Student Investigations of Forces in a Roller Coaster Loop.

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Abstract. How does the experience of a riding in a roller coaster loop depend on your position in the train? This question has been investigated by first year engineering physics students, using multiple representations of force and motion. Theoretical considerations for a circular loop show that the differences between the forces on a rider in the front, middle or back of the train depend on the ratio between train length and radius of the loop, which can be estimated from a photograph. Numerical computations complement the analysis of a video clip, accelerometer data and measurements of time needed for the train to move over the highest point. A roller coaster ride gives striking examples of Newton's laws applied to your own body, and demonstrates that the experience depends on the vector character of velocity and acceleration.

1. Introduction

Roller coaster queues are often longer for the front or back seat. How do the forces on your body depend on your position in the train? This is one of the questions investigated by students in an amusement park project during an introductory physics course. In this paper, a train's motion over the top (T) of a vertical roller coaster loop is analysed, with examples of challenges taken on by students as extensions of the basic assignments.

A vertical loop (figure 1) is the most basic of roller coaster inversions. The train moves essentially in a vertical plane, upwards, over the top and down again, completing a 360° rotation. At the top, the rider is upside down. Modern roller coaster loops are not circles: Clothoids (Cornu spirals) and other mathematical curves have been introduced to obtain a smaller radius of curvature at the top and to avoid the sudden onset of large forces on the rider [1, 2, 3, 4]. In this paper, the word "loop" will be used as a
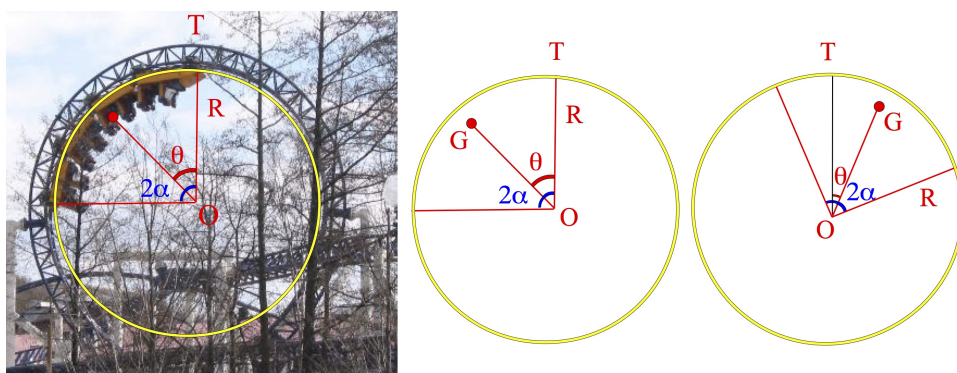
Figure 1. A photo of the vertical loop in the Kanonen roller coaster at Liseberg, where the front of the train of length L has just reached the top at T. Also shown is an approximating circle with origin at O and radius R, and the angle α relating the train length, L, to the radius: $L = 2R\alpha$. The angle θ is introduced to describe the location of the middle of the train in relation to the highest point. The drawings represent the situation in the photo and also a later time when the train has passed the highest point and is moving with increasing speed and angular velocity. G marks the approximate location of the centre of mass of the train.

shorthand for a vertical roller coaster loop. The focus is on the upper part of the loop, approximated by a circular arc.

The examples are presented in some detail and are intended as inspiration and support for teachers who want to use amusement park applications in their teaching. The examples in this paper go beyond the initial observation that the speed will be lowest as the middle of the train reaches T, since the centre of mass is highest at that moment, whereas both the front and the end of the train will move faster at T, and thus have a larger centripetal acceleration. Most results are presented in a form applicable for any train length and radius of curvature of the track.

