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The Quantum SI: A possible new International System of Units

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Publications

Redefinition of the kilogram: a decision whose time has come,
Metrologia 42, 71 (2005).

and

Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation 1 (CI-2005),
Metrologia 43, 227 (2006).

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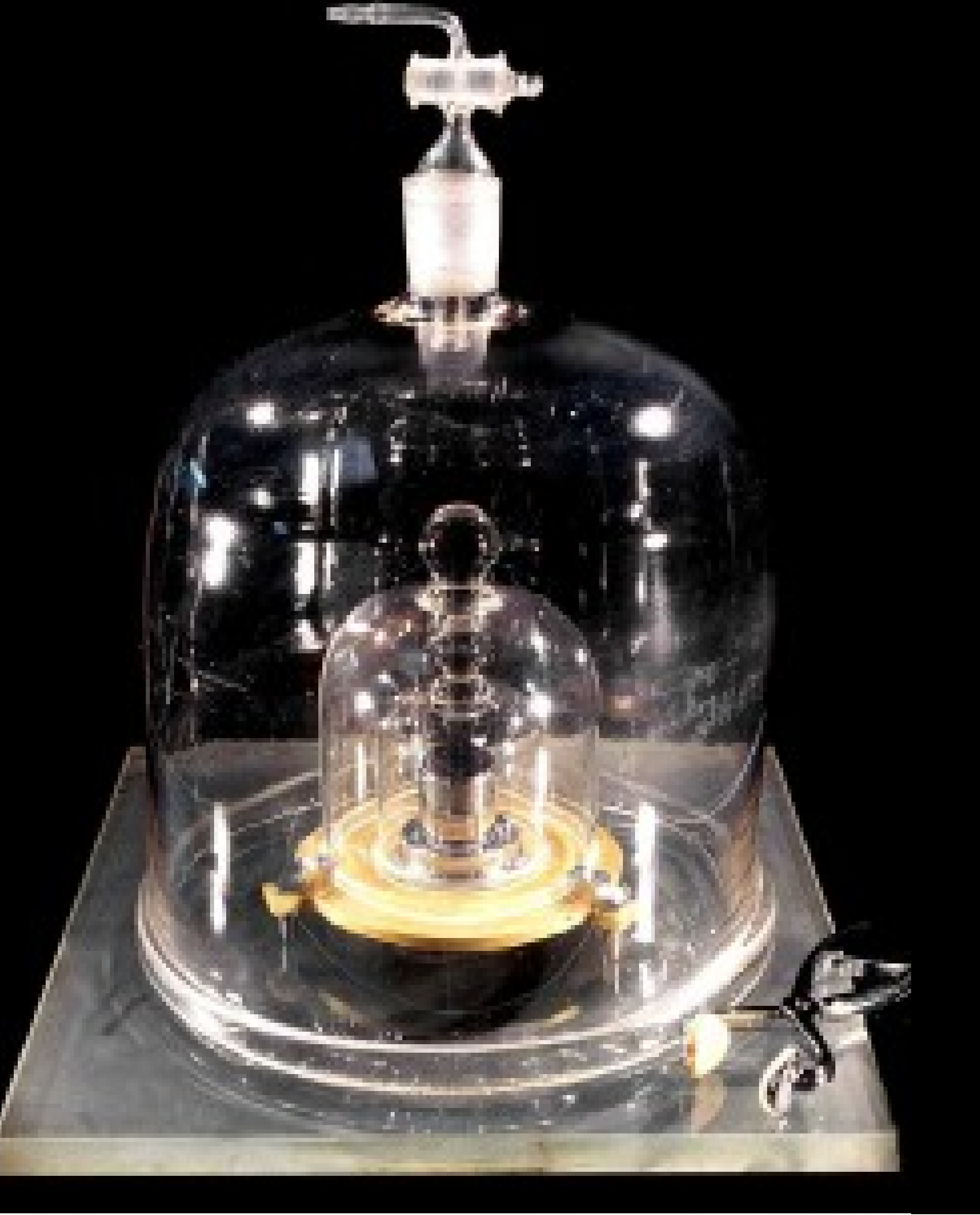
Outline

- International System of Units (SI)
- Problems with the SI
- Definition of the meter
- Possible new kilogram
- Possible new ampere
- Possible new kelvin and mole
- Effect on the fundamental constants
- The Quantum SI and unit democracy
- Rydberg constant and the second
- Timing of the changes
- Conclusion

International System of Units (SI)

