Many-Body Localization Transition

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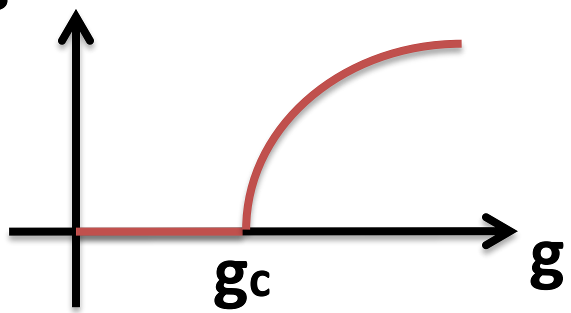
Contents of the talk

- Quantum phase transitions
- Anderson localization
- Many-body localization
- Scaling near the MBL transition

Quantum Phase Transitions

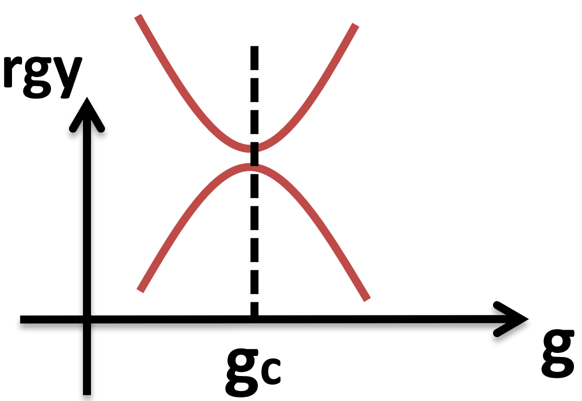
Quantum: $H(g), |GS(g)\rangle$ ➡ External parameter g is the control parameter

Magnetization



- Phase transition is captured by a **local** order parameter: $m = \sum_i \sigma_z^i$

Energy



- QPT is a **ground state** property

$\xi \sim |g - g_c|^{-\nu}$ ➡ $\langle S_i^z S_{i+x}^z \rangle \sim e^{-\frac{x}{\xi}}$

Order Parameter

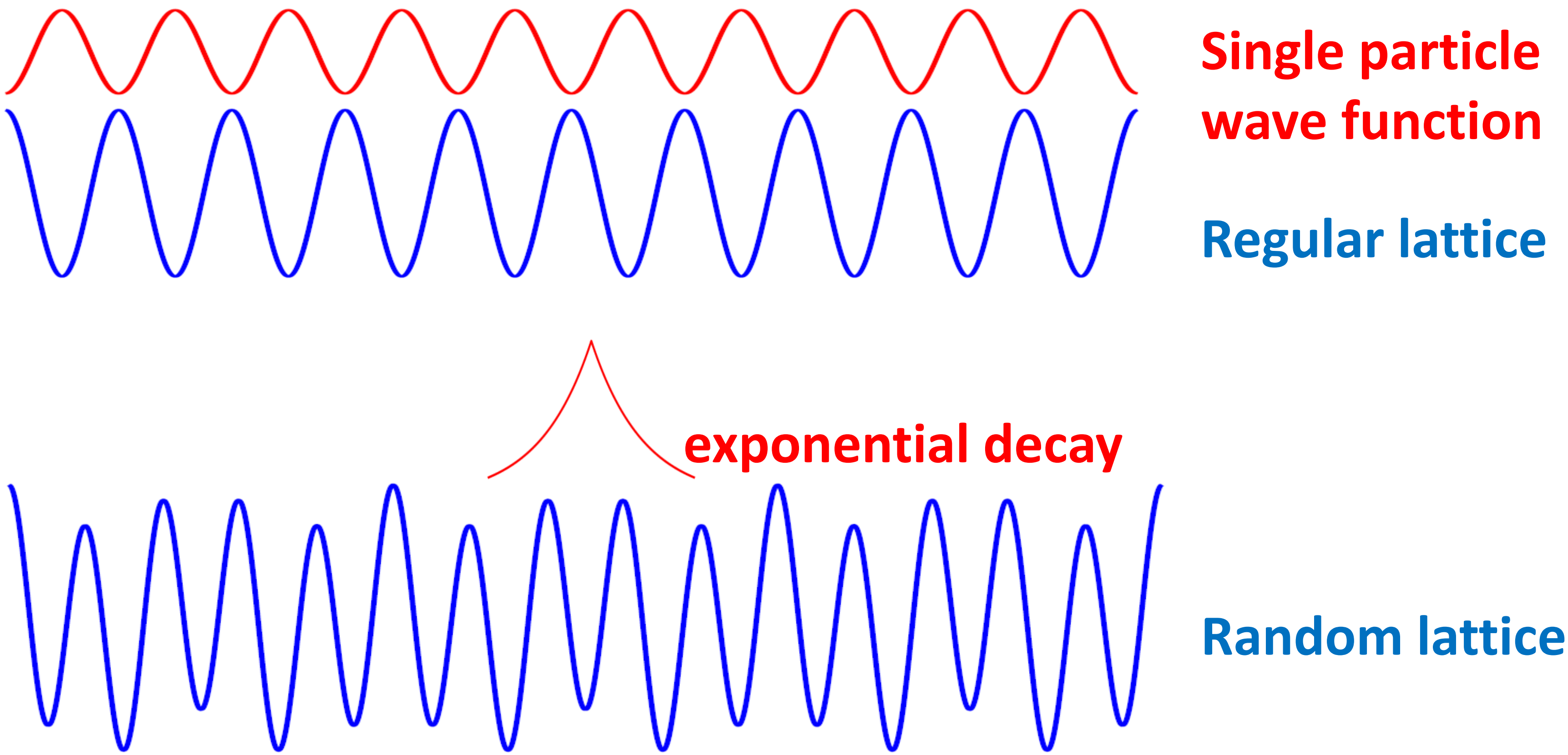
Order parameter is:

1- Observable
2- Is zero in one phase and non-zero in the other
3- Scales at criticality

Landau-Ginzburg paradigm:

4- Order parameter is local
5- Order parameter is associated with a spontaneous symmetry breaking

Anderson localization



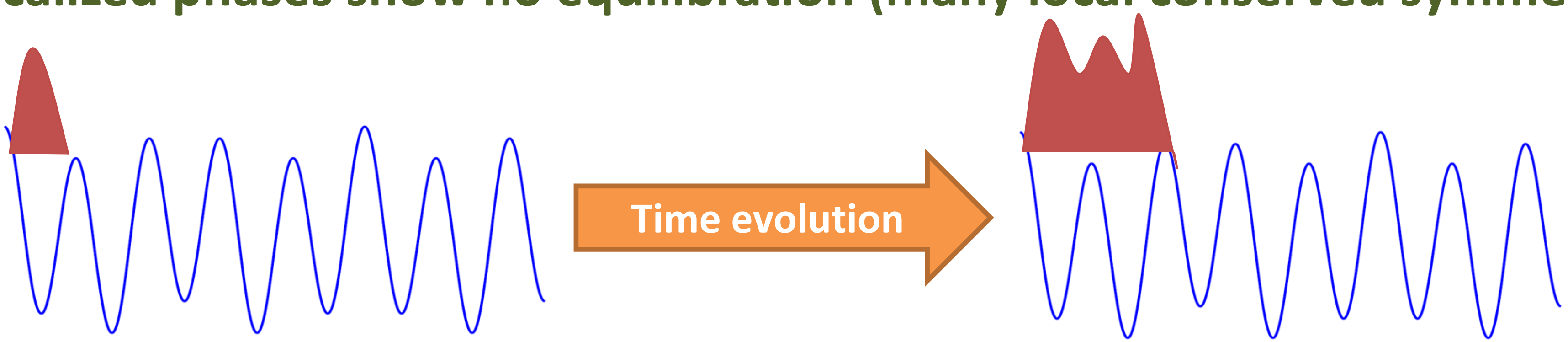
- **Any strength of randomness implies localization**
- **Localization is a property of the whole spectrum**