Studying forces on riders in roller coasters and other amusement rides has been part of an introductory course for engineering physics students at Chalmers during the period 2005-2012. Typically 120 students started every year and were divided in groups of about 6 students who were assigned parts of a roller coaster and one other ride. Preparation and presentations were done in smaller segments of 30 students, where the five groups had different assignments, including forces in a roller coaster loop. For preparation the students were given the photo of the Kanonen roller coaster loop, shown in figure 1, (although without annotations) and a few questions about forces in circular loops together with some data from the technical drawing. As background material they were also provided with a copy of an article about different shapes of roller coaster loops [2] and more details about the ride [5, 6, 7] which opened in 2005. The students were encouraged to suggest additional investigations. The amusement park visit took place at the end of the third week of term. After the visit, students used the data and assignments to practice newly acquired Matlab skills and to write a group report using LaTeX, to be read by an opposition group and presented 2-3 weeks after the visit.

During the visit, students could used their phones to measure the time needed for the train to pass the highest point, T. They could also take photos or video clips for presentation and analysis. Of course, they had an opportunity to ride and experience the forces. WDSS accelerometers [8] were available, including data vests to bring them safely on the rides. (SmartPhone accelerometers and rotation sensors are useful for smaller rides [9], but do not yet have sufficient range for large roller coasters.) A couple of times, three students managed to bring three accelerometers simultaneously in the front, middle and back of the same train. Data were then shared with all groups studying the same roller coaster. In the case of a launched roller coaster, such as the Kanonen [7, 10], the launch provides a well-defined time stamp to bring data from all accelerometers into a single graph, enabling comparisons between the forces experienced by riders in different positions ([10, 11]).

Accelerometer data, time measurements and analyses of photos and videos obtained during the visit, as well as the authentic experiences of the body, can be complemented with the results from a theoretical analysis of the motion represented in a number of different ways, as discussed below. For the analyses in this paper, sideways motions and energy losses have been neglected.

2. Energy conservation and forces on a train in a vertical roller coaster loop

2.1. Short train on a circular track

As an introduction, it is useful to approximate the train with a point particle, moving without energy losses on a circular track with radius R and passing the top at elevation R with speed v_0. Conversion between potential and kinetic energy gives

$$v_\theta^2 = {v_0}^2 + 2gR(1 - \cos\theta) \tag{1}$$

where v_θ is the speed at elevation $R\cos\theta$. Students may need some prompting to see that the mass is not needed for this analysis: When only gravity changes the speed of the train, the mass does not affect the motion, according to the principle of equivalence between gravitational and inertial mass.

After the train has reached the highest point, the speed increases again. For a very short train, the tangential acceleration is given by

$$a_\theta = R\ddot{\theta} = g\sin\theta$$

giving

$$\ddot{\theta} = \frac{g}{R}\sin\theta \tag{2}$$

The force from the track on a short roller coaster train, and the force from the train on the rider, can be approximated by normal forces alone (except in lift hills, launches or brakes). As we shall see below, this is not a good approximation of forces on a rider in the back or front of a longer train.

2.2. Energy considerations and centre of mass for an extended train

Consider a train of length L, approximated by a circular arc with radius R and angle 2α, where $L = 2R\alpha$ (Figure 1). The centre of mass is located a distance

$$d_{cm} = \frac{R}{2\alpha}\int_{-\alpha}^{\alpha}\cos\phi \, d\phi = R\,\frac{\sin\alpha}{\alpha}$$

from the centre of the circle. This is the also elevation of the centre of mass when the train is in the highest position. In the position in the photo where the front has just reached the top, the centre of mass is lower, at

$$h_{cm} = R\,\frac{\sin 2\alpha}{2\alpha}$$

Thus, the centre of mass rises a distance

$$\Delta h = d_{cm} - h_{cm} = R\,(\frac{\sin\alpha}{\alpha} - \frac{\sin 2\alpha}{2\alpha}). \tag{3}$$

This leads to a difference in centripetal acceleration at the top between a rider in the middle and a rider in the front or back of the train.

$$\Delta(\frac{v^2}{R}) = -2g\frac{\Delta h}{R} = 2g(\frac{\sin 2\alpha}{2\alpha} - \frac{\sin\alpha}{\alpha}). \tag{4}$$

(Students can be encouraged to try a series expansion for small angles, and also to discover how a graph of this function can be obtained in Wolfram Alpha [12], by typing e.g. "2sina/a-sin(2a)/a, a from -pi/2 to pi/2" into the search window.) In the case of Kanonen, $\alpha \approx \pi/4$, giving $\Delta(v^2/R) \approx 0.5g$. From this analysis we conclude that the normal force on the rider at the top differs by about $0.5mg$ at the top, depending on position in the train. E.g., if a rider in the front or back of the train is pushed downward at the top of the loop by a force of $0.3mg$ from the seat, a rider in the middle will instead be pushed upwards with a force of $0.2mg$ from the shoulder restraints ("negative g"). Longer trains (larger α) of course lead to even larger differences. For long trains the rider also experiences forces in the direction of the track, as discussed in section 3.2.