- **SI base units and symbols**
  - meter m (length)
  - kilogram kg (mass)
  - second s (time)
  - ampere A (electric current)
  - kelvin K (thermodynamic temperature)
  - mole mol (amount of substance)
  - candela cd (luminous intensity)
- **Some SI derived units and symbols**
  - hertz Hz (frequency)
  - newton N (force)
  - joule J (energy)
  - coulomb C (electric charge)
  - volt V (electric potential difference)
- **Non-SI units and symbols**
  - electron volt eV (energy)
  - unified atomic mass unit u (mass)



Limitations of the current kilogram prototype definition

- The prototype definition is not linked to an unchanging property of nature.
- The mass of the international prototype appears to be changing relative to the mass of its copies.
- The drift of the kilogram prototype together with its copies (relative to an unchanging standard) could be as large as 20 x 10^{-9} kg per year (Davis 2003).
- The prototype and its copies appear to gain mass over time and lose mass when washed for use in comparisons.
- The kilogram mass definition cannot be realized independently of the international prototype.

Limitations of the current ampere definition

9th CGPM in 1948:

- The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

- Modern voltage measurements are based on the Josephson effect: $K_\mathrm{J} = 2e/h$ $[8.5 \times 10^{-8}]$

- Modern resistance measurements are based on the quantum Hall effect: $R_K = h/e^2$ $[3.3 \times 10^{-9}]$

- To express measurements with better accuracy, arbitrary exact units (not SI) are used:
  - $K_\mathrm{J} \rightarrow K_\mathrm{J\text{-}90} = 483\,597.9$ GHz/V
  - $R_\mathrm{K} \rightarrow R_\mathrm{K\text{-}90} = 25\,812.807\ \Omega$

Possible redefinitions of the kilogram

- **The limitations on stability of the definition of the kilogram in terms of the international prototype could be eliminated if the kilogram were defined in terms of a fundamental constant in analogy with the definition of the meter.**
- **17th CGPM in 1983:**
  - **The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.**
- **As a consequence for the velocity of light *c*:**

$$c = \frac{1\ \mathrm{m}}{1/299\,792\,458\ \mathrm{s}} = 299\,792\,458\ \mathrm{m/s}$$

- **An alternative statement of the definition could be:**
  - **The meter is the length scaled such that the velocity of light is 299 792 458 m/s.**

Experimental realization of the kilogram (watt-balance experiment)

Kibble, Robinson, and Belliss (1990)
Williams Steiner, Newell, and Olsen (1998)
Steiner, Williams, Newell and Liu (2005)

- **Current interpretation:**
  - precise kilogram mass + watt-balance experiment → *h*
- **Alternative interpretation:**
  - precise kilogram mass ← watt-balance experiment + *h*
- **This suggests a possible new definition of the kilogram:**
  - The kilogram is the unit of mass scaled such that the Planck constant is exactly 6.626 069 3 x 10^{-34} J s.

Experimental realization of the kilogram (X-ray-crystal-density method)

Deslattes, Henins, Bowman, Schoonover, Carroll, Barnes,
Machlan, Moore, Shields (1974)

International Avogadro Project (2004-2010)

Current interpretation:

  - known mass silicon sphere + volume measurement + lattice spacing measurement + silicon isotopic composition measurement → N_A

- Alternative interpretation:
  - known mass silicon sphere ← volume measurement + lattice spacing measurement + silicon isotopic composition measurement + N_A
- This suggests a possible new definition of the kilogram:
  - The kilogram is the unit of mass scaled such that the Avogadro constant is exactly $6.022\ 141\ 527 \times 10^{23}\ \mathrm{mol}^{-1}$.