P. W. Anderson, Phys. Rev. 109, 1492 (1958).

Ergodicity vs. localization

Thermalization/Equilibration happens in ergodic phases

Localized phases show no equilibration (many local conserved symmetries)



M. Serbyn, Z. Papic, and D. A. Abanin, Phys. Rev. B 90, 174302 (2014)
M. Serbyn, Z. Papic, and D. A. Abanin, Phys. Rev. Lett. 111, 127201 (2013)

Eigenstate thermalization hypothesis

$$|\Psi(0)\rangle = \sum_n c_n |E_n\rangle \;\Rightarrow\; |\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle = \sum_n c_n e^{-iE_n t}|E_n\rangle$$

$$\langle\Psi(t)|A|\Psi(t)\rangle = \sum_{n,m} c_n c_m{}^* A_{mn} e^{-i(E_n - E_m)t}$$

Where $A_{mn} = \langle E_m|A|E_n\rangle$

$$\lim_{t\to\infty}\langle\Psi(t)|A|\Psi(t)\rangle = \sum_n |c_n|^2 A_{nn}$$

If A_{nn} is independent of n

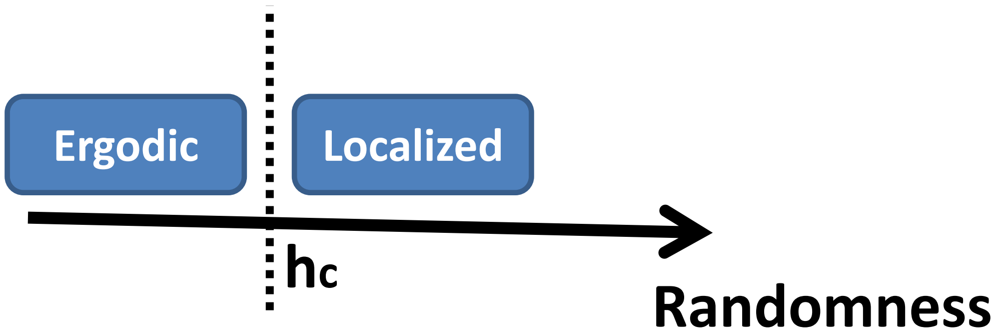
$$\lim_{t\to\infty}\langle\Psi(t)|A|\Psi(t)\rangle = A_{nn}$$

Independent of initial state

ETH means that $A_{nn} = \langle E_n|A|E_n\rangle$ **is independent of n.**

Many-body vs. Anderson localization

- Anderson localization **survives** in the presence of **interactions**
- Unlike Anderson localization the randomness has to be stronger than a **threshold** to induce localization.



Many-body vs. Anderson localization

Thermal phase	Single-particle localized	Many-body localized
Memory of initial conditions hidden in global operators at long times	Some memory of local initial conditions preserved in local observables at long times	Some memory of local initial conditions preserved in local observables at long times
Eigenstate thermalization hypothesis (ETH) true	ETH false	ETH false
May have nonzero DC conductivity	Zero DC conductivity	Zero DC conductivity
Continuous local spectrum	Discrete local spectrum	Discrete local spectrum
Eigenstates with volume-law entanglement	Eigenstates with area-law entanglement	Eigenstates with area-law entanglement
Power-law spreading of entanglement from nonentangled initial condition	No spreading of entanglement	Logarithmic spreading of entanglement from nonentangled initial condition
Dephasing and dissipation	No dephasing, no dissipation	Dephasing but no dissipation

Annu. Rev. Condens. Matter Phys. 2015. 6:15–38

Many-body localization

$$H = J\left(\sum_i \sigma_i . \sigma_{i+1} + h_i \sigma_i^z\right) \qquad h_i \in [-h, +h]$$

E_{n+1}

E_n

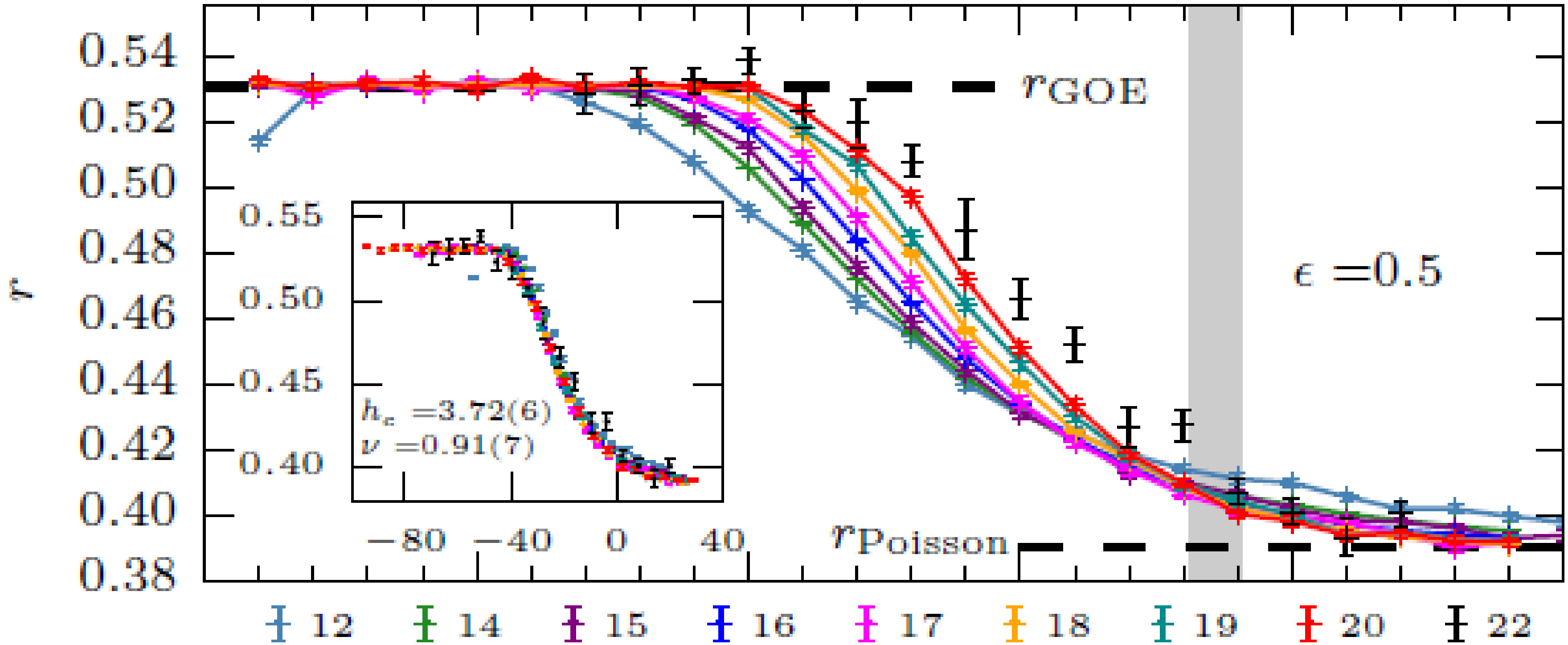
E_{n-1}

$$\delta_n = E_{n+1} - E_n \qquad r_n = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})}$$