3. A long train passing the top of a circular roller coaster loop

How fast does the train move during different parts of the ride? Figure 2 shows an analysis of the motion of the train from a short video clip, used in combination with the Logger Pro software [13]. Below, we consider the equation of motion for the train, analysing the forces on the rider in different situations. A numerical solution of the equation of motion gives the time dependence of the location of the train, which is compared to results from video analysis, accelerometers and time measurements.

3.1. Equations of motion for a roller coasters train on a circular track

After the middle of the train has passed the top, the tangential component of gravity causes an increase of train speed. The tangential acceleration of the train can be obtained by considering the time derivative of the angular momentum, $L = mRv$,
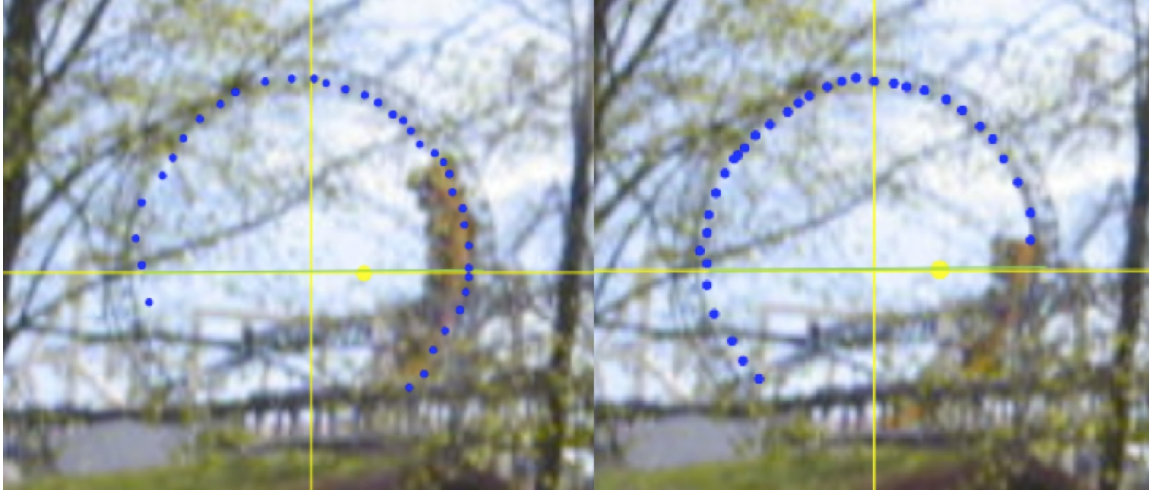
Figure 2. Frame by frame video analysis of the motion of the front (left) and back (right) of the train. Dots were inserted manually for each frame by identifying the front or back of the train in the video clip, using the LoggerPro3 software [13]. Note how the sequence of dots finish at the front or back end of the train.

where m is the mass of the train. Since all parts of the train are moving along the circle, orthogonal to the radius, the magnitude of the angular momentum can simply be written as

$$L = mR^2\dot{\theta} \tag{5}$$

The rate of change in angular momentum can be obtained as $\dot{L} = \tau$, where, τ is the torque exerted by gravity, giving

$$\dot{L} = mR^2\ddot{\theta} = \frac{mR}{2\alpha}\int_{\theta-\alpha}^{\theta+\alpha} g\sin\phi\, d\phi \tag{6}$$

This gives

$$\ddot{\theta} = \frac{g}{R}\frac{\cos(\theta-\alpha) - \cos(\theta+\alpha)}{2\alpha} = \frac{g}{R}\sin\theta\frac{\sin\alpha}{\alpha} \tag{7}$$

which is a modification of the equation (2) for short trains, with an additional factor $\sin\alpha/\alpha$.

