(II.16

LECTURE 11 FEBRUARY

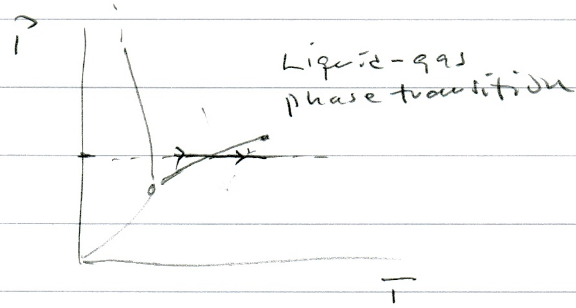
Today I'll do two things — connecting back to the experimental realization of the Haldane model

1. Brief intro to cold atoms in optical lattices, which has become an important platform for studying quantum matter $\boxed{\text{SLIDES}}$

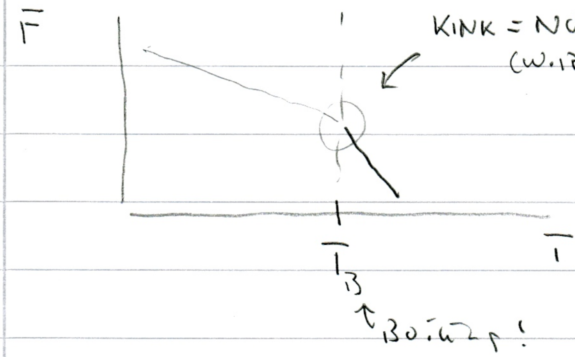
2. I'll introduce you to a very recent development, which has caused a lot of excitement, the so-called DYNAMICAL QUANTUM PHASE TRANSITIONS.

DYNAMICAL QUANTUM PHASE TRANSITIONS

Let me begin with something you already know:



$$F = -k_B T \log Z \quad , \quad Z = \sum_n e^{-\beta E_n} \quad , \quad \beta = 1/k_B T$$



KINK = NONANALYTICITY ≡ PHASE TRANSITION (W.R.T. A CONTROL PARAMETER)

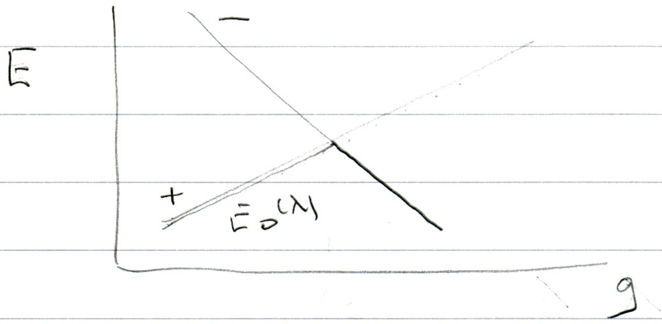
1st ORDER (like here) if F has a cusp

CONTINUOUS if $\frac{\partial^n F}{\partial T^n}$, $n \geq 1$ has a cusp

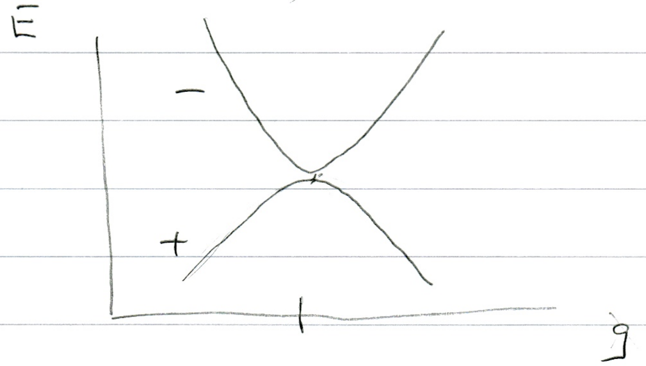
IN PASSING...

This definition generalizes to
(EQUILIBRIUM) QUANTUM PHASE TRANSITIONS (AT $T=0$)

$F \to E_0$, $T \to g$
coupling constant, external field, ...

