. Both operators may be combined with others from charge, spin, and SCA sectors to form new composite operators provided that these respect the constraints i) and ii) above. For the *isotropic* problem the leading behavior of the impurity susceptibility $\chi(T, h)$ is driven by the same operator (of dimension $\Delta = 3/2$) as for non-interacting electrons, giving rise to a logarithmic divergence as $T \to 0$ or $h \to 0$ [32]. In our construction the boundary operator driving the behavior is a combination of the first descendant of the pseudospin $j=1$ $SU(2)_4$ conformal tower with the $\Delta = 1/6$ NS field. To explore whether a faster divergence may result from any of the new composite operators generated from the broken pseudospin symmetry, we have to identify those of scaling dimension $\Delta < 3/2$ (since they may produce more leading contributions) and then test them against criteria i) and ii) above.

We can here identify two distinct classes of possible boundary operators. The first class contains operators with dimensions $\Delta < 3/2$, which do not depend on the Luttinger liquid interaction parameter K_c (thus containing the identity operator from the charge sector) and hence they should be present in the non-interacting limit as well. Since these contributions do not vanish in the non-interacting limit, the constraint ii) implies that the operators from this class are not present in the theory. The second class contains operators with dimensions $\Delta(K_c) < 3/2$, which depend on the Luttinger liquid interaction parameter K_c (thus containing non-trivial operators from the charge sector). A straightforward calculation, using the method described in ref. [32], reveals that the contributions to the observables from these operators do not vanish in the non-interacting limit. Thus, again using constraint ii), we conclude that these operators should also not be included in the theory.

It follows that the leading behavior of the differential capacitance $c(T, u)$ of our proposed setup exhibits the same logarithmic scaling as in the two-channel Kondo effect for non-interacting electrons,

$$c(T, u=0) = A \ln\left(\frac{T_K}{T}\right) + \cdots, \qquad T \ll T_K \tag{9}$$

and

$$c(T=0, u) = B \ln\left(\frac{T_H}{eu}\right) + \cdots, \qquad eu \ll T_H \tag{10}$$

but *with significantly larger Kondo temperatures* T_K *and* $T_H \approx T_K$. Here A and B are constants, and "..." indicate subleading terms. These subleading terms —produced by more irrelevant operators than those identified above— may turn out to be interaction dependent. To determine their scaling dimensions, however, requires a cumbersome analysis, beyond the scope of the present letter. It is here interesting to compare with the single-channel case where already the *leading* scaling behavior depends on the strength of the electron-electron interaction [30].

In conclusion, we have shown that charge fluctuations close to a degeneracy point of a 1D Coulomb blockaded

quantum box side-coupled to a quantum wire exhibit logarithmic two-channel Kondo divergences with temperature T (voltage u) for $T \ll T_K$ ($eu \ll T_H$). This leading behavior is *not* modified by the strong 1D electron correlations in the wire and the box (with possible interaction-dependent contributions at most showing up in subleading terms). The Kondo temperature T_K (or T_H in the case of voltage scaling) can be significantly larger compared to a device with non-interacting electrons. While design constraints for a semiconductor implementation probably only allow for crossover effects to be observed, the fabrication of a metallic device should yield access to the full two-channel charge Kondo effect.

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