Nanophysics: from fundamentals to applications
XIIIth Rencontres du Vietnam
Quy Nhon, August 2017

Robustness of symmetry-protected topological states against time-periodic perturbations

Oleksandr Balabanov and **Henrik Johannesson**

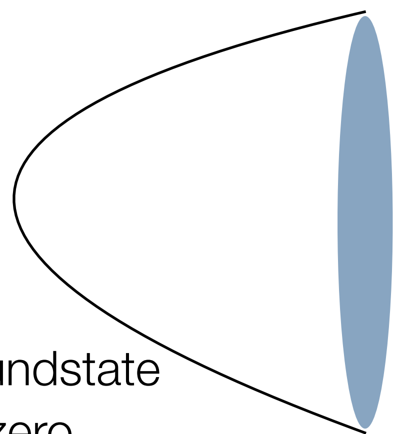
UNIVERSITY OF GOTHENBURG

Phys. Rev. B **96**, 035149 (2017) [arXiv:1704.00782]

supported by Vetenskapsrådet

Background and motivation...

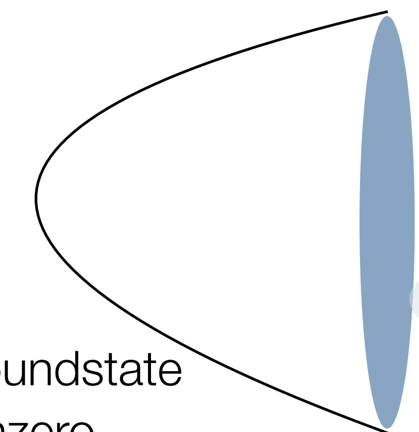
Symmetry-protected topological states



gapless boundary states

gapped bulk with groundstate characterized by a nonzero topological invariant

Symmetry-protected topological states



the boundary states are robust against symmetry-preserving local perturbations which do not close the bulk gap

gapless **boundary** states

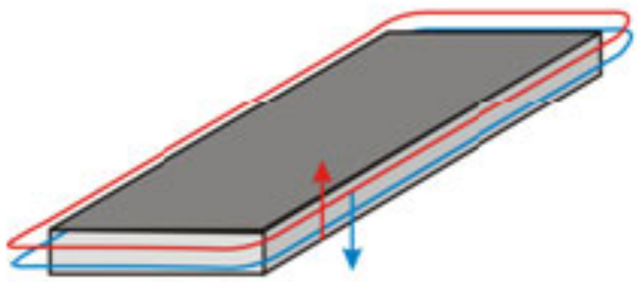
gapped **bulk** with groundstate characterized by a nonzero topological invariant

"bulk-boundary correspondence"

L. Fu and C. L. Kane, Phys. Rev. B **74**, 195312 (2006)

Symmetry-protected topological states

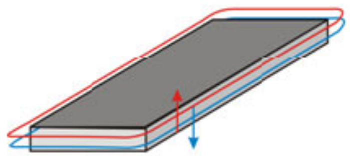
examples



Quantum spin Hall system:
helical edge states protected by time-reversal symmetry; Z_2 topological invariant

Symmetry-protected topological states

examples



Quantum spin Hall system:
helical edge states protected by time-reversal symmetry;
Z_2 topological invariant

1D topological superconductor:
Majorana zero-energy boundary modes protected by particle-hole symmetry;
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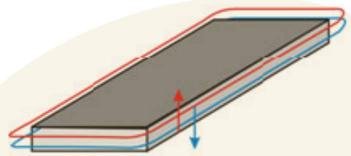
Symmetry-protected topological states

examples



Quantum spin Hall system:
helical edge states protected by time-reversal symmetry;
Z_2 topological invariant

1D topological superconductor:
Majorana zero-energy boundary modes protected by particle-hole symmetry;
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… and many more!

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A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

A. P. Schnyder *et al.*, Phys. Rev. B **78**, 196125 (2008)
A. Kitaev, AIP Conf. Proc. 1134, **22** (2009)

What about symmetry protection against *time-dependent* local perturbations?

What about symmetry protection against *time-periodic* local perturbations?

… can be addressed using Floquet theory!

