Acceleration and rotation in a pendulum ride, measured using an iPhone 4.

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Abstract. Many modern cell phones have built-in sensors that may be used as a resource for physics education. Amusement rides offer examples of many different types of motion, where the acceleration leads to forces experienced throughout the body. A comoving 3D-accelerometer gives an electronic measurement of the varying forces acting on the rider, but a complete description of a motion, also requires measurement of the rotation around the three axes, as provided e.g. by the iPhone 4. Here we present and interpret accelerometer and gyroscope data that were collected on a rotary pendulum ride.

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Figure 1. The Rockin' Tug family ride combines a pendulum and circular motion. The little tug moves back and forth along a circular rail and at the same time rotates along its own axis orthogonal to the rail.

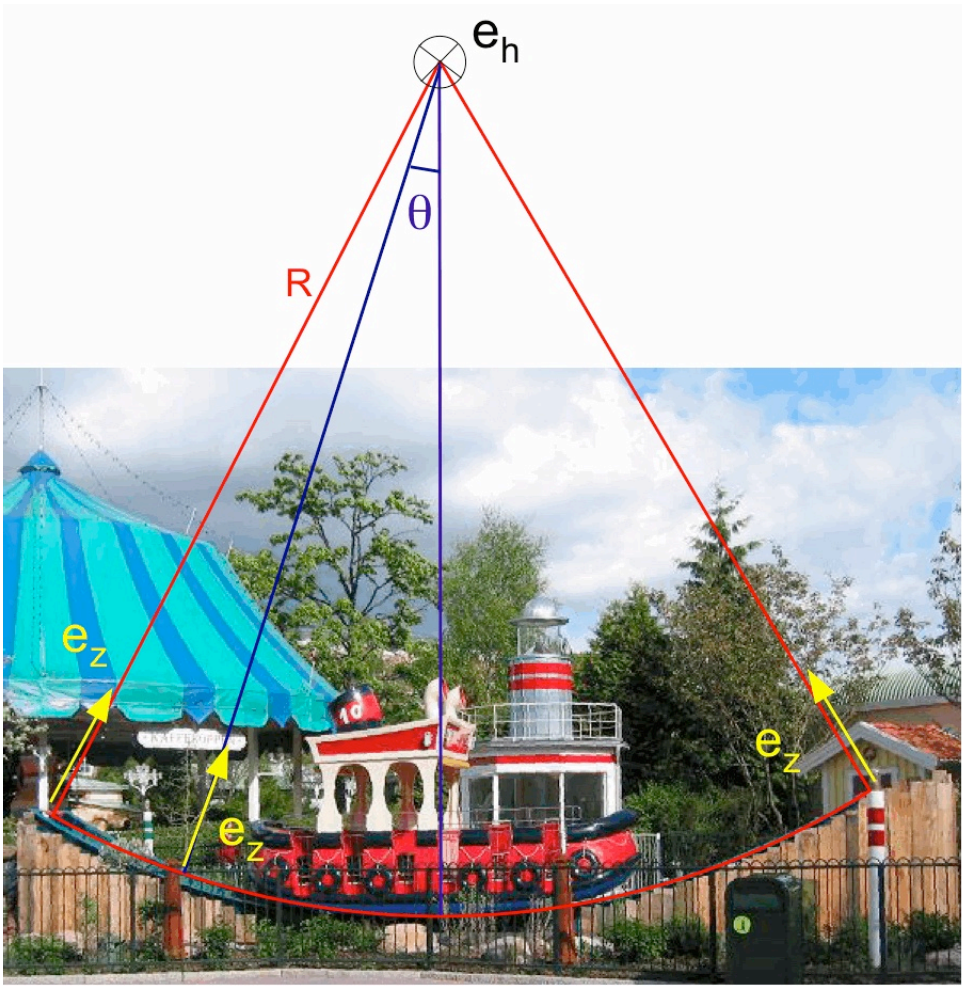

Figure 2. The radius, R, of the circular arc of the rails (red). The blue lines mark the angle θ to the vertical at particular point of the track. The yellow arrows mark the direction of the z-axis as it varies with the motion along the track. The horizontal axis of rotation, e_h, for the pendulum motion is shown at the centre of the circle, at the top of the figure.