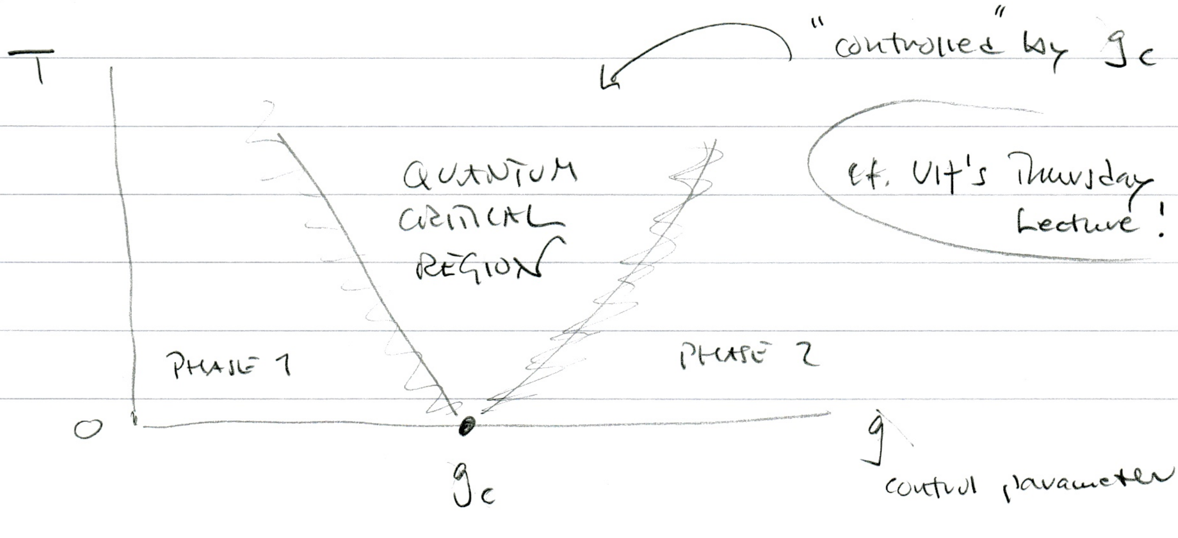


LEVEL CROSSING → FIRST ORDER QPT
(possible also for finite systems)

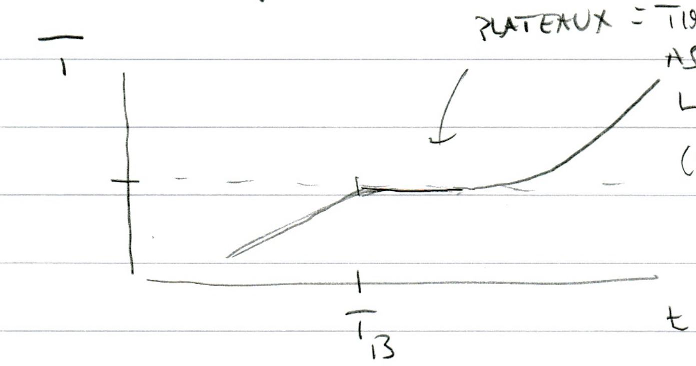


AVOIDED LEVEL CROSSING
→ CONTINUOUS QPT
(IN THE THERMODYNAMIC LIMIT)

WHY CARE ABOUT $T=0$?



Now back to the thermal phase transition. What about the time dependence?



PLATEAUX = TIME FOR THE LIQUID TO ABSORB THE REQUIRED LATENT HEAT (1st order transition)

$T(t)$ ANALYTIC !

N.B. The rate $\frac{\partial T}{\partial t}$ may have physical consequences, like in the KIBBLE-ZUREK MECHANISM.

Kibble: domain formation in the early universe

Zurek: quantitative theory for condensed matter applications

describes formation of topological defects in a system driven through a continuous phase transition at a finite rate $\frac{\partial T}{\partial t}$

$T(t) = vt$ (assuming a linear rate)

relaxation time: $\tau \sim |T - T_c|^{-z\nu}$, z = "dynamical exponent"

ν = correlation length exponent

$\xi \sim |T - T_c|^{-\nu}$

$T_c (= T_B)$

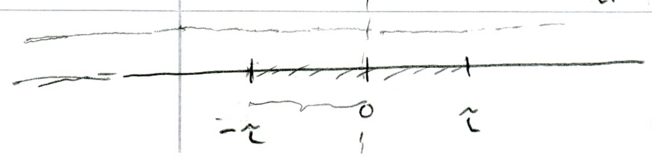
$\bar{t}$ = time to reach the critical point

$\bar{t} = \tau = \text{const} \times |T(\bar{t}) - T_c|^{-z\nu}$

freeze-out time, put $T_c = 0$ for simplicity $\Rightarrow$ $\bar{t} \sim v^{-z\nu/(1+z\nu)}$