Disorder average: $r = \langle r_n \rangle$

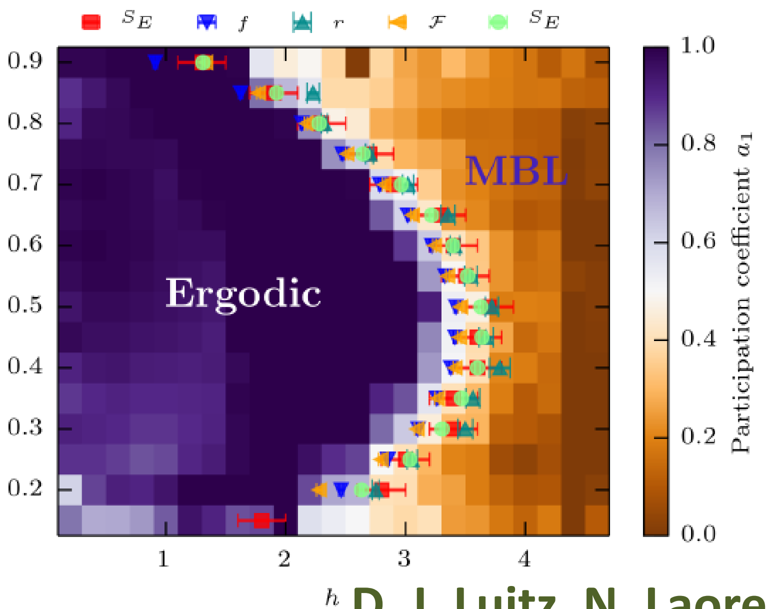
- **Ergodic phase: rn has a Gaussian distribution (mean=0:5307)**
- **Localized phase: rn has a Poisson distribution (mean=0:3863)**

Level statistics



D. J. Luitz, N. Laorencie, and F. Alet, Phys. Rev. B, 91, 081103 (2015)

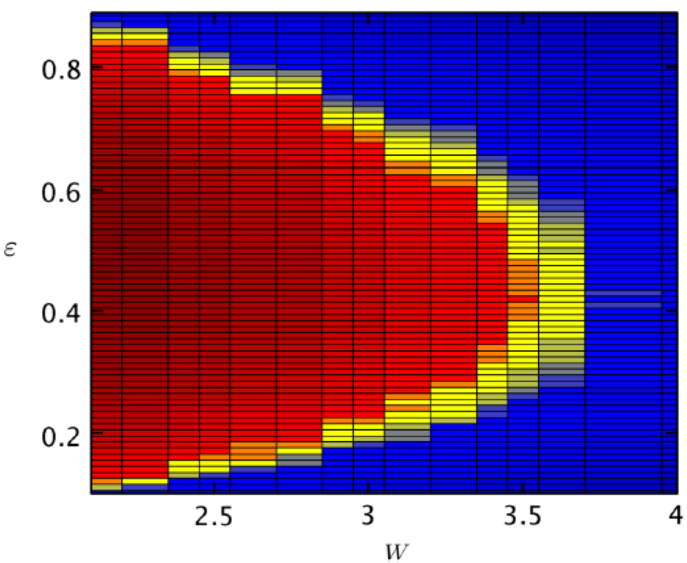
MBL transition



$$H = J\left(\sum_i \sigma_i.\sigma_{i+1} + h_i\sigma_i^z\right) \quad h_i \in [-h, +h]$$

- **MBL occurs throughout the spectrum**
- **It can be observed at all temperatures**

D. J. Luitz, N. Laorencie, and F. Alet, Phys. Rev. B, 91, 081103 (2015)



M. Serbyn, Z. Papic, and D. A. Abanin, Phys. Rev. X 5, 041047 (2015)

Scaling

Is there scaling at MBL transition? $\xi \sim |h - h_c|^{-\nu}, \quad r = f\left(\frac{N}{\xi}\right)$

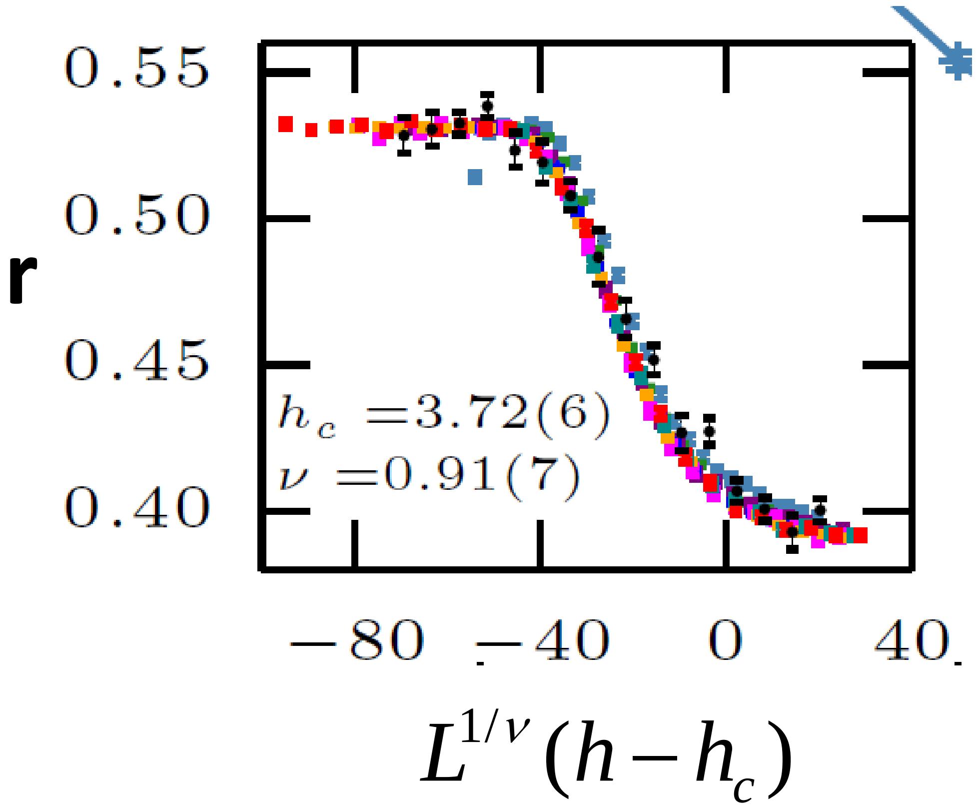
Yes, it seems so!!