1. Introduction

Inertial motion tracking in three dimensions requires accelerometer and gyroscopic data around three axes. The ability to track motion has many applications [1], including movie making, crash analysis, sports, virtual reality - and games. The iPhone 4 offers a user-friendly interface to a motion tracking MEMS (MicroElectroMechanical System) sensor [2], offering new possibilities not only for games but also for the physics classroom, where the data collection should be combined with an analysis of the motion studied.

Amusement parks are among the most favourite school trips. In an amusement park the visitor may experience many examples of acceleration and rotation, from simple children's carousels rotating around a vertical axis, to looping and twisting roller-coasters, with changing acceleration and rotation in three dimensions. The rides can be studied at many different levels of difficulty and involvement, from observation of motion and the interplay between kinetic and potential energies in pendulum rides and roller coasters, to electronic data collection and detailed analysis of the motion in the rides.

The pendulum is a classic textbook example, which can be studied using ordinary playground swings [3] and also in many amusement rides. In this paper we focus on a pendulum ride, with an added rotation: the "Rockin' Tug" family ride from Zamperla et al [4]. As seen from figures 1-2 the pendulum string is replaced by a rail, which is known to have a radius of curvature of R=11.5m. The boat moves along the circular rail and can also rotate around its own axis with a rotation speed 11 turns per minute according to specifications. How do the pendulum and rotation motions combine? What are the resulting forces on the rider - and how can we understand the data from the iPhone, collected using the free app SensorLog? The motion is relatively simple and can be described mathematically. Still, the combination of pendulum motion and rotation around an additional axis leads to non-zero results for all axes, both for the accelerometer and gyroscopic sensors, as discussed in this work.

2. The pendulum motion

During the ride the Iphone was held in place on the seat of the ride, inside a closed pocket. The resulting accelerometer data are shown in figure 3. Since the the sensors move along with the rider, the directions of the axes change all the time, with the z axis pointing up from the seat. Figure 2 shows the ride and the radius of the circular arc of the rail, which takes the place of the string in a pendulum. If maximum angle of the pendulum is θ_0, the time dependence of the angle in the pendulum motion can be written as $\theta(t) = \theta_0 \cos pt$ with $p^2 = g/R$ (using the standard approximation valid for small angles). The vertical accelerometer data in the top graph of figure 3 shows a periodic variation, corresponding to half a period of the pendulum motion. It is consistent with the period $T = 2\pi\sqrt{R/g} = 6.8$s for a mathematical pendulum of length R= 11.5m.

It should be noted that an accelerometer does not measure components of acceleration, but of the vector $\mathbf{a} - \mathbf{g}$, where $\mathbf{a} = \mathbf{F}/m$ is the acceleration. The expression $\mathbf{a} - \mathbf{g} = (\mathbf{F} - m\mathbf{g})/m$ corresponding to the force per kilogram *from the ride* on the rider. The acceleration along the rail is caused only by gravity and gives no contribution to the accelerometer data. For a pure pendulum ride only the "vertical" axis would show non-zero results. The "vertical" component of the accelerometer (i.e. the component along the co-moving z-axis in figure 2) depends on the angle θ and on the maximum angular displacement, θ_0. At the turning point, the vertical component will be $g \cos\theta_0$ and as the pendulum passes the lowest point it will be $g\,(3 - 2\cos\theta_0)$. (The interpretation of accelerometer data from a swing is discussed in more detail in [3].) From the vertical accelerometer data in figure 3, we can conclude that the maximum angular displacement θ_0 of the pendulum motion is about 25^o. The longitudinal and lateral accelerometer