Relation between the Avogadro constant N_A and the Planck constant *h*

- **Rydberg constant definition:**

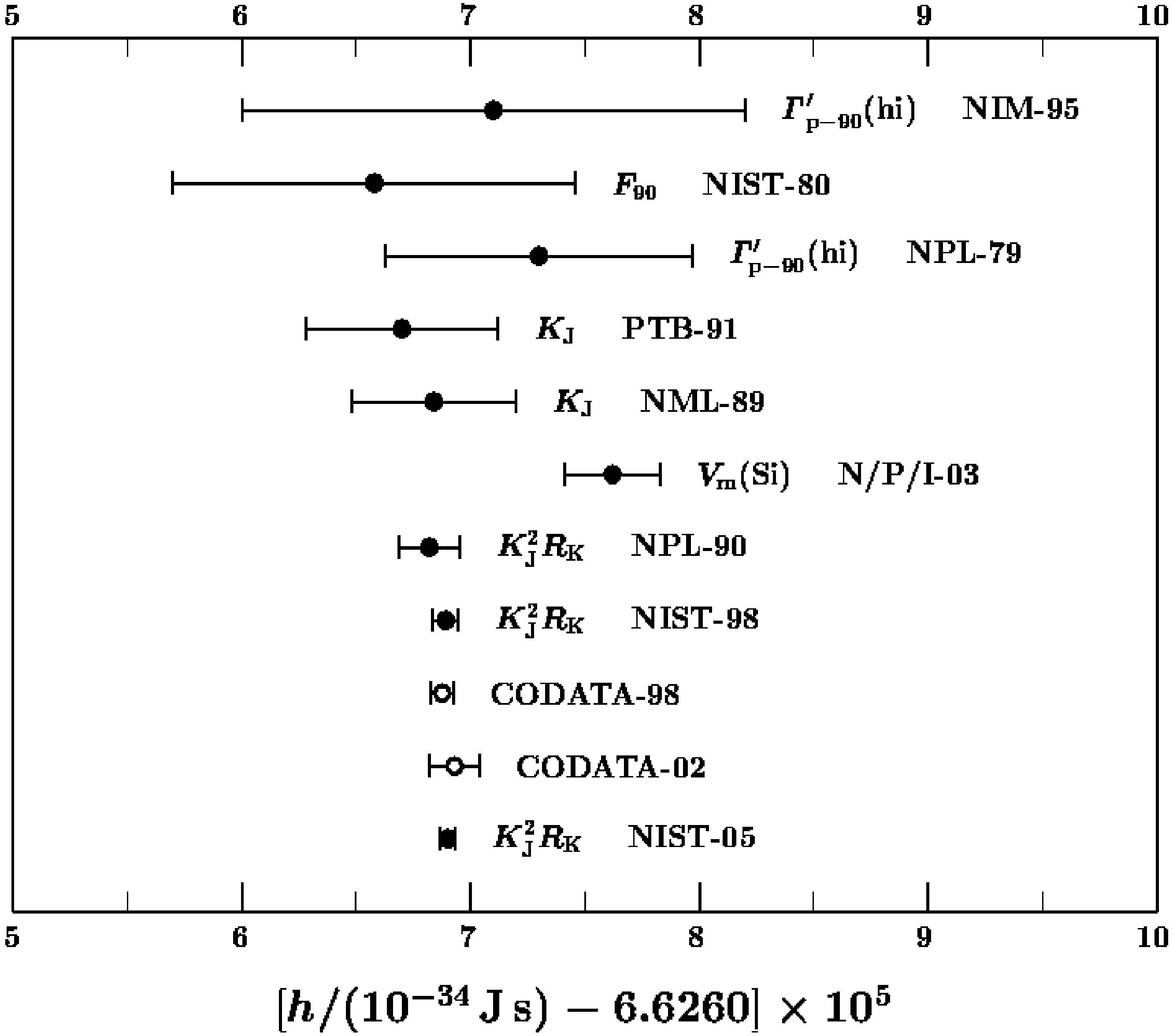
$$R_\infty = \frac{\alpha^2 m_\mathrm{e} c}{2h} \quad \Rightarrow \quad \frac{1}{m_\mathrm{e}} = \frac{\alpha^2 c}{2hR_\infty}$$

- **unified atomic mass unit u:**

$$m_\mathrm{e} = A_\mathrm{r}(\mathrm{e})\ \mathrm{u} \quad \Rightarrow \quad \frac{1}{\mathrm{u}} = \frac{A_\mathrm{r}(\mathrm{e})}{m_\mathrm{e}}$$

- **Avagadro constant:**

$$N_\mathrm{A} = \frac{10^{-3}\ \mathrm{kg/mol}}{1\ \mathrm{u}} = A_\mathrm{r}(\mathrm{e}) \left(\frac{\alpha^2 c}{2hR_\infty} \right)\ 10^{-3}\ \mathrm{kg/mol}$$


5
6
7
8
9
10
$\Gamma'_{\mathrm{p-90}}$(hi) NIM-95
F_{90} NIST-80
$\Gamma'_{\mathrm{p-90}}$(hi) NPL-79
K_{J} PTB-91
K_{J} NML-89
V_{m}(Si) N/P/I-03
$K_{\mathrm{J}}^2 R_{\mathrm{K}}$ NPL-90
$K_{\mathrm{J}}^2 R_{\mathrm{K}}$ NIST-98
CODATA-98
CODATA-02
$K_{\mathrm{J}}^2 R_{\mathrm{K}}$ NIST-05
$[h/(10^{-34}\,\mathrm{J\,s}) - 6.6260] \times 10^5$