It has been mathematically proved that $\nu \geq 2$

J. T. Chayes, L. Chayes, D. S. Fisher, and T. Spencer, Phys. Rev. Lett. 57, 2999 (1986).

A. Chandran, C. R. Laumann, and V. Oganesyan, arXiv:1509.04285

Finite size scaling



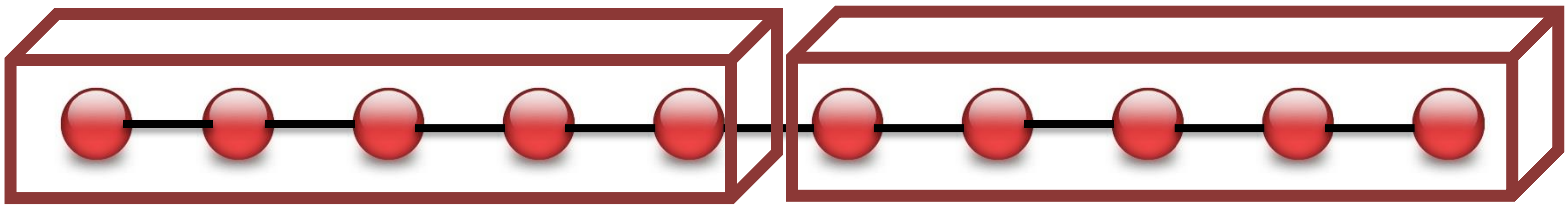
D. J. Luitz, N. Laorencie, and F. Alet, Phys. Rev. B, 91, 081103 (2015)

Extensive numerical studies end up at $\nu \sim 1$ instead of $\nu \geq 2$

System sizes are too small to capture right exponents

Entanglement Spectrum

Left side Right side



$$|GS\rangle = \sum_{i,j} \alpha_{ij} \left|\tilde{L}_i\right\rangle \otimes \left|\tilde{R}_j\right\rangle$$

$$|GS\rangle = \sum_k \sqrt{\lambda_k} |L_k\rangle \otimes |R_k\rangle, \quad \lambda_k \geq 0$$ Schmidt decomposition

$$\rho_L = \sum_k \lambda_k |L_k\rangle\langle L_k|, \qquad \rho_R = \sum_k \lambda_k |R_k\rangle\langle R_k|$$

Entanglement spectrum: $\lambda_1 \geq \lambda_2 \geq \ldots$

Von Neumann entropy: $s(\rho_L) = s(\rho_R) = -\sum_n \lambda_n \log(\lambda_n)$

Schmidt gap: $\Delta_S = \lambda_1 - \lambda_2$

Schmidt gap as an order parameter

PRL **109**, 237208 (2012) PHYSICAL REVIEW LETTERS week ending 7 DECEMBER 2012

Entanglement Spectrum, Critical Exponents, and Order Parameters in Quantum Spin Chains

G. De Chiara,[1,2] L. Lepori,[1] M. Lewenstein,[3,4] and A. Sanpera[3,1]

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[2]*Centre for Theoretical Atomic, Molecular and Optical Physics, School of Mathematics and Physics, Queens University Belfast, Belfast BT7 1NN, United Kingdom*
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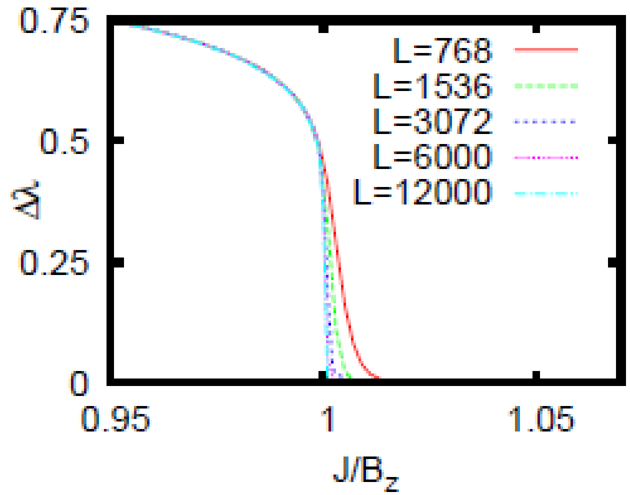
[4]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Castelldefels, Spain*

(Received 5 July 2011; published 5 December 2012)

$$H_{Ising} = -J\sum_i \sigma_x^i \sigma_x^{i+1} - B_z \sum_i \sigma_z^i$$

$$H = J\sum_{i=1}^{L-1}\left[\cos(\theta) S_i \cdot S_{i+1} + \sin(\theta)(S_i \cdot S_{i+1})^2\right] + D\sum_{i=1}^{L} S_{zi}^2$$



Schmidt gap as an order parameter

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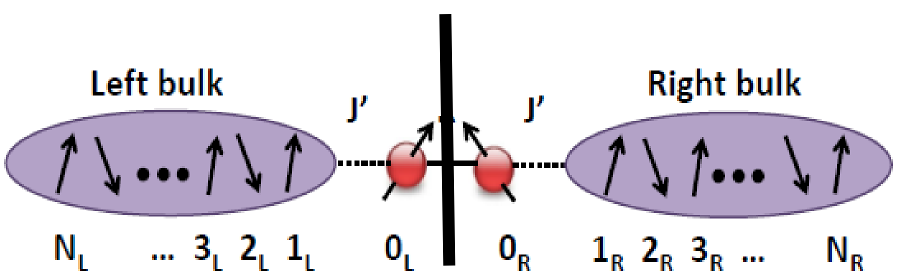
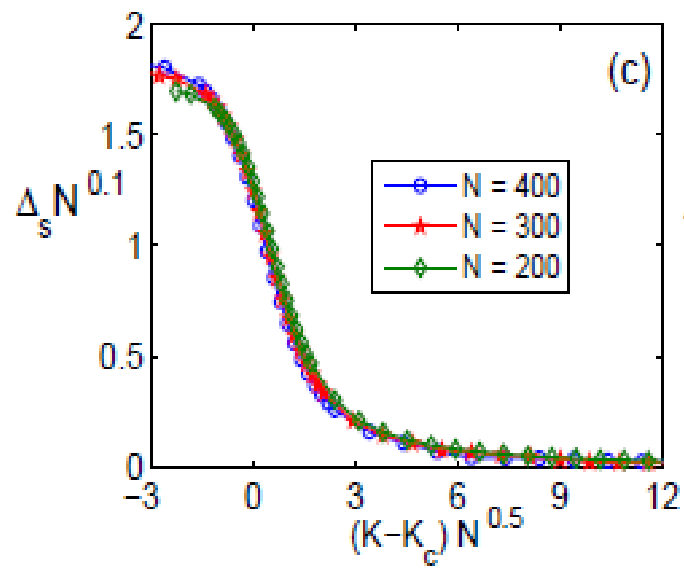
An order parameter for impurity systems at quantum criticality

Abolfazl Bayat[1], Henrik Johannesson[2], Sougato Bose[1] & Pasquale Sodano[3,4,5]

