(II. 21a

EXTRA LECTURE 19/2 (DQPTs CONT'D)
$10^{00}-11^{45}$ F7103

So, today I'll wrap up the subject of DQPTs.

"quantum phase transition in time"

Central object: LOSCHMIDT AMPLITUDE $G(t) = \langle\Psi_0|e^{-iHt}|\Psi_0\rangle$ $\quad H=H(g)$ (II.32)

QUENCH
$H(g_0) \to H(g)$

sudden change of $g_0 \to g$ at $t=0$

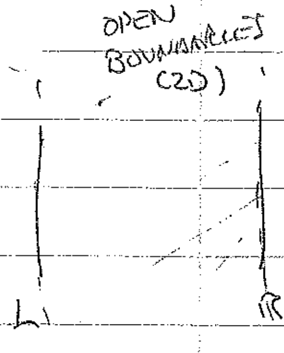
initial state (typically g.s. of $H_0 = H(g_0)$)

LOSCHMIDT ECHO $\mathcal{L}(t) = |G(t)|^2$ (II.33)

There's a formal similarity between $G(t)$ and the so called boundary partition function Z_B of an equilibrium quantum many-body system with boundaries:

PERIODIC BOUNDARY CONDITIONS
$$Z = \sum_n e^{-\beta E_n} = \sum_n \langle\psi_n|e^{-\beta H}|\psi_n\rangle$$

OPEN BOUNDARIES (2D)
$$Z = \sum_n \langle\psi_n|e^{-\beta H}|\psi_n\rangle + \underbrace{\langle\Psi_L|e^{-\beta H}|\Psi_R\rangle}_{Z_B}$$

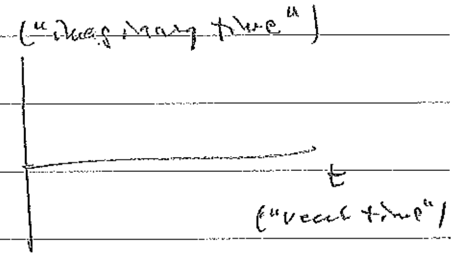


$|\Psi_L\rangle, |\Psi_R\rangle$ encodes the boundary conditions
$|\Psi_L\rangle = |\Psi_R\rangle \doteq |\Psi_B\rangle$ if open boundaries to L & R

important computational analogy:
$$Z_B = \langle\Psi_B|e^{-\beta H}|\Psi_B\rangle \xrightarrow[\substack{\beta \to it \\ |\Psi_B\rangle \to |\Psi_0\rangle}]{} \underbrace{\langle\Psi_0|e^{-iHt}|\Psi_0\rangle}_{G(t)}$$

We can take this similarity one step further, and define a "RATE FUNCTION" as the analogy of a free energy density.*
But first, let's "complexify" time t

$$t \rightarrow z = t + i\tau \in \mathbb{C} \qquad (\text{III}.34)$$



For a finite system, a Loschmidt amplitude is ANALYTIC.
To see why, insert an eigenbasis $|E_\nu\rangle$ of H into (II.32):

$$G(z) = \sum_\nu |\langle E_\nu | \psi_0\rangle|^2 e^{-iE_\nu z}$$

finite sum of analytic functions
$\rightarrow G(z)$ analytic

$\Downarrow$ Weierstrass factorization theorem

$$G(z) = e^{\mu(z)} \prod_j (z_j - z) \qquad (\text{III}.34)$$

$\mu(z)$ analytic

Now define the "RATE FUNCTION"

$$g(z) = -\frac{1}{N}\log G(z) = -\frac{1}{N}\mu(z) - \underbrace{\frac{1}{N}\sum_j \log(z_j - z)}_{g_s(z)}$$

"singular part of the rate function"

ANALOGY

SLIDE OF
"FISHER ZEROS"
("YANG-LEE ZEROS")

Discuss analogy with 2D electrostatics!

* Free energy density $f = \frac{1}{N}F$. Cf. $f_B = -\frac{1}{N}k_B T \log(Z_B)$

here: a divergence of $g_s(z)$

IN $g_s(z)$ AS A FUNCTION

NONANALYTICITY OF REAL TIME t WHEN A FISHER ZERO CROSSES THE REAL AXIS IN THE COMPLEX PLANE

$\Downarrow$

$$g_s(z) \xrightarrow[t \to t_c]{} \infty \Rightarrow G(z) \to 0$$

$\Downarrow$

NO OVERLAP BETWEEN $|\psi_0\rangle$ AND $|\psi(t^*)\rangle$

Important feature that follows from the analogy with Fisher zeros in the case of equilibrium phase transitions:

DQPTs are robust against $\underbrace{\text{symmetry-preserving perturbations}}_{\text{can't melt the lines or areas of Fisher zeros.}}$

Before proceeding, note that if E_0 is degenerate, then $\mathcal{L}(t)$ must be replaced by