From these expressions we can also express the force, $\mathbf{X}$, from the train on the rider in different parts of the train. Applying Newton's second law gives $m\mathbf{g} + \mathbf{X} = m\mathbf{a}$, i.e. $\mathbf{X}/m = \mathbf{a} - \mathbf{g}$. Figure 3 shows free-body diagrams for a rider in the front, middle and back of the train and for two different train positions.

Eq. (7) can be solved numerically to obtain a description of the motion. In figure 4, the position has been marked with crosses for positions at constant time intervals. (We chose the speed for the middle of the train at the top to be $v_0 = \sqrt{gR}$, corresponding to a centripetal acceleration of g, making the rider weightless at the top.) These can be compared to the frame by frame video analysis shown in figure 2.

The numerical solution can also be used to compare the time development of the components of the motion for the different positions, as shown in figure 5.
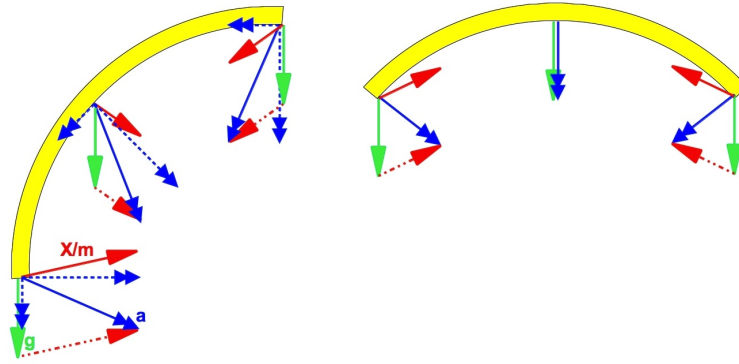
Figure 3. Free-body diagrams of the forces on a rider in the front, middle and back of a roller coaster train. The picture to the left shows the situation where the front of the train is just reaching the top. The picture to the right shows the middle of the train at the highest point. Single arrows mark forces, with gravity always downwards. The acceleration is marked with lines with double arrows and the tangential and normal components of the acceleration are indicated with dashed lines. The force from the train on the rider, expressed as $\mathbf{X}/m = (\mathbf{a} - \mathbf{g})$, is constructed (dashed line) to satisfy Newton's second law: $m\mathbf{g} + \mathbf{X} = m\mathbf{a}$. The speed is chosen to make a rider in the middle of the train weightless at the top.
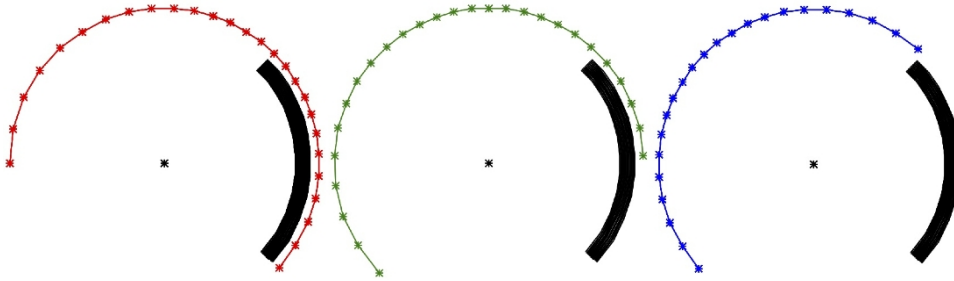

Figure 4. The position of the front, middle and back of the train with marks at equal time intervals. The motion of the middle of the train was obtained by numerical integration of equation (7), giving the angle as a function of time. The corresponding positions for the front and back were obtained by adding or subtracting the angle $\alpha = L/2R$. The sequence of dots finish at at the same time, but at different positions in the train for the three graphs, for comparison with the results from the frame by frame analysis shown in figure 2.