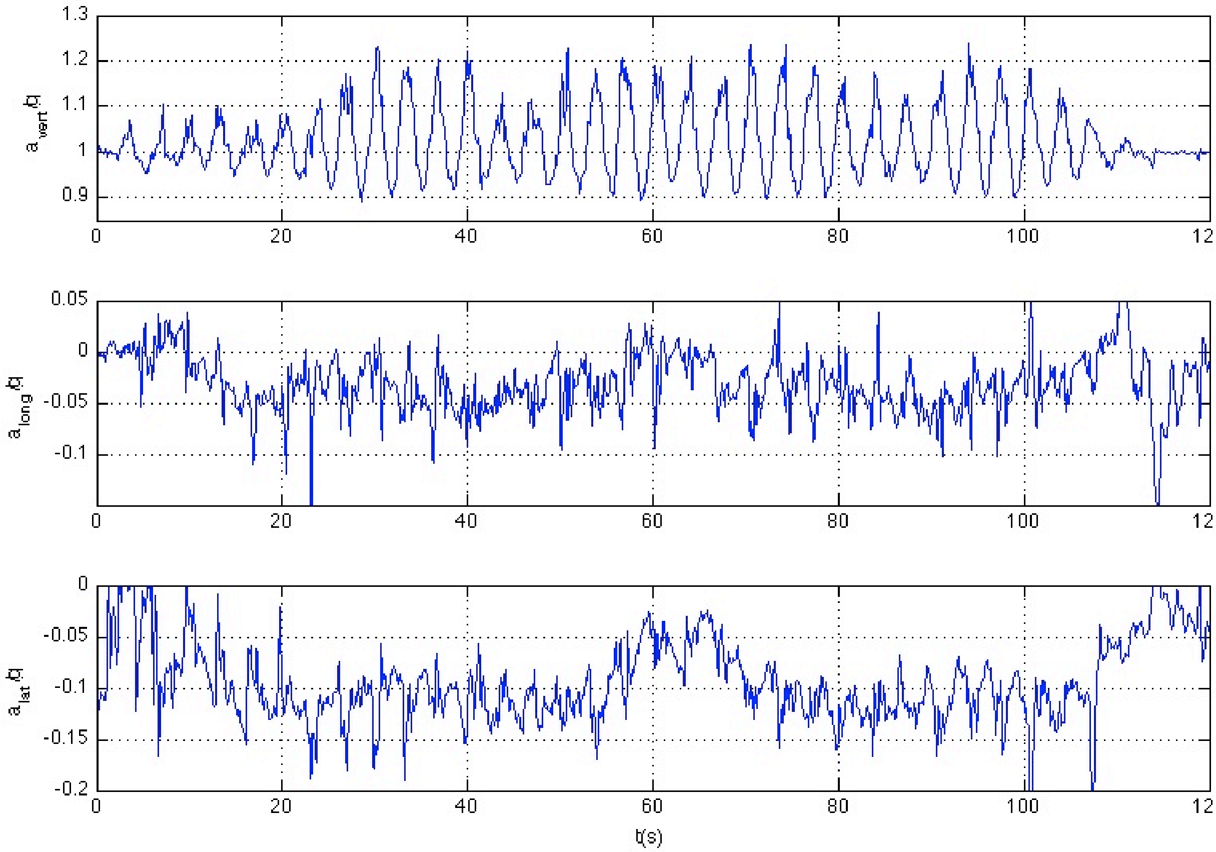

Figure 3. Accelerometer data from the iPhone4, which was held in place on the seat of the ride. The "vertical" z-axis points up from the seat. The sensor was placed with the "longitudinal" x-axis pointing to the back of the rider and the "lateral" y axis pointing to the right.

components in Figure 3 arise from the rotation around the "vertical" axis, discussed in section 3.

