Condensed Matter Physics

homework problems, set 2, fall 2015

deadline 4 December

Please structure your solutions carefully. All essential steps in your analysis and calculations should be made explicit. You are encouraged to study the problems together with your fellow students. However, you must take full responsibility for your own work, and, if you get questions about your solutions, you should be able to explain what you have done.

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1. Consider the Thomas-Fermi expression for a screened Coulomb potential,

$$V_{TF} \approx \frac{\exp(-r/\xi_{TF})}{r},$$

with ξ_{TF} the *Thomas-Fermi screening length*. What is a typical value of ξ_{TF} for an ordinary metal? How does this value compare to the average distance between the conduction electrons in the metal? What about a semiconductor, e.g. Si? Discuss your findings! Why are the screening lengths for metals and semiconductors so different?

2. One of your friends is taking her first undergraduate course in solid state physics. She is puzzled that the lecturer has nothing to say about the electron-electron interaction in a metal: "Lots of talk about periodic potentials, bla, bla,.... but nothing about the really strong Coulomb interaction between the conduction electrons! How can one simply ignore this important piece of physics?!" As a Masters student taking a more advanced course, you offer to explain to her how things *really* work. What do you say? Build your explanation around the concepts of *screening, reduced scattering rates*, and *quasiparticles*. Use simple language, analogies, and pictures. Keep formulas to a minimum! Make sure to reference the texts/literature that you use.

3. Consider a two-dimensional crystal with lattice potential

$$V(\boldsymbol{r}) = 2V(\cos gx + \cos gy),$$

and a 1BZ given by a square with side g. Assuming that the potential is weak, calculate the bandgap (i) along the edges of the 1BZ; (ii) at the corners of the 1BZ. Given your results, do you have sufficient information to decide whether the crystal is an insulator or a metal? If not, what additional information must be supplied to figure this out?

Hint: Part (i) is very similar to the one-dimensional problem on nearly-free electrons that we did together in class. Part (ii) is a direct generalization of the same type of analysis but now with two more equations. When you figured out how, the rest will be simple. To simplify the math, it is sufficient to find the bandgap exactly at a corner of the 1BZ.