# of degenerate ground states

$$\mathcal{L}_{deg}(t) = \sum_{\alpha=1}^{\nu} \underbrace{|\langle \Psi_\alpha | e^{-iHt} | \Psi_\alpha \rangle|^2}_{\mathcal{L}_\alpha(t)} = \sum_{\alpha=1}^{\nu} \mathcal{L}_\alpha(t) \quad (\text{III}.35)$$

We can now define the corresponding rate function, call it $\lambda(t)$, for $\mathcal{L}(t)$:

$$\mathcal{L}(t) = e^{-N\lambda(t)}, \quad \lambda_s(z) = 2\,\mathrm{Re}\, g_s(z) \quad (\text{III}.36)$$

can be measured experimentally doing a quench with TRAPPED IONS (Jurcevic et al., PRL 119, 080501 (2017))

Model system: TRANSVERSE FIELD ISING CHAIN WITH LONG-RANGE INTERACTION

$$H = -\sum_{l>m} J_{lm} \sigma_l^z \sigma_m^z - h \sum_{l=1}^{N} \sigma_x^l \quad (\text{III}.37)$$

$$J_{lm} \sim \frac{1}{|l-m|^\alpha} \quad \text{for } |l-m| \gg 1$$

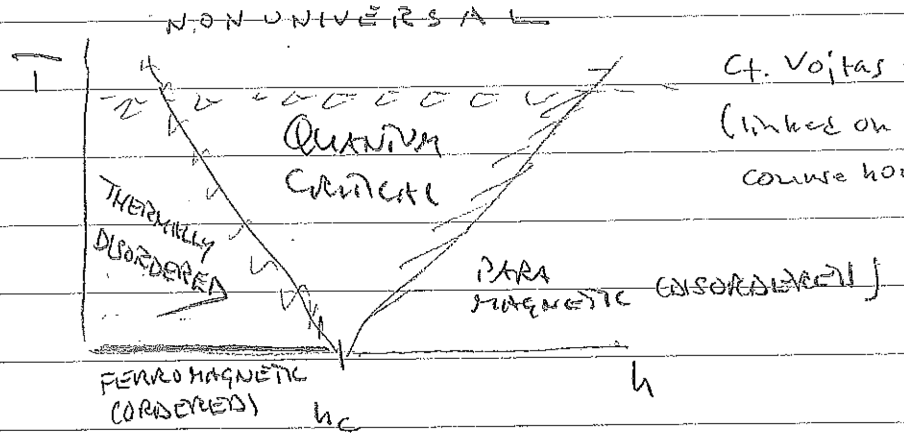
α tunable from 0 to 3

The quench is done across the quantum critical point separating ferro- and paramagnetic phases

$J_{\ell m} = J_{\ell,\ell+1}$

For the nearest-neighbor transverse Ising chain,

RECALL... $h = h_c = J_{\ell,\ell+1}$ is a quantum critical point



Cf. Vojtas review (linked on the course home page)

For the long-range model $h_c = h_c(\alpha)$, more complicated...

Initially, the system is prepared in the fully polarized state (ferromagnetic)

groundstate when $h=0$ $\longrightarrow$

$$|\Psi_0\rangle = |\Uparrow\rangle \equiv |\uparrow\rangle \otimes |\uparrow\rangle \otimes |\uparrow\rangle \otimes \ldots$$

The system is then quenched by quickly changing h from 0 to a value $> h_c$.

Since $|\Psi_0\rangle$ is degenerate (at $h=0$) with $|\Downarrow\rangle \equiv |\downarrow\rangle \otimes |\downarrow\rangle \otimes |\downarrow\rangle \otimes \ldots$ we must use the generalization (III.35) for the Loschmidt echo:

$$\mathcal{L}(t) = \mathcal{L}_\uparrow(t) + \mathcal{L}_\downarrow(t)$$, with corresponding $\lambda_{\uparrow,\downarrow}(t)$

$$\begin{cases} \mathcal{L}_\uparrow(t) = |\langle \Uparrow | \Psi(t)\rangle|^2 \\ \mathcal{L}_\downarrow(t) = |\langle \Downarrow | \Psi(t)\rangle|^2 \end{cases}$$

One can show that $\lambda(t) \overset{N \gg 1}{\approx} \min_{\eta=\uparrow,\downarrow} \lambda_\eta(t)$

EXP

THEORY

SLIDE

good agreement

Many questions (currently being addressed!):

- Does the nonanalyticity really signal a phase transition? If so, how to characterize the phases BEFORE and AFTER the appearance of the nonanalyticity? ("Phase-transition in time") Orderparameter (broken symmetry) or a topological invariant? Or something else?