3. The rotation of the boat

After a few oscillations, the little tug starts to rotate as shown in figure 1. The rotation of the boat leads to a centripetal acceleration of the rider towards the centre of the boat, depending on the distance to the centre of rotation and on the angular velocity, Ω. Figure 4 shows the coordinates for a rider in the moving x-y coordinate system attached to the boat. The centripetal accelerations in x and y directions, corresponding to the "longitudinal" and "lateral" accelerations in figure 3, will thus be $a_x = -b\Omega^2$ and $a_y = -c\Omega^2$ (using the coordinates from figure 4). The specified rotation of 11 turns per minute corresponds to an angular velocity of Ω =1.15 rad/s. The accelerometer data

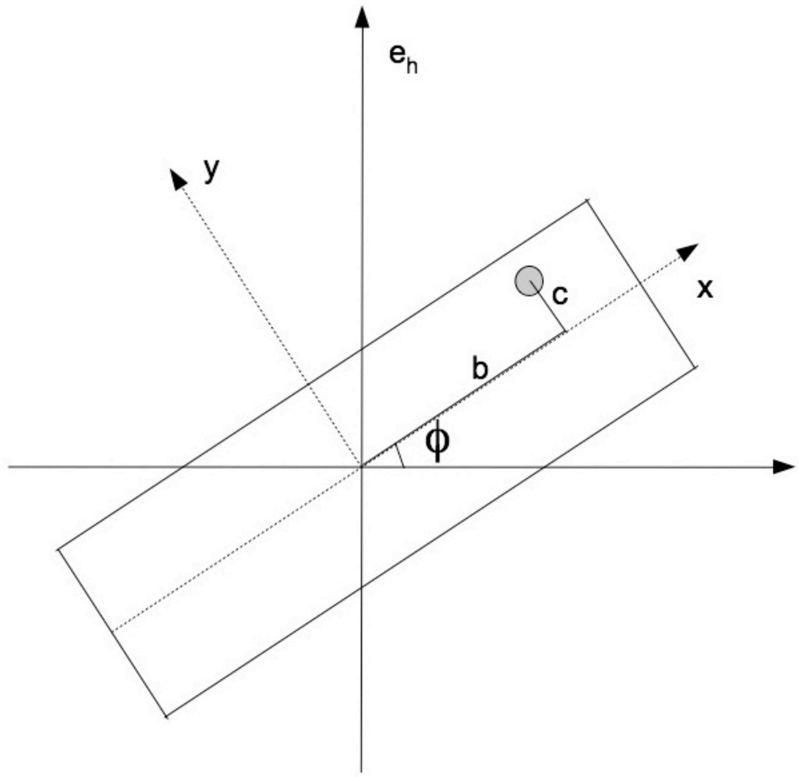

Figure 4. Coordinate system used to describe the motion of the rider relative to the centre of the boat, as seen from above, looking in the negative z direction. The fixed axis of rotation for the pendulum motion, seen also in figure 2, can be expressed as $e_h = \sin\phi\ e_x + \cos\phi\ e_y$. The angle ϕ changes as the tug rotates around the z axis.

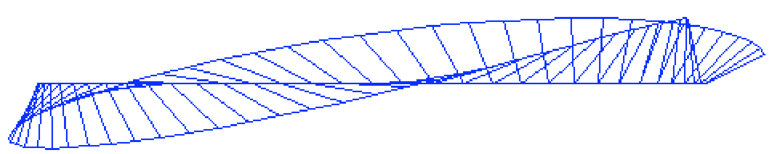

Figure 5. The straight horizontal line in the graph represents the motion of the centre of the Rockin' Tug ride. The superimposed circular motion, seen from above, is illustrated by the lines going out from the position of the centre of the boat, at 0.1s time intervals. The resulting curve, connecting the outer points of these lines, represents the path of the rider through one period of the pendulum motion.

(averaged over the times 20-50 s and 70-100 s) from figure 3 indicate that the sensor was placed at a point located b=0.8m in the x direction and c=0.3m in the y-direction away from the centre.