Floquet theory (Gaston Floquet, 1847-1920)

applied to a time-periodic Hamiltonian

$$H(t) = H(t + T)$$

”Floquet states”:

$$|\psi_n(t)\rangle = \exp(-i\varepsilon_n t)|u_n(t)\rangle$$

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eigenstates of the time-evolution operator

$$U(t, t+T)|u_n(t)\rangle = \exp(-i\varepsilon_n T)|u_n(t)\rangle$$

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"quasienergies"
defined mod $2\pi/T$

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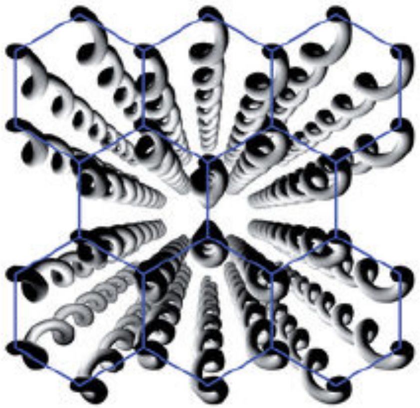
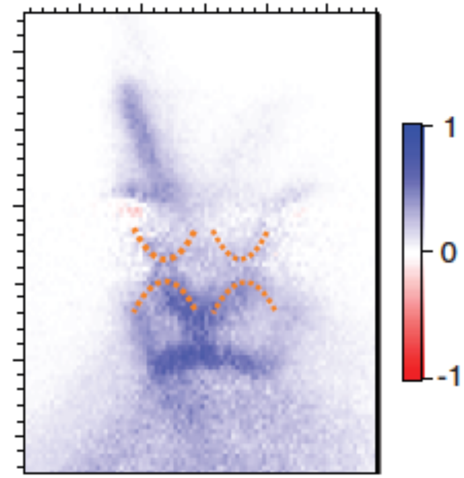
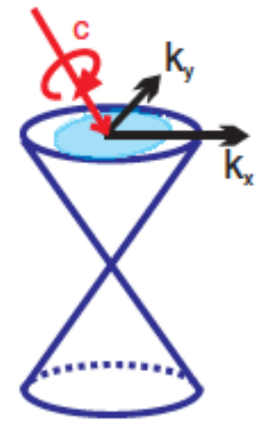
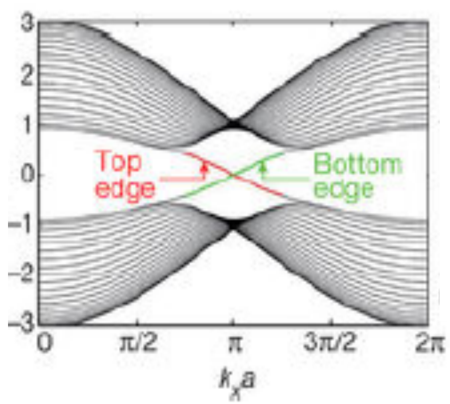
Nontrivial topological structure of quasienergy spectra:
"Floquet topological insulators/superconductors"