The time dependence of angle for the middle of the train makes it possible to work out how the speed as well as centripetal and tangential accelerations vary with time. At any given time, these are the same for the whole train, but the different angular positions for the different parts of the train leads to differences when combined with the gravitational force, as shown in figure 3. Below, we discuss the resulting accelerometer graphs for long trains.
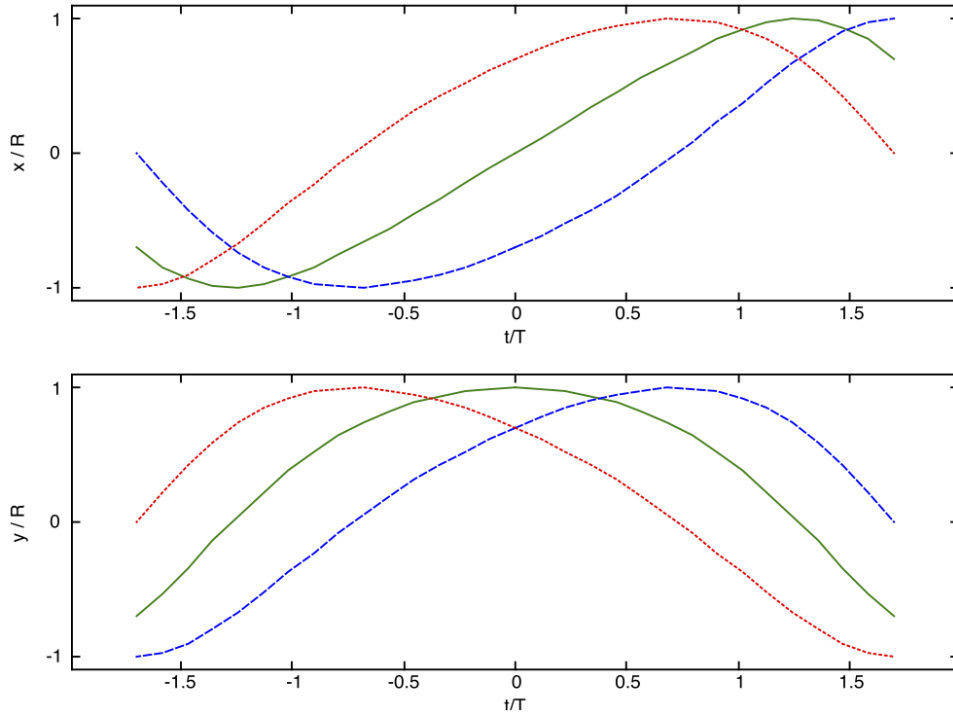
Figure 5. Time dependence of the horizontal (x) and vertical (y) positions for the front, middle and back of the train (the dotted, solid and dashed curves, respectively). The positions are expressed in dimensionless quantities x/R and y/R and the time in terms of the dimensionless quantity t/T where $T = \sqrt{R/g}$ and $t = 0$ corresponds to the time when the middle of the train passes the highest point.

3.2. Accelerometer graphs for different positions in the train

In spite of their name, accelerometers do not measure acceleration, but the "g force", or more precisely, components of the vector $(\mathbf{a} - \mathbf{g})$, often presented in the unit g. If the accelerometer is aligned according to conventions for amusement rides, the z component is directed up from the seat, along the spine of the rider, measuring the normal force X_n/m. The x component should be in the direction of motion, measuring the tangential force, X_t/m. For a short train, the tangential acceleration is equal to the tangential component of the acceleration of gravity, and only the normal component of $\mathbf{X}$ will be non-zero. For a longer train, there will be a small upwards tangential component even for the middle of the train: $X_t = mg \sin\theta(1 - \sin\alpha/\alpha)$, in the direction of motion as the train moves upwards, and opposing the motion on the way down. For the front and back of the train, the differences will be larger, $X_t = mg(\sin(\theta \pm \alpha) - \sin\alpha/\alpha)$. The normal force on a rider in the middle of the train is given by