$\Downarrow$ $\Longrightarrow$ DOMAIN (DEFECT) FORMATION

Adiabaticity is lost (recovered) at time $-\bar{t}$ $(+\bar{t})$



τ symmetry broken phase

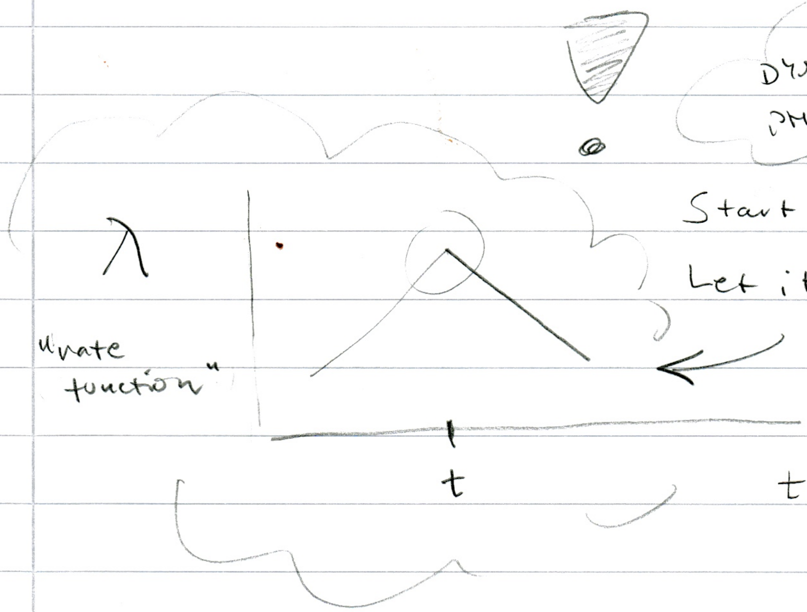
TIME MEASURED W.R.T. THE TIME WHEN CROSSING CRITICAL POINT

<u>Surprise discovery</u> in 2010 (Pollmann et al., Phys. Rev. E <u>81</u>, 020101(R))

<u>Theoretical ansatz</u>: Heyl et al., PRL <u>110</u>, 135704 (2013)

<u>NONEQUILIBRIUM</u> CLOSED QUANTUM MANY-BODY SYSTEMS MAY EXHIBIT NONANALYTICITIES IN TIME.

DYNAMICAL QUANTUM PHASE TRANSITION (DQPT)



Start the system <u>out of equilibrium</u>

Let it evolve (unitarily!) with time

λ plays the role of a free energy F (or, ground state energy E_0, in case of an equilibrium QPT). To see how, let's consider a

<u>QUANTUM QUENCH</u> (simplest way to take a quantum system out of equilibrium): sudden change of a Hamiltonian parameter at $t=0$ from g_0 to $\underline{g}$:

$$|\Psi_0\rangle \longrightarrow |\Psi_0\rangle \longrightarrow |\Psi(t)\rangle$$

$H_0 \equiv H(g_0)$ $H \equiv H(g)$ $|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle$

$t<0$ $t=0$

$(H_0|\Psi_0\rangle = E_0|\Psi_0\rangle)$

typically the groundstate of H_0.

nontrivial dynamics whenever the initial state $|\Psi_0\rangle$ is not an eigenstate of H.

Central object within the theory of DQPTs:

LOSCHMIDT AMPLITUDE (Johann Loschmidt* 1821–1895)
("return amplitude", "fidelity", vacuum persistence amplitude")

$$G(t) = \langle \Psi_0 | \Psi(t) \rangle = \langle \Psi_0 | e^{-iHt} | \Psi_0 \rangle \qquad (\text{II}.32)$$

measures the deviation of the time-evolved state from the initial condition. The corresponding probability

$$\mathcal{L}(t) = |G(t)|^2 \qquad (\text{II}.33)$$

is called the LOSCHMIDT ECHO.

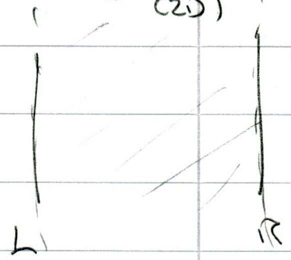
There's a formal similarity between $G(t)$ and the so called boundary partition function Z_B of an equilibrium quantum many-body system with boundaries:

PERIODIC BOUNDARY CONDITIONS

$$Z = \sum_n e^{-\beta E_n} = \sum_n \langle \psi_n | e^{-\beta H} | \psi_n \rangle$$

OPEN BOUNDARIES (2D)

$$Z = \sum_n \langle \psi_n | e^{-\beta H} | \psi_n \rangle + \underbrace{\langle \Psi_L | e^{-\beta H} | \Psi_R \rangle}_{Z_B}$$



$|\Psi_L\rangle, |\Psi_R\rangle$ encodes the boundary conditions
$|\Psi_L\rangle = |\Psi_R\rangle = |\Psi_B\rangle$ if open boundaries to L & R

important computational analogy!

$$Z_B = \langle \Psi_B | e^{-\beta H} | \Psi_B \rangle \xrightarrow[\substack{\beta \to it \\ |\Psi_B\rangle \to |\Psi_0\rangle}]{} \underbrace{\langle \Psi_0 | e^{-iHt} | \Psi_0 \rangle}_{G(t)}.$$

MORE NEXT WEEK

* Loschmidt's "reversibility paradox" prodded Boltzmann to develop his statistical concept of entropy, $S = k_B \ln \Omega$.