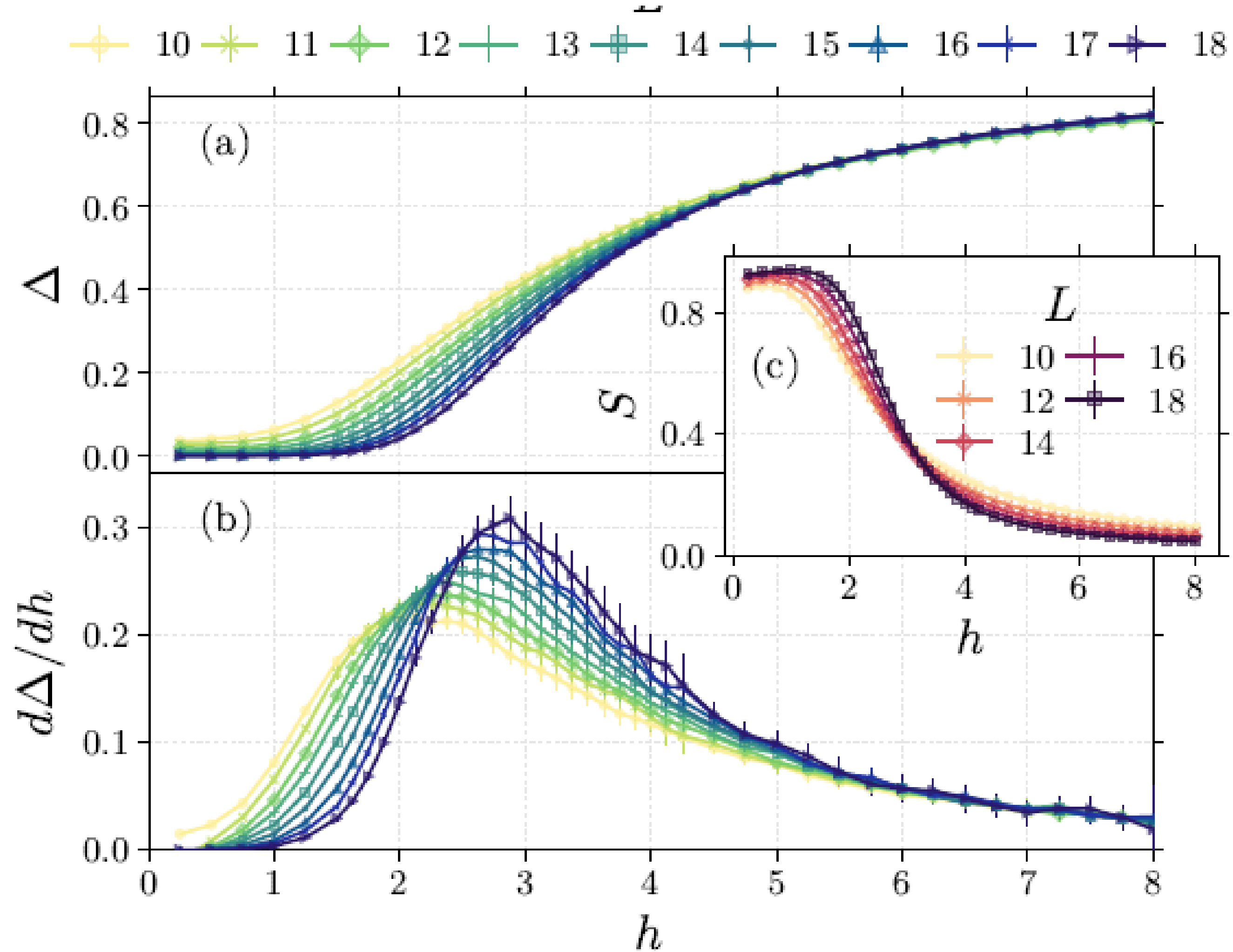




Schmidt gap for characterizing MBL

J. Gray, S. Bose, A. Bayat, Phys. Rev. B 97, 201105 (2018)

Schmidt gap

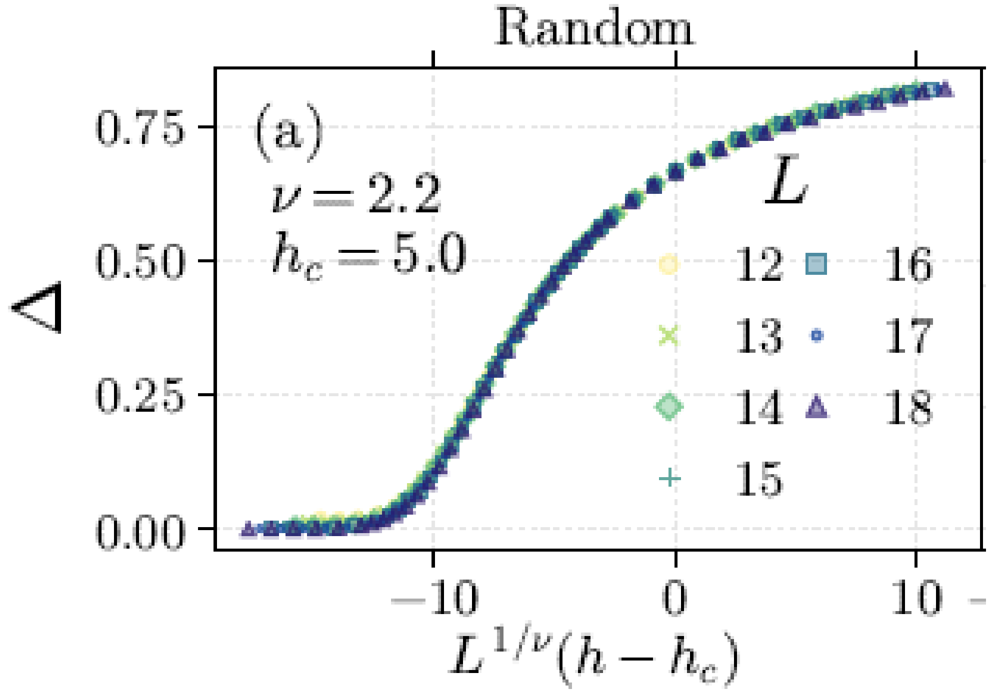


Schmidt gap can capture the transition point by its derivative

J. Gray, S. Bose, A. Bayat, Phys. Rev. B 97, 201105 (2018)

Finite size scaling for Schmidt gap

$$\Delta = f(L^{1/\nu}(h - h_c))$$



$\nu = 2.2 \geq 2$

J. Gray, S. Bose, A. Bayat, Phys. Rev. B 97, 201105 (2018)

Logarithmic Negativity

For any density matrix: $\rho \Rightarrow \rho^{T}$ is also a density matrix

Separable:

$$\rho_{AB} = \sum_i p_i \, \rho_i^A \otimes \rho_i^B \Rightarrow \rho_{AB}^{T_B} = \sum_i p_i \, \rho_i^A \otimes \left(\rho_i^B\right)^T \Rightarrow \rho_{AB}^{T_B} \geq 0 \checkmark$$