How does the rider move when the circular motion is added to the pendulum motion? Figure 5, illustrates the motion during one pendulum period starting at the highest point. The shape of the combined pendulum and circular motion depends on

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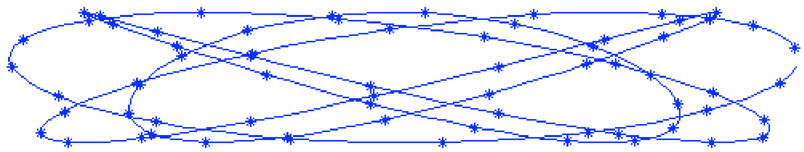

Figure 6. The figure shows the path of a rider through a few oscillations and turns, seen from above. The path starts and finishes at the upper right part of the figure. An asterisk marks the position every 0.4s.

the relation between the periods for the pendulum and the circular motion. Figure 6 shows the combined motion for four pendulum periods which corresponds to five circular motions. (The little gap left in the upper right part of the figure marks the start and finish of the drawing.) Closed orbits result when the ratio is a rational number, as in this case.

4. Angular velocities during the combined motion

The analysis above uses only the accelerometer data in three dimensions. These can be obtained using many types of equipment, including the Vernier WDSS sensor [5, 6] and many modern mobile phones. The possibility to record also the rotation offers new possibilities, including "motion tracking" [7], and is now available e.g. with the iPhone 4, used for the measurements presented here. Figure 6 shows the time dependence of the angular velocities around the axes of the comoving sensor. The rotation is measured around the three axes relative to the body, and are often referred to as yaw (around the "vertical" z-axis), pitch (around the "lateral" y-axis) and roll (around the "longitudinal" x axis). The yaw data shows the rotation of the boat, increasing to a constant angular velocity and then turning around to rotate in the other direction. The data are consistent with the specified rotational speed of 11 turns/minute.

The angular velocities for pitch and roll exhibit more complicated patterns. They should account for the rotations corresponding to the pendulum motion around a fixed horizontal axis, $e_h = \sin\phi \ e_x + \cos\phi \ e_y$, shown in figures 3 and 4. The angle of the rotating x-y coordinate system in figure 4 can be written as $\phi = \Omega t$. The time dependence of the angular velocity associated with the pendulum motion can be written as $\omega_h = \omega_0 \sin pt$. However, due to the rotation of the boat (and sensor), this angular velocity corresponding to this rotation, has both x and y components (figure 4), giving

$$\omega_x = \omega_0 \sin pt \sin\phi = \omega_0 \sin pt \sin \Omega t$$
$$\omega_y = \omega_0 \sin pt \cos\phi = \omega_0 \sin pt \cos \Omega t$$

Using the properties of the trigonometric function, these expressions can be rewritten as

$$\omega_x = \frac{\omega_0}{2} \left[\cos(p - \Omega)t - \cos(p + \Omega)t\right]$$

$$\omega_y = \frac{\omega_0}{2}\left[\sin(p-\Omega)t + \sin(p+\Omega)t\right]$$

The combination of the two periodic motions results in an oscillation with a larger angular frequency $\Omega + p$ added to an oscillation with a smaller angular frequency $\Omega - p$, in this case corresponding to the periods 27.5 s and 3.0 s, respectively. These periods are clearly visible in the rotation data in figure 7.

The total horizontal angular velocity is the vector sum of these two rotations, with a modulus $|\omega_h| = \sqrt{\omega_x^2 + \omega_y^2}$ shown in bottom graph of figure 8, where the top graph shows the angular velocities around the x and y axes.