$$X_n = m(\frac{v_0^2}{R} + 2g(1 - \cos\theta)\frac{\sin\alpha}{\alpha}).$$

For the front and back, we get instead

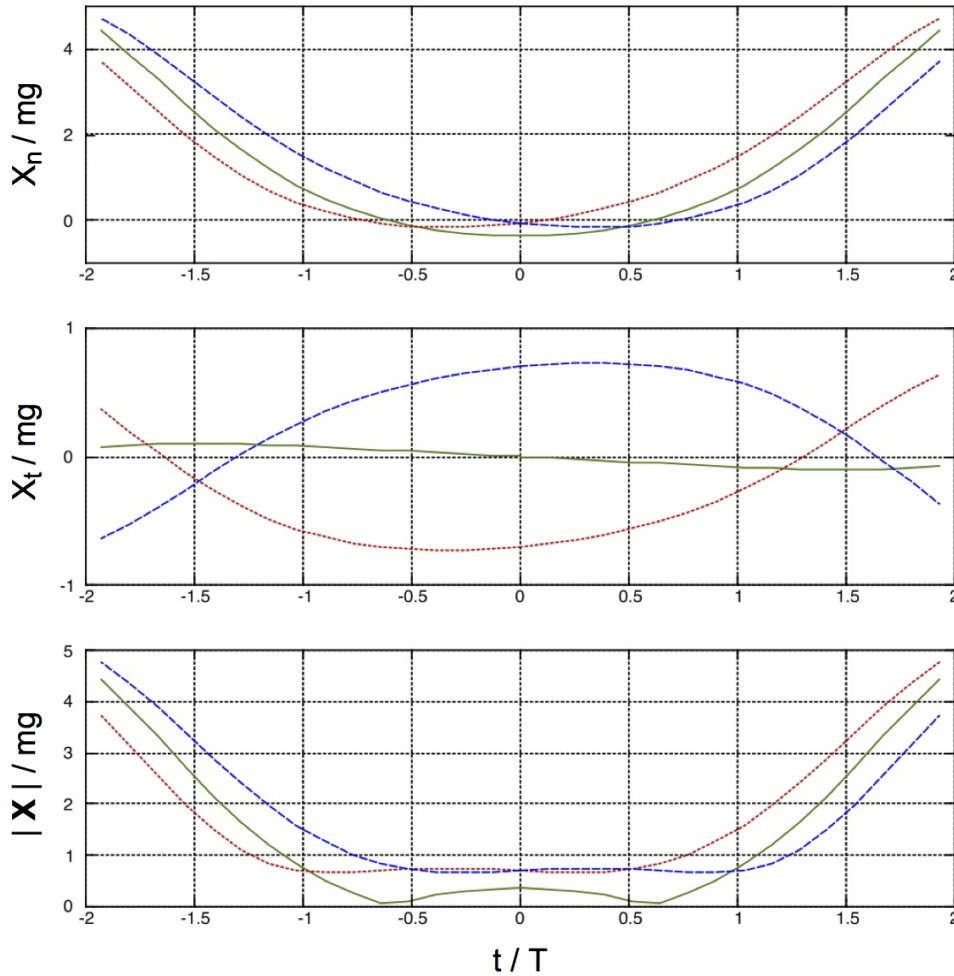$$X_n = m(\frac{v_0^2}{R} + 2g(1 - \cos\theta)\frac{\sin\alpha}{\alpha} + g\cos(\theta \pm \alpha)).$$

Figure 6. Time dependence of the force $\mathbf{X}$ from the train on the rider, divided by the force of gravity, mg. The normal component X_n is defined as positive in the direction from the seat towards the head of the rider and the tangential force, X_t, is positive in the direction of motion. The third graph shows the absolute value, $|\mathbf{X}|/mg$. The time has been expressed in terms of the dimensionless quantity $T = t/\sqrt{R/g}$. The solid curve corresponds to the middle of the train, whereas the dotted and dashed curves give the forces for riders in the front and back of the train. The speed at the top for the middle of the train was chosen to be $v_0 = 0.8\sqrt{gR}$, resulting in "negative g" on the top.

as shown in figure 3 for two different positions.