T. Kitagawa *et al.*, Phys. Rev. B **82**, 235114 (2010)
N. H. Lindner *et al.*, Nature Phys. **7**, 490 (2011)









Observation of Floquet-Bloch States on the Surface of a Topological Insulator

Y. H. Wang,* H. Steinberg, P. Jarillo-Herrero, N. Gedik†

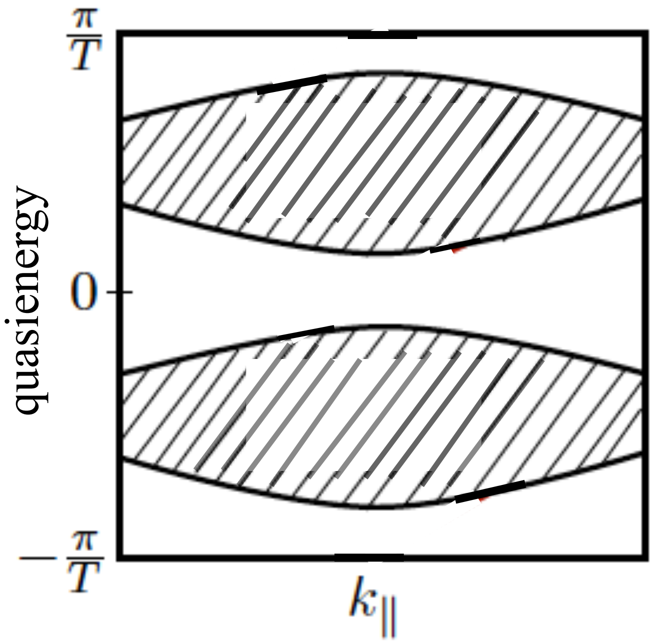
SCIENCE VOL 342 25 OCTOBER 2013 **453**

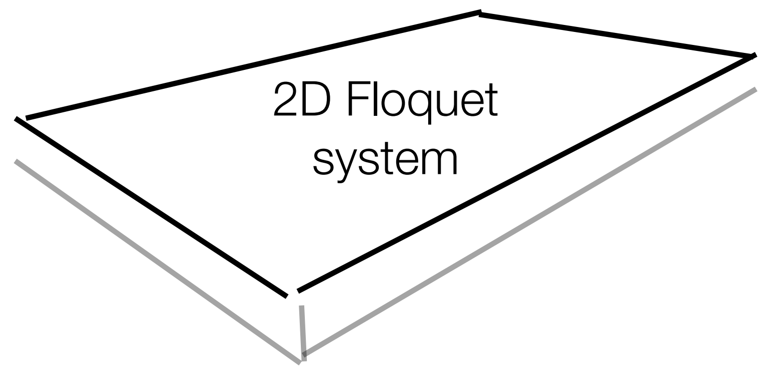
Photonic Floquet topological insulators

Mikael C. Rechtsman, Julia M. Zeuner, Yonatan Plotnik, Yaakov Lumer, Daniel Podolsky, Felix Dreisow, Stefan Nolte, Mordechai Segev & Alexander Szameit

Nature **496**, 196–200 (11 April 2013) doi:10.1038/nature12066

Nontrivial topological structure of quasienergy spectra:





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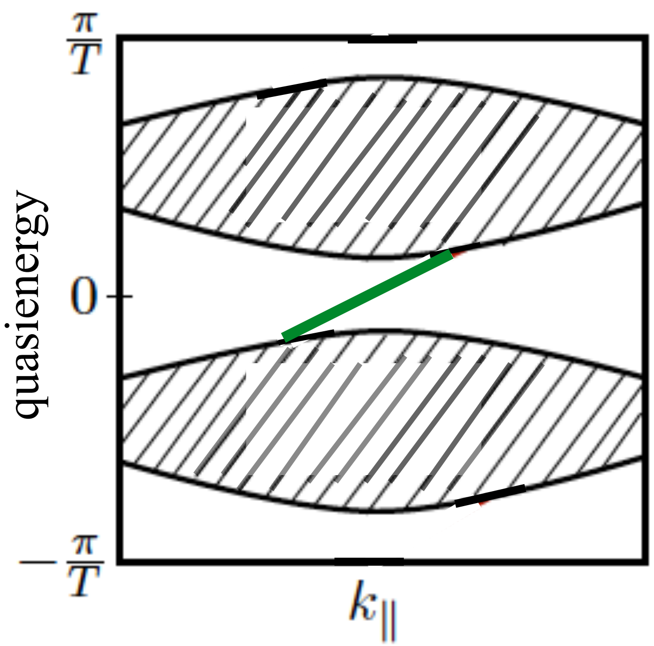