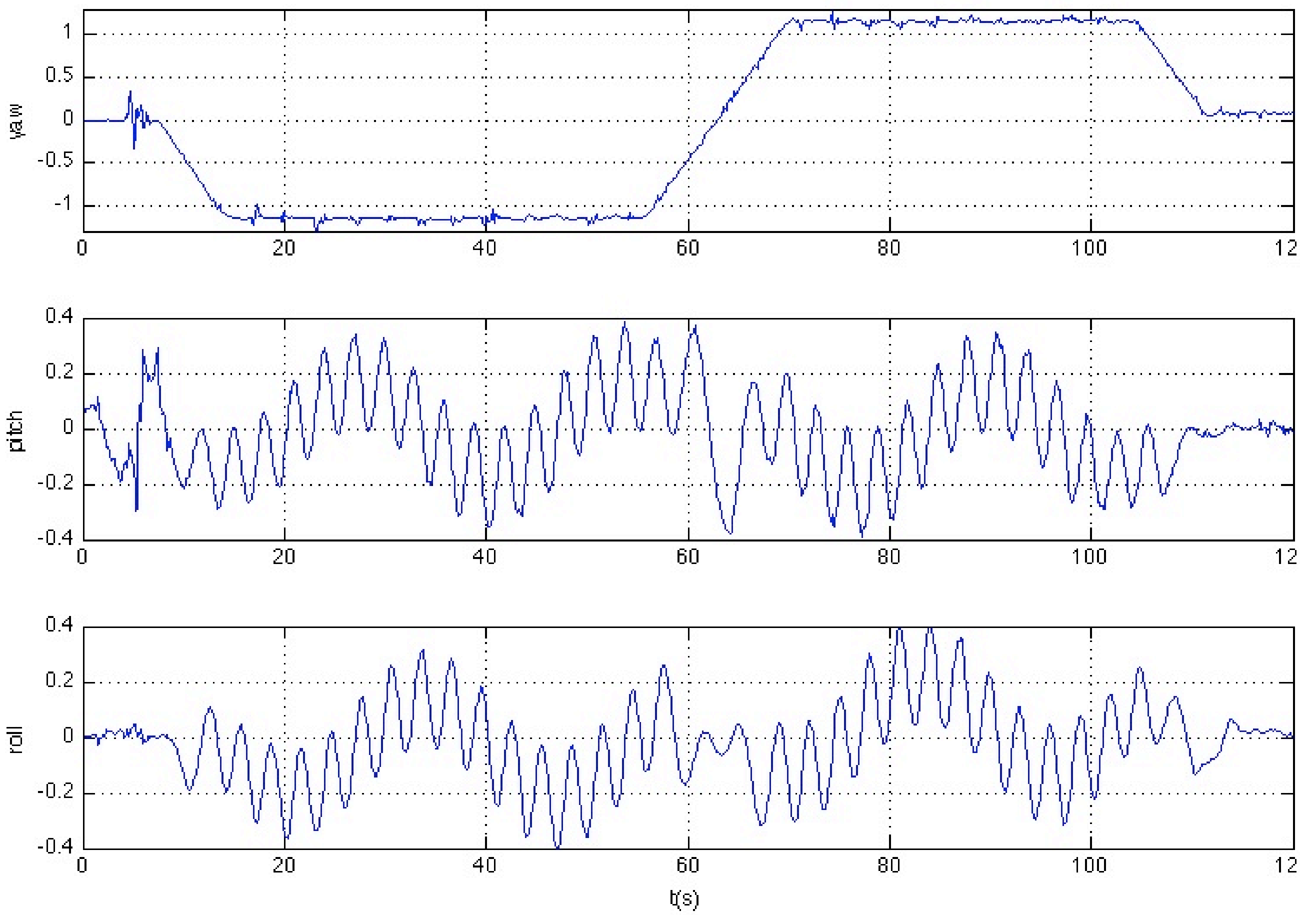

Figure 7. Measured angular velocities around the three axes of the iPhone4 during the ride.