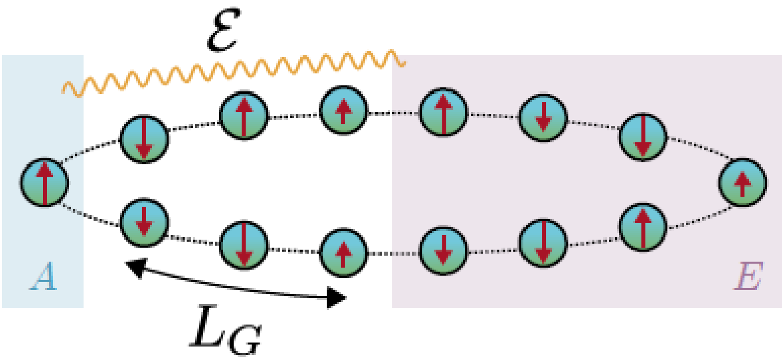
Valid density matrices

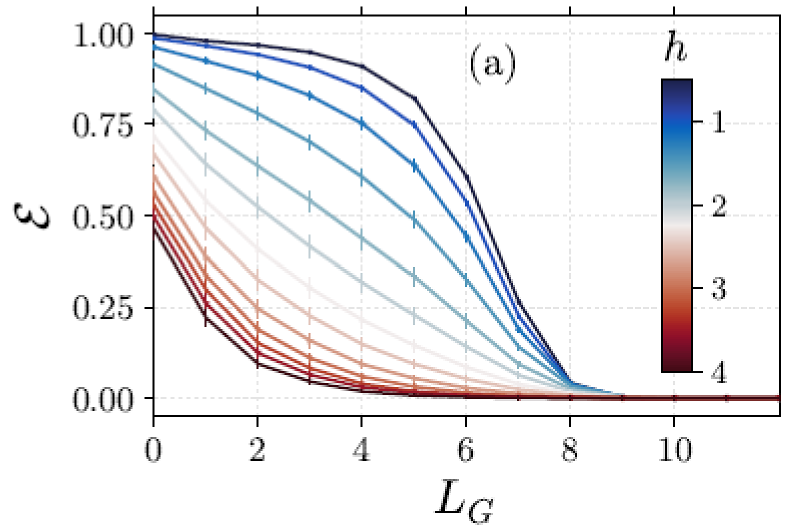
Entangled: $$\rho_{AB}^{T_B} |\lambda\rangle = \lambda |\lambda\rangle \quad (\lambda < 0)$$

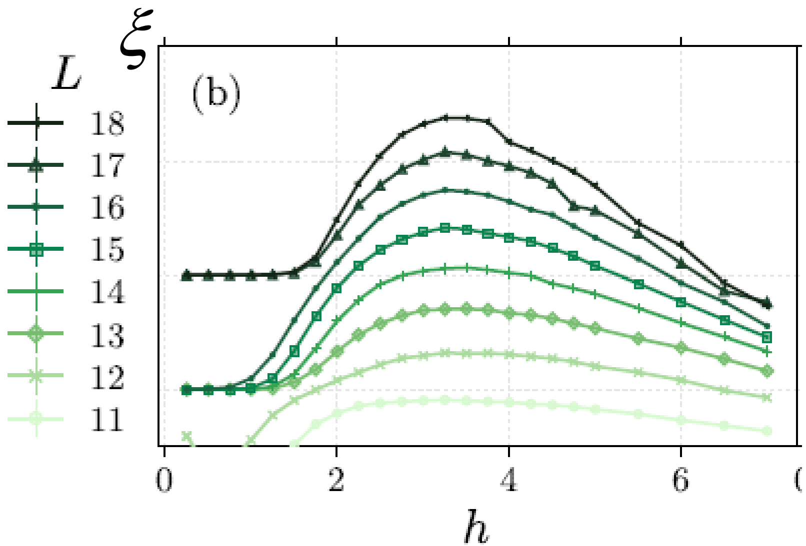
Logarithmic Negativity: $$N(\rho) = \log \left| \rho_{AB}^{T_B} \right| = \log \left(\sum_k |\lambda_k| \right)$$

Length scale

What is the physical meaning of the length scale? $\xi \sim |h - h_c|^{-\nu}$







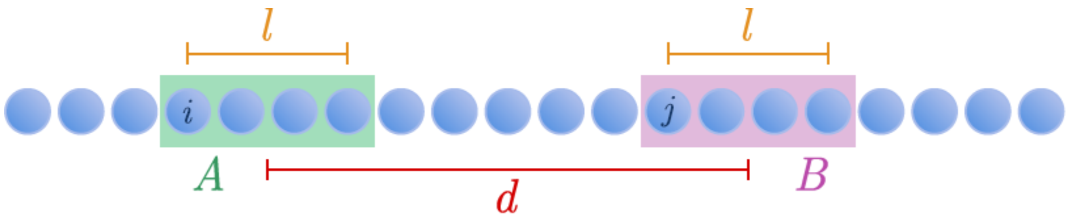
Negativity length peaks at the transition just as expected for ξ

J. Gray, S. Bose, A. Bayat, Phys. Rev. B 97, 201105 (2018)

Scale Invariance

J. Gray, A. Bayat, A. Pal, S. Bose, arXiv:1908.02761

Normalized Entanglement and Mutual Information



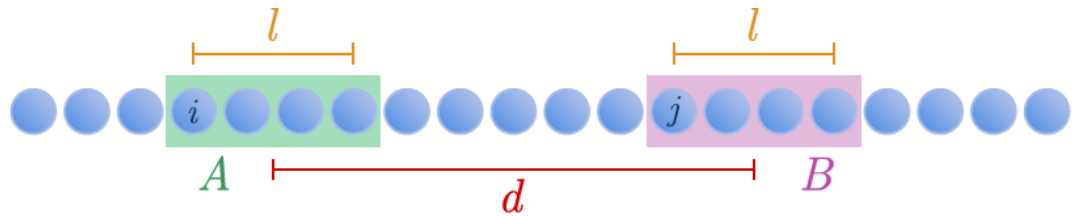
Negativity between blocks A and B: $\varepsilon_{AB} \Longleftrightarrow \tilde{\varepsilon}_{AB} = \varepsilon_{AB}/l$

Entanglement between A and the rest: $\varepsilon_A = S(\rho_A) \Longleftrightarrow \tilde{\varepsilon}_A = \varepsilon_A/l$

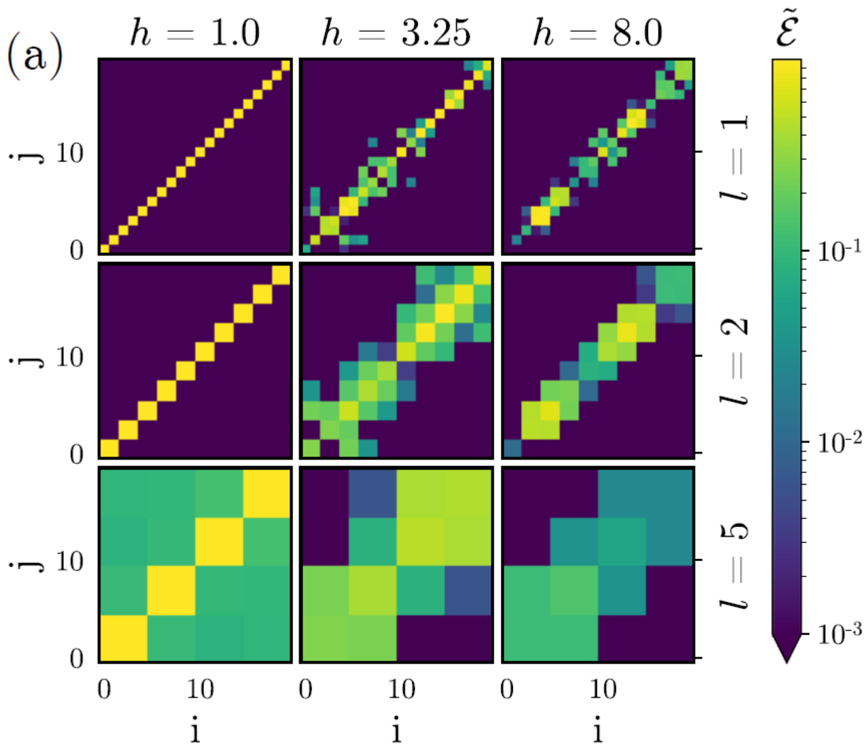
Mutual information between blocks A and B:

$$I_{AB} = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \Longleftrightarrow \tilde{I}_{AB} = I_{AB}/l$$