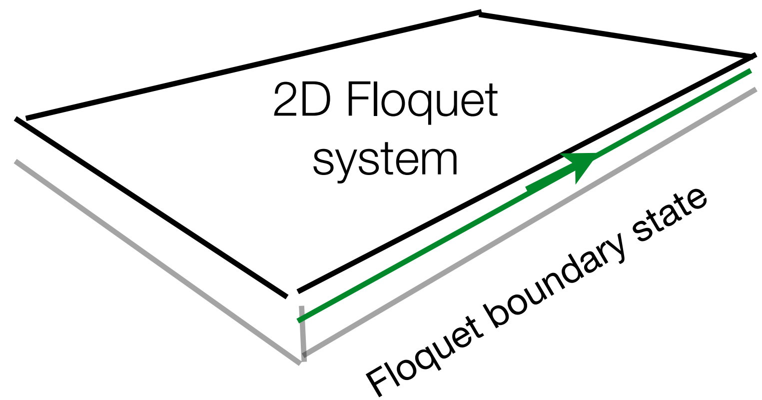

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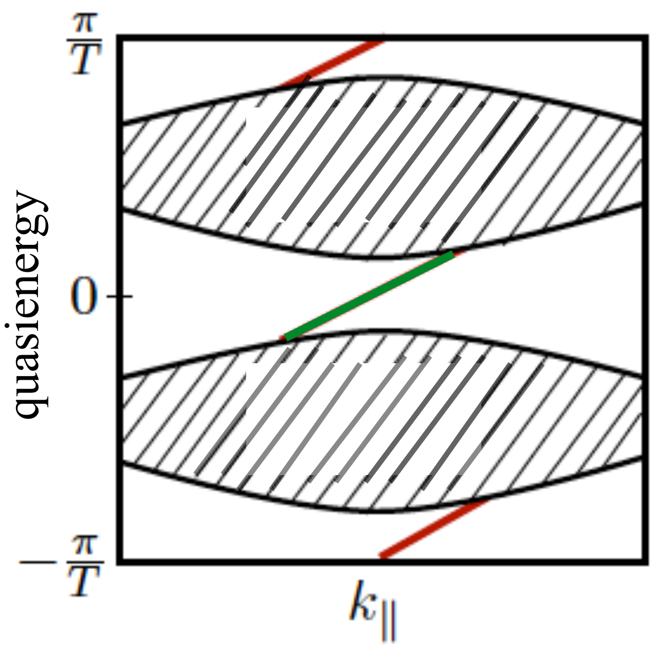

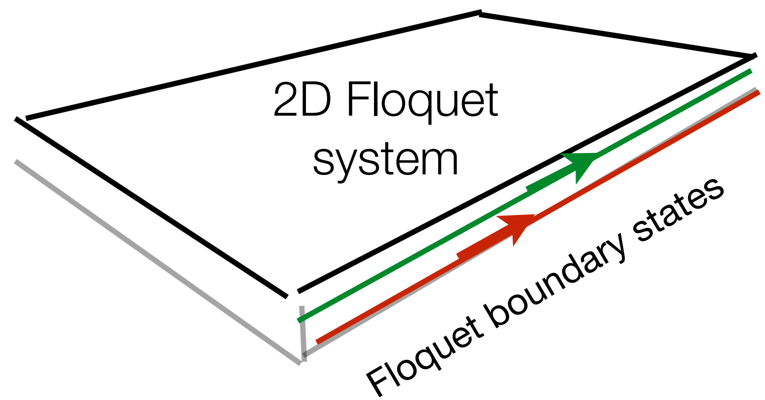

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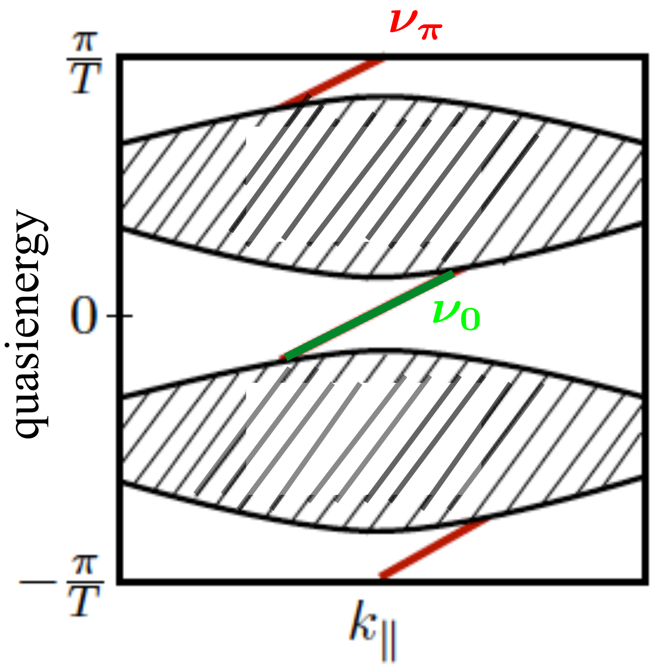