Perfect alignment of accelerometers taken on a ride is not easily accomplished. Sometimes, the dominating z component from a reasonably aligned accelerometer includes most of the interesting information. Alternatively, the absolute value $|\mathbf{X}|/m$ can be evaluated, although this obviously hides any "negative g", that may be visible in the z component. Figure 6 shows the theoretical values for the tangential and normal forces for a rider in different part of a train passing over the top of the loop, as well as the absolute value.
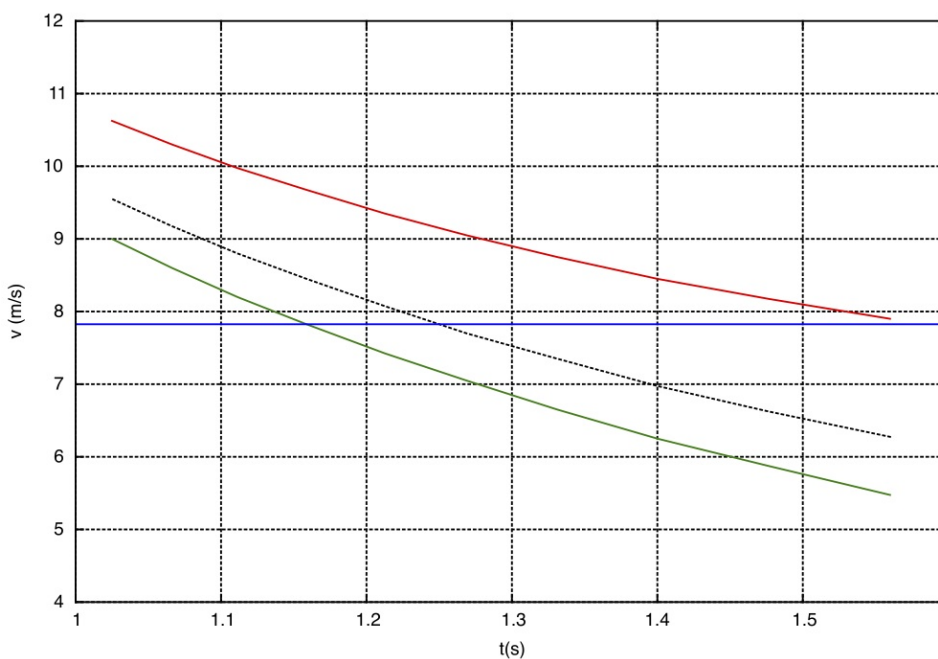
Figure 7. The relation between the speed and the time, t, taken for the Kanonen train to move over the top of the loop. The lowest curve shows the speed in the middle of the train. The upper curve shows the speed when the front or back of the train passes the highest point. The dotted curve in the middle shows the average speed, L/t, during the passage. The horizontal line marks the speed which gives a centripetal acceleration equal to g, taking the radius to be $2L/\pi$, where $L = 9.784m$.

The results above are presented in a form that could be used with any loop, where the train covers about 90° of a circular arc. The equations are, of course, valid for any train length.

In the next section we compare theoretical results with observations and data for the Kanonen roller coaster.

3.3. On the top of the Kanonen loop

By measuring the time, t, needed for the train to move over the top, an estimate of the speed can be obtained if the train length, L, is known or measured. For the Kanonen train, $L = 9.784$m. All students in a group have been asked to bring out their mobile phones and measure t. Working out averages and standard deviations, they may also consider if they can detect any possible difference between the trains, within their measurement uncertainty. The groups typically get average times, t, between 1.1s and 1.2s, giving speeds, $L/t \approx 8.5$m/s $= 31$km/h, consistent with data from the technical drawing. The speed required for weightlessness, $v_0 = \sqrt{gR}$ can be estimated using $R \approx L/\pi/2 = 6.2$m, giving $v_0 \approx 7.8$m/s.