Possible ampere redefinition

to make *e* exact

- The ampere is the electric current corresponding to the flow of 6.241 509 468 x 10^{18} elementary charges per second.

or

- The ampere is the electric current scaled such that the elementary charge is 1.602 176 53 x 10^{-19} coulomb.
- Consequences:
  - for ε_0

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \quad \Rightarrow \quad \epsilon_0 \text{ not exact in redefined SI}$$

  - Josephson constant (voltage measurements)

$$K_\mathrm{J} = \frac{2\,e}{h} \qquad \text{(exact in redefined SI)}$$

  - von Klitzing constant (resistance measurements)

$$R_\mathrm{K} = \frac{h}{e^2} \qquad \text{(exact in redefined SI)}$$

Possible kelvin redefinition

to make *k* exact

- The kelvin, the unit of thermodynamic temperature, is scaled such that the Boltzmann constant is exactly 1.380 650 5 x 10^{-23} joule per kelvin.

Possible mole redefinition

to make N_A exact

- The mole, the unit of amount of substance, is scaled such that the Avogadro constant is exactly 6.022 141 5 x 10^{23} per mole.

Effect of new SI on the uncertainties of some fundamental constants

G: $1.5 \times 10^{-4} \rightarrow 1.5 \times 10^{-4} + \varepsilon$

N_A: $1.7 \times 10^{-7} \rightarrow$ exact

m_e: $1.7 \times 10^{-7} \rightarrow 6.6 \times 10^{-9}$

h: $1.7 \times 10^{-7} \rightarrow$ exact

α: $3.3 \times 10^{-9} \rightarrow 3.3 \times 10^{-9}$

m_e (amu): $4.4 \times 10^{-10} \rightarrow 4.4 \times 10^{-10}$

R_∞: $6.6 \times 10^{-12} \rightarrow 6.6 \times 10^{-12}$

m(K): exact $\rightarrow$ parts in 10^9

Some of the other quantities that become exact in the new SI

Faraday constant
molar gas constant
Stefan-Boltzmann constant
Josephson constant
von Klitzing constant
eV-joule conversion factor
Hz-joule conversion factor
kelvin-joule conversion factor

Quantum SI and Unit democracy

- In the Quantum SI, the fundamental constants, c, h, e, k, N_A, v_{Cs}, ... are given fixed numerical values.
- These constants determine the scales of the base units.
- The defined set of base units determines all the units in the SI.
- In the Quantum SI, the fundamental constants, c, h, e, k, N_A, v_{Cs}, ... are given fixed numerical values which determine all the units in the SI.
- The distinction between base units and other units is unnecessary.

The Rydberg constant and the second

- **The Quantum SI uses the constants, c, h, e, k, N_A, ν_{Cs}, ...**
- **Evidently, the second definition is out of character with the rest of the constants which could be called universal (with the possible exception of the Avogadro constant).**
- **The cesium hyperfine frequency, which refers to a particular atom, could be replaced by the Rydberg constant, which is universal.**
- **However, at the moment, the precision of the theory needed to relate the Rydberg constant to measured frequencies is not sufficient to make this change.**

Time scale for possible redefinitions

- At its meeting in October 2004, the International Committee on Weights and Measures (CIPM) asked the Consultative Committee on Units (CCU) to study the possiblity of a fundamental constant-based definition of the kilogram.

- At its meeting in June 2005, the CCU requested that the CIPM approve preparation possible new definitions of the kilogram, ammpere, and kelvin in terms of fundamental constants and also consider redefinining the mole at the same time.

- At its meeting in October 2005, the CIPM approved, in principle, preparation of the new definitions, as requested by the CCU, for possible adoption by the General Conference on Weights and Measures (CGPM) in 2011, provided the results of experiments over the next few years are acceptable.

Conclusion

- The SI can be improved by modernizing the way units are defined.
- In particular, the definitions of the kilogram, ampere, kelvin and mole are based on 19th century science and technology and can be replaced by ones that take into account subsequent progress in physics.
- If an update of the SI is done by specifying values of the fundamental constants as discussed, the concepts of base units and derived units would not be not necessary.
- If there are no persistent problems with experiments, the changes could be made in 2011.