Condensed Matter Physics

homework problems, set 3, fall 2015

deadline 11 December

Please structure your solutions carefully. All essential steps in your analysis and calculations should be made explicit. You are encouraged to study the problems together with your fellow students. However, you must take full responsibility for your own work, and, if you get questions about your solutions, you should be able to explain what you have done.

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1. Calculate the cyclotron frequency ω_c for a Bloch electron in 2D $\boldsymbol{k}$-space living on an energy contour given by

$$\mathcal{E}_{\boldsymbol{k}} = \frac{\hbar^2}{2m_1^*}k_x^2 + \frac{\hbar^2}{2m_2^*}k_y^2.$$

The magnetic field is perpendicular to the plane of the contour. Also sketch the orbit of the electron in coordinate space. Explain how there can be *two* effective masses, m_1^* and m_1^*, associated with a single electron!

2. *Bloch oscillations* are one of the simpler predictions of the semiclassical theory of electron dynamics in a crystal. The phenomenon is hard to observe, however, since the electrons are likely to scatter off impurities and phonons before a Bloch oscillation cycle is completed.

(a) Assume that you have a piece of Cu (average time between scattering events $\sim 2 \times 10^{-13}$ s, typical lattice spacing ~ 0.4 nm) and that you hook it up to a voltage source. How large voltage would you need in order to see Bloch oscillations? Could you do it?

(b) A possible problem with applying a large electric field (as in part (a)) is that this may kick an electron into a higher energy band *(Zener tunneling)*, invalidating the assumptions of the theory. As you can see from your analysis in (a), one way to reduce the demand for a very strong field is to build an artificial crystal with a huge lattice spacing *(semiconductor superlattice)*. Still, Bloch oscillations are hard to come by, and require highly sophisticated experimentation. One complication is that the Zener tunneling tends to increase as one increases the lattice spacing. Can you explain why, simply by sketching the qualitative band structures in the 1BZ for large and small lattice spacings respectively (assuming that the strength of the crystal potential is roughly the same in the two cases)?

3. A Bloch electron in the lowest band of a 1D crystal with potential

$$V(x) = 2V\cos gx$$

is scattered from k_x to $-k_x$ by an *impurity potential*

$$V_{imp}(x) = U\exp(-(gx/4)^2).$$

Here g, V, and U are constants. Use the Born approximation to explore how the transition probability $Q(k_x, -k_x)$ for this process varies with k_x. *See next page for a hint!*

Hint: Use "nearly-free-electrons". Begin by calculating $Q(k_x, -k_x)$ far from the zone boundaries (easy!). *Then write an Ansatz for the approximate wave function which is valid close to a zone boundary* (think Bragg scattering!). *How can you determine the coefficients that go into your Ansatz?* (Think strong Bragg scattering = opening of a band gap!) *To round up, you may wish to throw away some very small contributions to $Q(k_x, -k_x)$ so as to obtain a nice-looking expression. Now compare your two expressions for $Q(k_x, -k_x)$, one valid far away from the zone boundaries, the other applicable close to a zone boundary. Conclusion?*