5. Discussion

The analysis of the acceleration and rotation data for the Rockin' Tug ride shows the richness of mathematics and physics examples exhibited already in a simple family ride. Many amusement parks feature similar rides in various sizes. The accelerometer

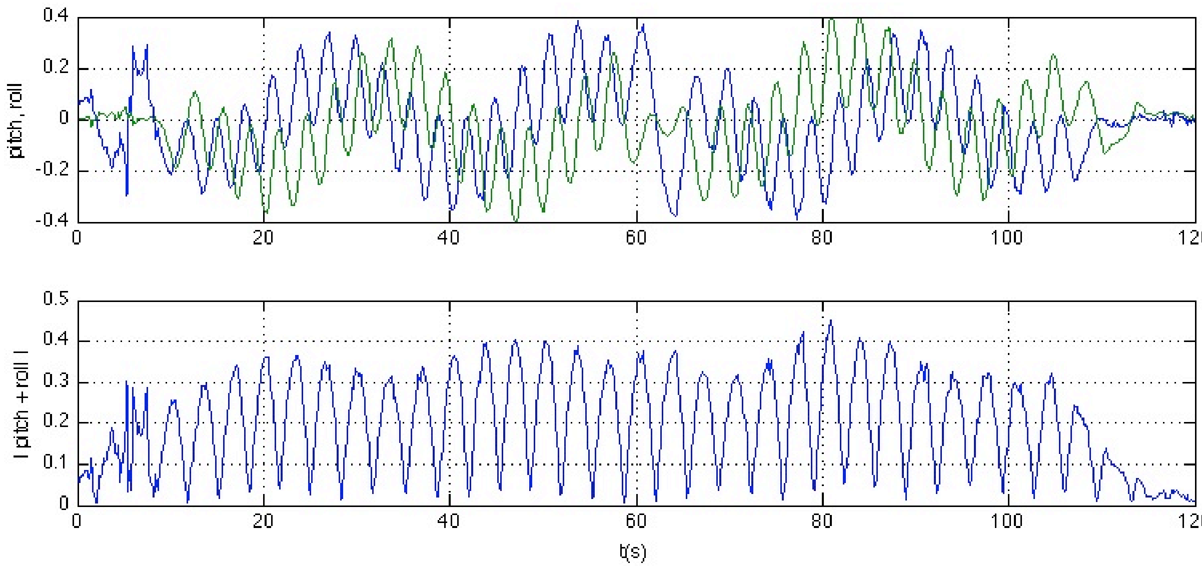

Figure 8. The "roll" and "pitch" rotations around the moving x and y axes in figure 4 are shown together in the top graph. Together they make up the rotation around the horizontal axis in the pendulum motion, and the bottom graph shows the magnitude of the vector sum in radians/s.

of the iPhone is limited to about 2g, which is insufficient for many rides. (The gyro can measure up to 2000^o/sec, which does not pose a limitation for measurements in amusement rides.) However, an advantage of using a smaller ride is that it is easier to place the sensor so that the axes point in the desired directions. Had this not been the case, it would also have been necessary to perform an initial transformation of the coordinates. The access to rotation data, made possible e.g.by the iPhone 4, offers illuminating examples of rotations around different axes.

Detailed analyses of rides are suitable tasks for group projects in connection with amusement park visits. Amusement rides can also take the role as previous shared experiences in combination with data already available. (The dataset used in this paper is available on request, as are data for many other rides of different types.) The students may also e.g.write small programs to animate the motion of the rider, or synchronize data with video recordings of a ride. We have found that assigning 2-3 rides to groups of 4-6 students is a suitable format leading to many challenging discussions of the physics involved. The learning is enhanced if the groups are required to write a report and present it orally to the other groups in the class, with different ride assignments. Asking each group to read the report of at least one other group and to prepare questions and feedback, invites additional reflection on their own work. The connection between the experience of forces on and in the body and the mathematical description of motion provides additional aspects and a can lead to a deepended understanding of classical mechanics and its relevance outside the classroom.

Acknowledgements

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