Condensed Matter Physics

homework problems, set 5, fall 2015

deadline Friday January 8; please put your solutions in the box marked "Condensed Matter Physics FKA091", floor 3, outside the elevator, south wing of the Origo building

Please structure your solutions carefully. All essential steps in your analysis and calculations should be made explicit. You are encouraged to study the problems together with your fellow students. However, you must take full responsibility for your own work, and, if you get questions about your solutions, you should be able to explain what you have done.

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1. One important topic that we never got to in the course is the *semiclassical theory of transport in metals and semiconductors*. All three recommended text books have excellent discussions of this subject (as does many other texts that you will find by searching the web!). In this problem you are asked to write a mini essay that summarizes what you have learned by a "self study" of semiclassical transport theory. Try to be succinct. Focus on essentials. Write simple and pedagogic. Pictures are appreciated.

For a full score your essay should include the concepts (with discussions): *Boltzmann equation, relaxation time, impurity scattering, scattering by phonons, AC/DC, thermal conductivity*, and *hole transport*. Make sure to reference the source(s) that you use.

2. As I discussed in class, a number-conserving theory for a conventional superconductor can be obtained by making coherent superpositions of BCS ground states of definite phase,

$$|\,\psi(N)\rangle = \int_0^{2\pi} d\phi\, e^{i\phi N/2}\,|\,\psi(\phi)\rangle.$$

a) Show by an explicit calculation that $|\,\psi(N)\rangle$ contains N electrons.
b) In class I claimed that the distribution of terms with different particle numbers in the BCS ground state peaks sharply about the average number $\langle N\rangle$ when $\langle N\rangle$ is macroscopic. In other words, the distribution of a_N in

$$|\,\psi\rangle = \sum_N a_N\,|\,\psi(N)\rangle$$

is sharply peaked about its mean value $\langle N\rangle$ for large $\langle N\rangle$. Can you calculate a_N?
c) The fact that a definite (indefinite) phase of the superconducting ground state implies an indefinite (definite) particle number (and vice versa) reflects the fact that phase and particle number are canonically conjugate variables and are subject to a Heisenberg uncertainty relation. This fact has consequences for the *Josephson effect* in mesoscopic junctions. What is the Josephson effect? And how is the mesoscopic Josephson effect influenced by the phase - particle number uncertainty? (Reference the source(s) that you use for this part of problem!)

3. In order to arrive at the BCS gap equation, one drops the fourth-order off-diagonal terms in the Bogoliubov transformed interaction term of the BCS Hamiltonian (see lecture notes or Eq. (7.3.6) in the hand-out on superconductivity). Show that this OK by showing that the fourth-order terms have a negligible expectation value in the BCS ground state.
Hint: The coefficients $u_{\boldsymbol{k}}$ and $v_{\boldsymbol{k}}$ in the BCS ground state are O(1).