Apart from uncertainties in the timing, the resulting speed will be an average, overestimating the speed for the middle of the train and an underestimating the speed
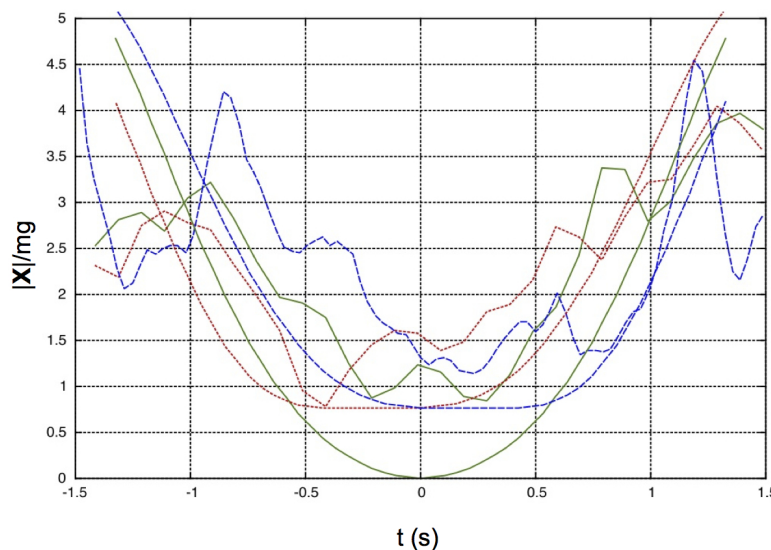
Figure 8. Experimental and theoretical data for the total force on the rider from the train passing the loop in Kanonen, in the front (dotted), middle (solid) or back (dashed) of the train. Vibrations in the roller coaster train leads to considerable noise in the experimental data.

for the front and back. A few of the student groups who were assigned the Kanonen ride took on the additional challenge of working out how the measured time, t, can be converted to a value for the speed of the middle of the train at the highest point. Figure 7 shows the relation between the time, t, and the actual speed of a rider in the middle of a train. For comparison, we have also included the average speed L/t, as well as the speed for the front or back of the train, obtained by inserting the expression (3) for Δh giving

$$v_{end} = \sqrt{v_{middle}^2 + 2gR(\frac{\sin \alpha}{\alpha} - \frac{2\sin \alpha}{2\alpha})}.$$

From the graph in figure 7 we note that a time of 1.2s to move over the top corresponds to a speed too low for weightlessness in the middle of the train, leading to "negative g", whereas 1.1s gives a centripetal acceleration larger than g at all times.

Forces on the rider can be measured with accelerometers taken on the ride. Figure 8 shows accelerometer data taken by three students in September 2012, riding front, middle back on the same Kanonen train. The launch at the start provides a well defined time for synchronization. Theoretical accelerometer values have been included for comparison. The rapidly varying motions in roller coasters cause measured data to be quite noisy, as can be seen from the graphs. Discerning the relevant differences is not always easy. To know the feeling as your body experiences the near-weightlessness at the top of the loop you have to go on the ride and compare the different positions.

4. Results and Discussion

A photo of a roller coaster loop combined with a knowledge of the length of the train offers many possibilities for investigations of forces, even before riding. The mass of the train is not needed for the equations of motion in the loop, as long as energy losses are neglected. The analysis of forces on a rider shows that weightlessness can only be obtained in the middle of the train. Although loops offer particular simplification through the approximate circular shape, similar investigations can be extended to other roller coaster elements. Different ways to investigate the motion offer a number of qualitatively different representations. Marton and Booth [14] emphasize the importance of this variation for the development of deeper understanding, transferrable to other situations. Airey and Linder [15] have studied how students acquire a disciplinary discourse involving a complex of representations, tools and activities of the discipline.

The engineering physics programme at Chalmers is highly competitive, and attracts some of the best students leaving high school. The Force Concept Inventory [16, 17] has been administered before the start of the course, and the cohorts then consistently score around 80%. Post tests scores have been around 90%, corresponding to a normalized gain of 50% [17, 18]. Other tests for this student group, administered in connection with project presentations, have included comparisons between forces in a roller coaster loop and in the classical Rainbow ride (with nearly uniform circular motion and weightless riders on top) [19], requiring students to discern similarities and differences between the two types of circular motion in a vertical plane. The follow-up of these questions leads to many clarifying discussions and to new insights for many students.

Using roller coasters and other amusement rides in physics teaching brings an enjoyable connection between the experiences of your own body, Newton's laws and motion in two and three dimensions, and contributes to a deeper student understanding of the vector character of force, velocity and acceleration.

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