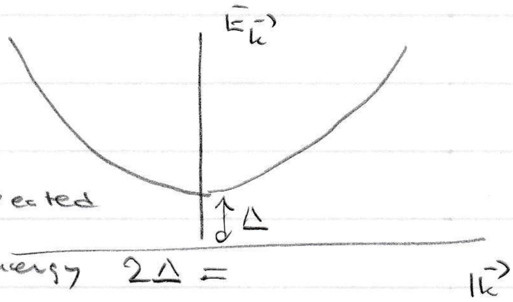
SUPPLEMENTARY LECTURE NOTES (THURSDAY DEC 17)

"bogolons"

Quasiparticle excitations above the BCS groundstate are created by acting with $\gamma^+_{\vec{k}}$ and $\gamma^+_{-\vec{k}}$. Energy:

$$E_{\vec{k}} = \left(\tilde{\varepsilon}^{\,2}_{\vec{k}} + \Delta^2 \right)^{1/2}$$



The bogolons can only be created in pairs. Need to supply an energy 2Δ = energy to break up a Cooper pair.

What happens at finite temperature T?

$\Delta \to \Delta(T)$. To see how, go back to $H_{BCS} = H_0 + H_V$

$$H_0 = \sum_{\vec{k}} \varepsilon_{\vec{k}} \left(2 v^2_{\vec{k}} + (\ldots)\left(\underbrace{\gamma^+_{\vec{k}} \gamma_{\vec{k}}}_{m_{\vec{k}}} + \underbrace{\gamma^+_{-\vec{k}} \gamma_{-\vec{k}}}_{m_{-\vec{k}}} \right) \right) + \ldots$$

OFF-DIAGONAL TERMS CANCELED OR DROPPED

$$H_V = \sum_{\vec{k},\vec{k}'} V_{\vec{k}\vec{k}'} \left(\text{CONST.} + (\ldots)\left(\overbrace{\gamma^+_{\vec{k}} \gamma_{\vec{k}}}^{m_{\vec{k}}} + \overbrace{\gamma^+_{-\vec{k}} \gamma_{-\vec{k}}}^{m_{-\vec{k}}} \right) \right) + \ldots$$

The energy required to create a bogolon AT FINITE T will be (think definition of chemical potential!)

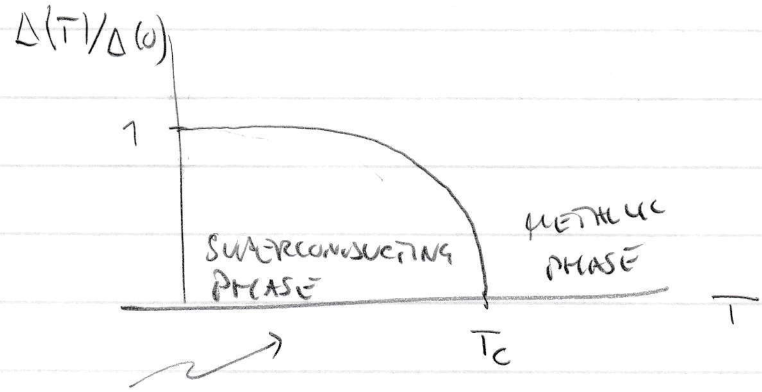
$$E_{\vec{k}} = \frac{\partial \langle H_{BCS} \rangle}{\partial \langle m_{\vec{k}} \rangle}$$

where $\langle \ldots \rangle$ is a THERMAL AVERAGE.

Assuming (for simplicity, at this level of discussion) that the bogolons (fermions, as we know!) are noninteracting we can employ the Fermi-Dirac distribution, and write

$$\langle m_{\vec{k}} \rangle = \langle m_{-\vec{k}} \rangle = \frac{1}{1 + e^{E_{\vec{k}}/k_B T}}$$

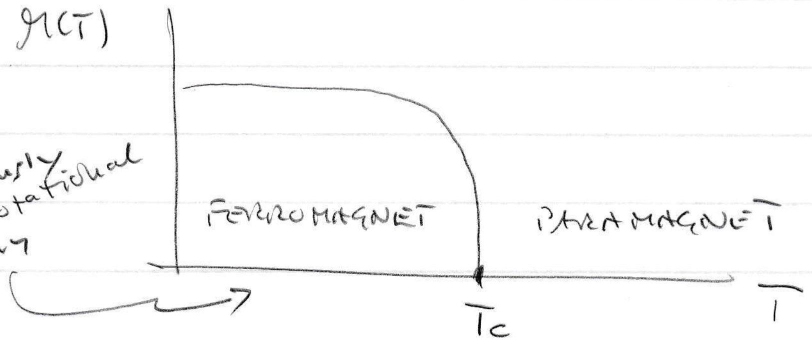
Working things out (see HAND-OUT, p 248 f) one finds a temperature-dependent $\Delta(T)$:



U(1) phase symmetry spontaneously broken

CRITICAL TEMPERATURE for the phase transition between metallic and superconducting phase.

$\Delta(T)$ (independent of $\vec{k}$, and hence of $\vec{r}$ in the BCS mean-field approximation where $V_{\vec{k}\vec{k}} \rightarrow V = \text{CONST}$) plays the role of an ORDER PARAMETER, much in the same way as the MAGNETIZATION $M(T)$ plays the role of an order parameter for the phase transition to a ferromagnetic phase in a magnet.



Spontaneously broken rotational symmetry

Given the BCS theory, one still has to work very hard to explain the two hallmarks of a superconductor: the VANISHING RESISTIVITY (perfect conductor!) and the MEISSNER EFFECT.

$\lightning$

Phenomenological theory (Fritz & Heinz London, 1935)

Assumption: Diamagnetic currents, in response to an externally applied magnetic field, can flow without resistance in the surface layer of the superconducting sample $\rightarrow$ "strong" diamagnetic response (linear in the applied vector potential):

$$\vec{j} = -\left(\frac{c}{4\pi\lambda^2}\right)\vec{A}, \qquad \vec{B} = \nabla \times \vec{A} \qquad (1)$$

constant, from dimensional analysis
$[\lambda]$ = length

Maxwell: $\nabla \times \vec{B} = \left(\frac{4\pi}{c}\right)\vec{j}$ (2)

cgs units

$\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$

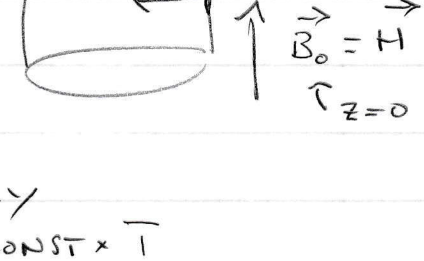
$$(1) \,\&\, (2) \Rightarrow \nabla \times \vec{B} = \left\{\nabla\cdot\vec{A} = 0 \text{ choosing Coulomb gauge}\right\}$$

$$= -\nabla^2\vec{A} = \left(\frac{4\pi}{c}\right)\vec{j} = -\frac{1}{\lambda^2}\vec{A}$$

$$\nabla^2\vec{A} = \frac{1}{\lambda^2}\vec{A} \Rightarrow \vec{A} = \vec{A}_0\, e^{-z/\lambda}$$

$\vec{H}$ constant along $\hat{x}$ and $\hat{y}$

$$\Rightarrow \vec{B} = \vec{B}_0\, e^{-z/\lambda} = \vec{H}\, e^{-z/\lambda}$$



BCS theory correctly predicts $\lambda(T) \sim \text{CONST} \times T$