Quantum Matter: Concepts and Models

homework problems, course week 4, fall 2019

Deadline: December 10

Please structure your solutions carefully. All essential steps in your analysis and calculations should be made explicit.

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1. In my lecture 26/11 I sketched the famous argument by R. Jackiw and C. Rebbi (Phys. Rev. D **13**, 3398 (1976)) that the 1(+1)D Dirac equation with a spatially varying mass may support fractionally charged states. We obtained this theory from the SSH model by some hand-waiving: First taking a continuum limit with a linearized spectrum close to one of the BZ boundaries (a maneuver for which some of you expressed a healthy skepticism!), then sending the mass in one region to infinity (although our parameterization from the SSH model required the mass parameter to be $\ll 1$!), and finally, a formal regularization of the charge operator (which those of you not having been through QFT must have perceived as only a convenient trick to obtain the desired result!). All in all, can we really trust that this kind of loose analysis really predicts charge fractionalization in the original SSH model? Do some literature study to find out! You may wish to begin with reading the brief little review by Jackiw (linked on the course home page) where he gives a "counting" argument for fractional charges (as a "poor man's alternative" to the formal QFT argument). Do you agree with him? Also have a look at the authoritative (but quite compact) review by Charlie Kane, p 9 (also linked on the course homepage) where he argues for charge fractionalization in the SSH model by connecting the end charge to the electric polarization (which we derived in one of the preceding lectures). Do you understand his argument? Have fractional charges ever been experimentally observed in a system described by the SSH model?