Condensed Matter Physics

homework problems, set 4, fall 2015

deadline Friday December 18, 17:00; please put your solutions in the box marked "Condensed Matter Physics FKA091", floor 3, outside the elevator, south wing of the Origo building

Please structure your solutions carefully. All essential steps in your analysis and calculations should be made explicit. You are encouraged to study the problems together with your fellow students. However, you must take full responsibility for your own work, and, if you get questions about your solutions, you should be able to explain what you have done.

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1. Using the *Supplementary Material* to this set of homework problems (see the course home page), calculate the electronic bandstructure of graphene based on nearest-neighbor tight-binding wave functions. Obtain a simplified expression for the band structure close to the Dirac point using a linear band approximation.

Hint: Start by separately multiplying Eq. (2.10) in the Supplementary Material *by Φ_A^* and Φ_B^*. Study the condition for nontrivial solutions to the resulting system of equations. Determine the matrix elements $H_{ij} \equiv \langle \Phi_i | H | \Phi_j \rangle$ and $S_{ij} \equiv \langle \Phi_i | \Phi_j \rangle$ that appear in your equations. Here you can exploit the equivalence of the two sublattices of graphene. (Part of this problem was also discussed by Ermin in his guest lecture; see the course homepage for his slides.)*

2. (a) In my derivation of the Fröhlich Hamiltonian in the lecture Dec. 11, I threw away a term that didn't contain the displacement vectors $\boldsymbol{y_l}$ (and hence doesn't know about the phonons!). I called this term the "Bloch Hamiltonian". Obtain an explicit expression for this object, with the sum taken over $\boldsymbol{k}$ and $\boldsymbol{g}$, where the $\boldsymbol{g}$ are reciprocal lattice vectors. Why the name "Bloch Hamiltonian"? Discuss!

(b) In the same lecture I derived, to second order in perturbation theory, an expression $\mathcal{E}_2$ for the change of the longitudinal phonon spectrum due to the interaction with the electrons. I claimed that my expression can be simplified to

$$\mathcal{E}_2 = \sum_{\boldsymbol{k},\boldsymbol{k}'} |M_{\boldsymbol{k},\boldsymbol{k}'}|^2 \langle n_{\boldsymbol{k}} \rangle \left[\frac{2(\mathcal{E}_{\boldsymbol{k}} - \mathcal{E}_{\boldsymbol{k}'})\langle n_{\boldsymbol{q}} \rangle}{(\mathcal{E}_{\boldsymbol{k}} - \mathcal{E}_{\boldsymbol{k}'})^2 - (\hbar\omega_{\boldsymbol{q}})^2} + \frac{1 - \langle n_{\boldsymbol{k}'} \rangle}{\mathcal{E}_{\boldsymbol{k}} - \mathcal{E}_{\boldsymbol{k}'} - \hbar\omega_{\boldsymbol{q}}} \right]. \tag{1}$$

Here $M_{\boldsymbol{k},\boldsymbol{k}'}$ is the same amplitude as defined in my lecture, having the property that $M_{\boldsymbol{k},\boldsymbol{k}} = 0$. Show how to obtain the formula in (1). (This formula is important. Among other things, it can be used to derive the *Kohn anomaly* which imprints an image of the Fermi surface on the phonon spectrum.)

Hint: Expectation values containing products of three number operators vanish. Why? Moreover, you may assume that $\omega_{\boldsymbol{q}} = \omega_{-\boldsymbol{q}}$, implying that $\langle n_{\boldsymbol{q}} \rangle = \langle n_{-\boldsymbol{q}} \rangle$.