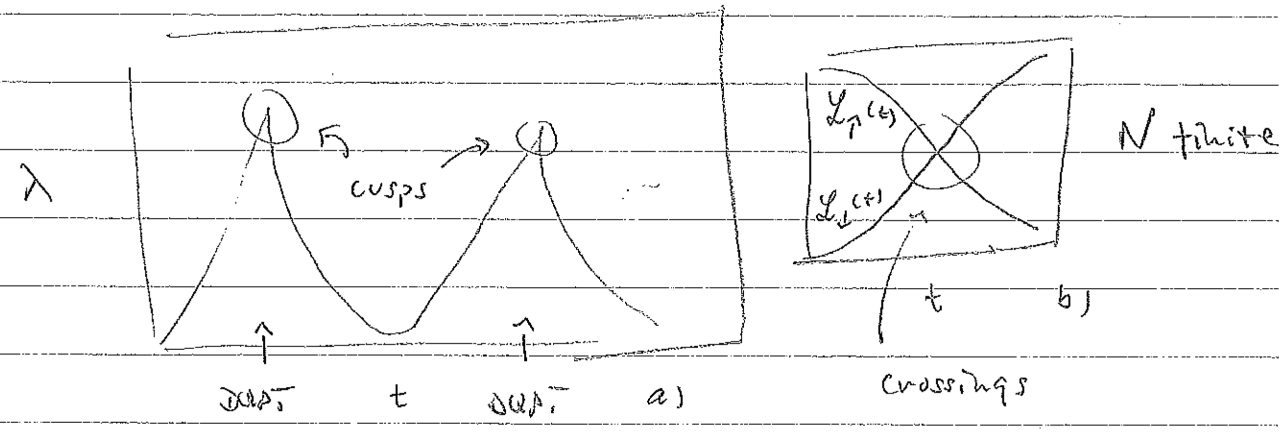
- Under what conditions do the nonanalyticities appear? Frequently when crossing a quantum critical point! (But not always.) Maybe best understood class of systems exhibiting DQPTs (so far): Noninteracting fermions in 1D with symmetry-protected topological phases.

- ... and much more.

SEE REVIEW ARTICLE BY MARKUS HEYL LINKED ON THE COURSE HOME PAGE.

"After-lecture addendum" (in response to two questions...)

1. SLIDE #3

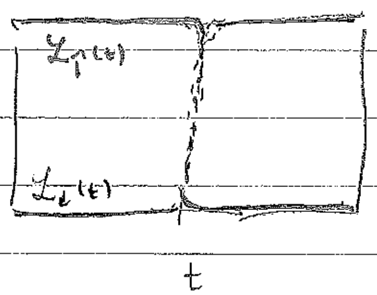


The cusps in Fig a) reflect the crossings in Fig b).

Q: "Doesn't this mean that the nonanalyticities (cusps → DQPT) are simply artefacts of having crossed $\mathcal{L}_\uparrow(t)$ and $\mathcal{L}_\downarrow(t)$?"

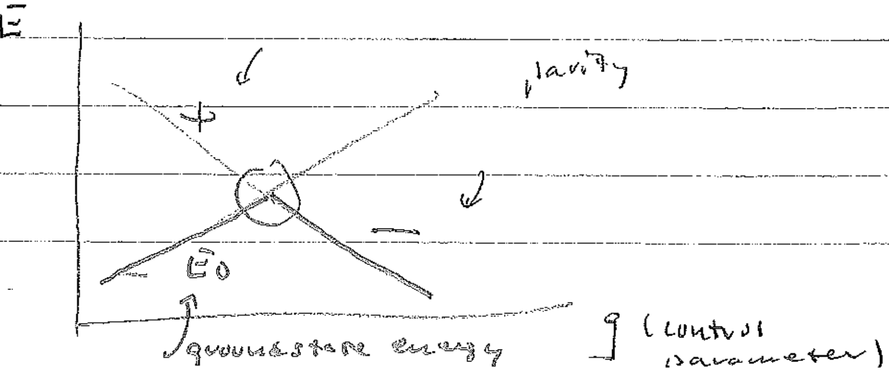
A: $\lambda(t) \xrightarrow{N\to\infty} \min_{\eta=\uparrow,\downarrow} \lambda_\eta(t)$ with $\mathcal{L}_\eta(t) = e^{-N\lambda_\eta(t)}$, $\eta=\uparrow,\downarrow$

$\Downarrow$



$N\to\infty$ ⇒ sudden change of type of overlap as measured by $\mathcal{L}$!

"Loose analogy": First-order phase transition



11. My statement: "DQPTs are the BIG thing in the theory of phase transitions happening after thermal (classical) phase transitions and (equilibrium) QPTs"

Q: "But what about the new exciting stuff on dissipative phase transitions?

A: Right! Should have mentioned this!
Please have a look at the paper by Kessler et al. linked on the course homepage, in particular the nice Table I comparing "classical", "quantum", and "dissipative" phase transition (but not DQPTs... !)