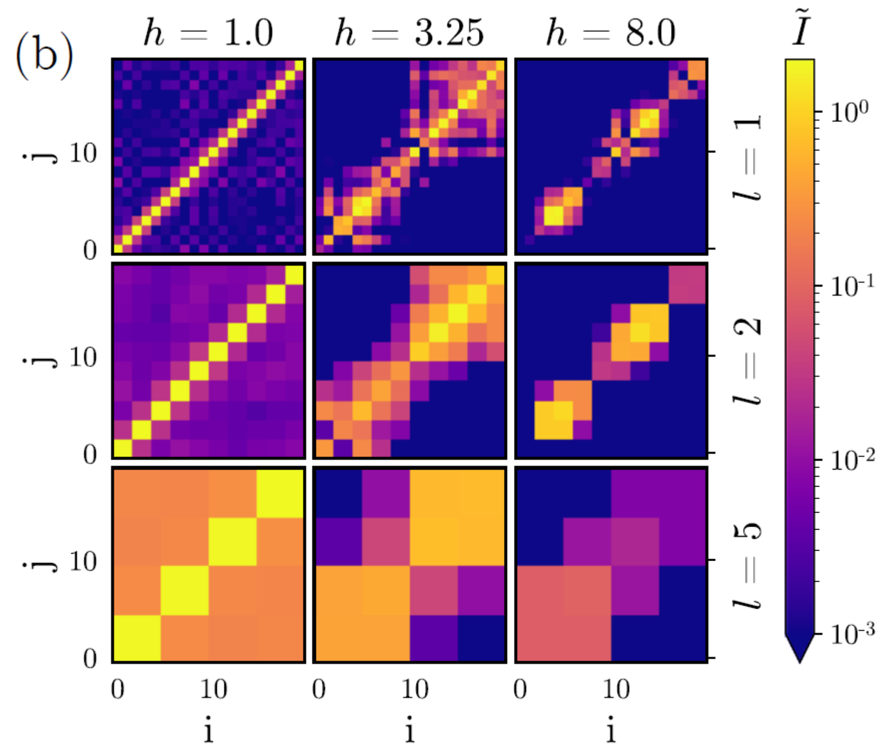
Normalized Negativity & Mutual Information



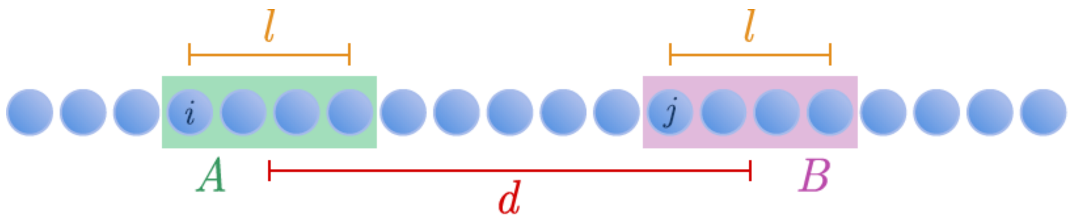
Negativity



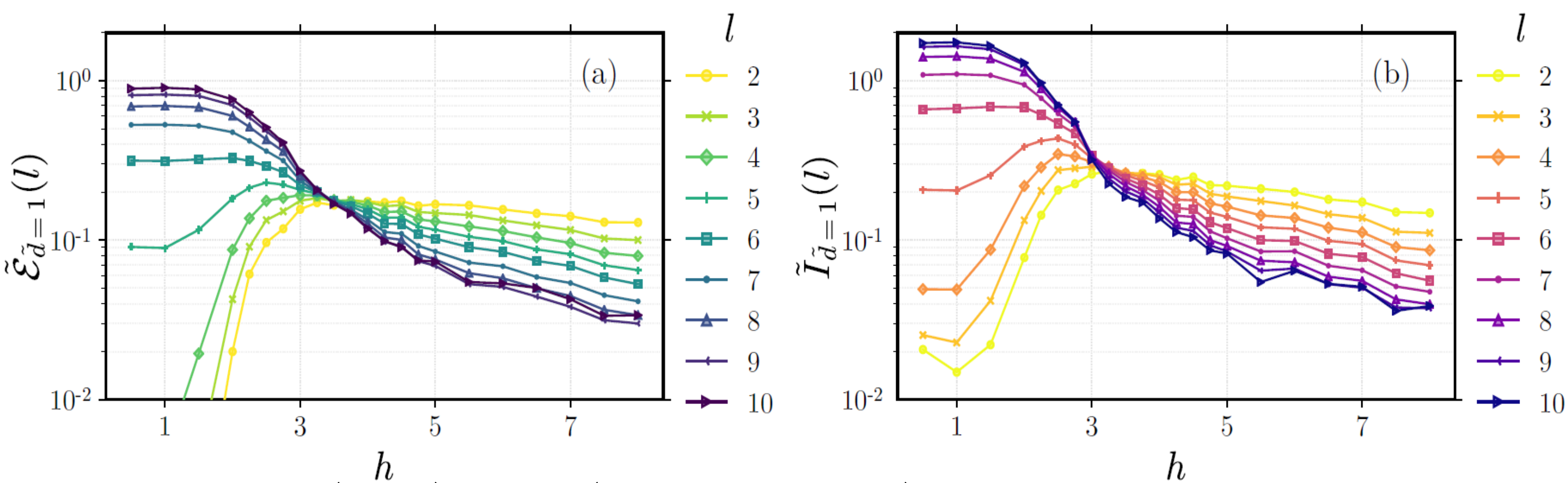
Mutual Information



Scale invariance



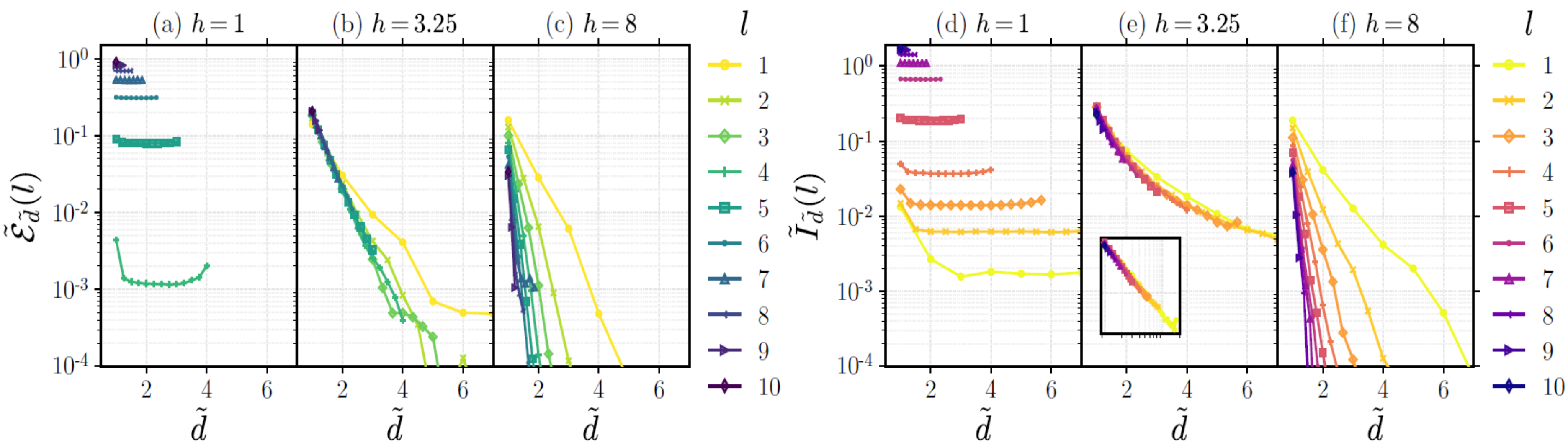
$$\tilde{d} = d/l \Rightarrow \begin{cases} \tilde{\mathcal{E}}_{\tilde{d}}(l) = \dfrac{1}{\mathcal{N}} \displaystyle\sum_{\{i,j:\ \frac{|i-j|}{l}=\tilde{d}\}} \left\langle \tilde{\mathcal{E}}_{A_i B_j} \right\rangle \\ \tilde{I}_{\tilde{d}}(l) = \dfrac{1}{\mathcal{N}} \displaystyle\sum_{\{i,j:\ \frac{|i-j|}{l}=\tilde{d}\}} \left\langle \tilde{I}_{A_i B_j} \right\rangle \end{cases}$$



$$\tilde{\varepsilon}_{\tilde{d}}(l) = \tilde{\varepsilon}\left(\frac{l}{d}, \frac{l}{\xi}\right) = \tilde{\varepsilon}\left(\frac{l}{d}, l\,|h - h_c|^{\upsilon}\right)$$

Data collapse at the transition

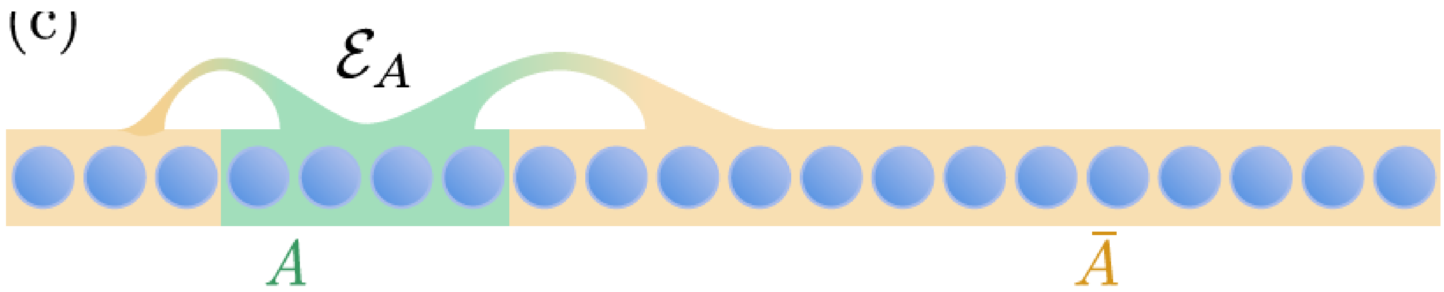
$$\tilde{\varepsilon}_{\tilde{d}}(l) = \tilde{\varepsilon}\left(\frac{l}{d}, \frac{l}{\xi}\right) = \tilde{\varepsilon}\left(\frac{l}{d}, l\,|h - h_c|^{v}\right)$$



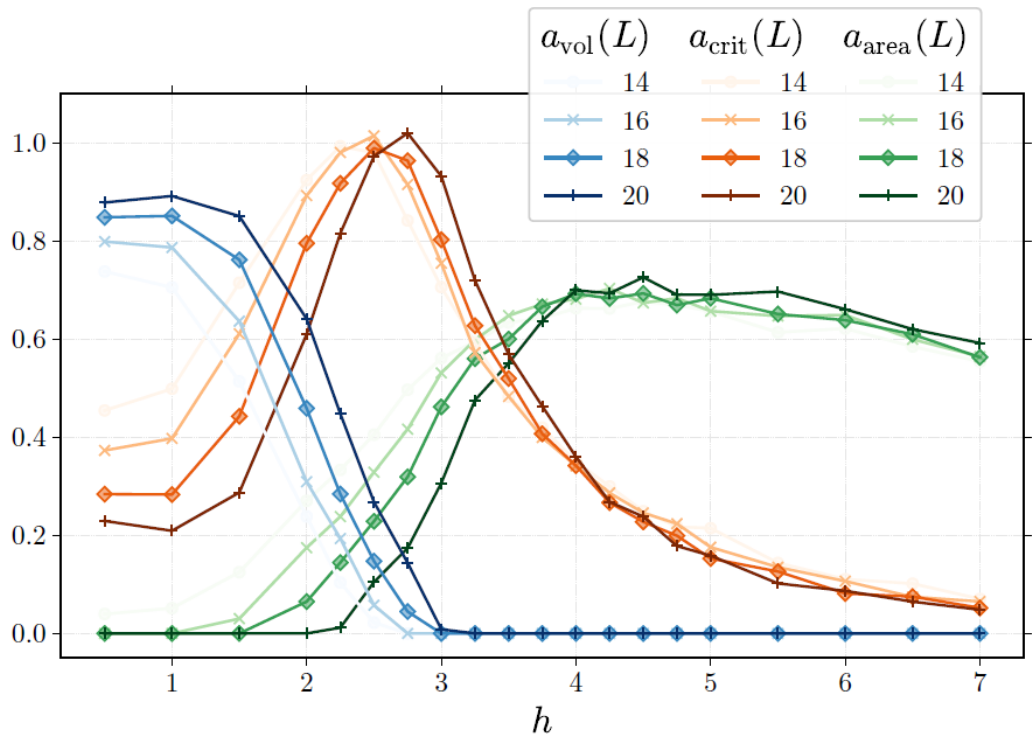
At criticality → Negativity decays **exponentially**

Mutual information decays **algebraically**

Emergence of log(l) scaling



$$\langle \mathcal{E}_A(l) \rangle = a_{\mathrm{vol}}\, l + a_{\mathrm{crit}} \log_2 l + a_{\mathrm{area}}$$



Summary

1. Schmidt gap can capture and characterize both:
   - Quantum phase transitions (even in the absence of spontaneous symmetry breaking)
   - Many-body localization transition

2. Negativity provides a physical picture for the diverging length scale at QPT and MBL transitions.

3. MBL shows scale invariance behaviour in both mutual information and negativity

References

- **Scale Invariant Entanglement Negativity at the Many-Body Localization Transition**
  J. Gray, A. Bayat, A. Pal, S. Bose
  arXiv:1908.02761

- **Many-body Localization Transition: Schmidt Gap, Entanglement Length & Scaling**
  J. Gray, S. Bose, A. Bayat
  Phys. Rev. B 97, 201105 (2018)

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