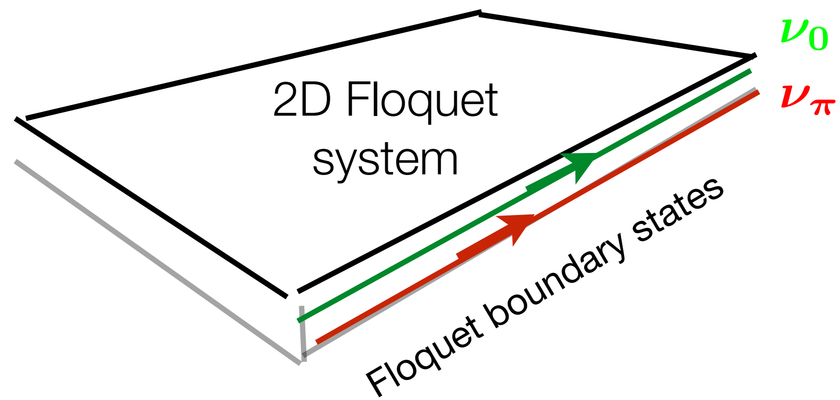

The type of Floquet topological invariants depends on dimensionality and protecting symmetries (similar to static systems)

F. Nathan and M. S. Rudner, New J. Phys. **17**, 125014 (2015)

Using Floquet theory to study symmetry protection against time-periodic perturbations

$$H(t) = H_0(t) + V(t)$$

time-periodic perturbation

$V(t) = V(t+T)$

periodically driven Floquet topological insulator

$H_0(t) = H_0(t+T)$

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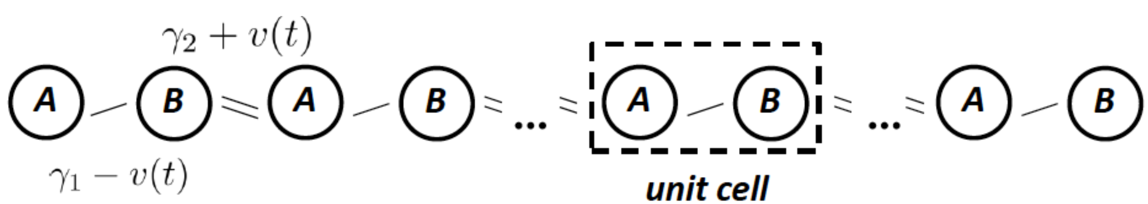
$H_0(t) = H_0(t+T)$

Case study: Harmonically driven *Su-Schrieffer-Heeger (SSH) model*

$$H_0(t) = -\sum_{j=1}^{N} \Big((\gamma_1 - v(t)) c^\dagger_{A,j} c_{B,j} + (\gamma_2 + v(t)) c^\dagger_{B,j} c_{A,j+1} + \text{H.c.} \Big) , \quad v(t) \sim \cos(\Omega t)$$



The ordinary time-independent SSH model ($v(t) = 0$) has time-reversal, particle-hole and **chiral symmetry**

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BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
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$H_0 = \sum_k (c^\dagger_{A,k}\ c^\dagger_{B,k}) H_0(k) (c_{A,k}\ c_{B,k})$

$$\sigma_z H_0(k) \sigma_z = -H_0(k)$$

All eigenstates have a partner with opposite energy (1)

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The **open SSH chain** has a localized zero-energy state at each boundary when the intracell hopping $\gamma_1 = 0$ **(2)**

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DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
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CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
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$$H_0 = \sum_k (c^\dagger_{A,k}\ c^\dagger_{B,k})\, H_0(k) \begin{pmatrix} c_{A,k} \\ c_{B,k} \end{pmatrix}$$

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(1) - (3)

symmetry protected zero-energy boundary states
in the topological phase $\gamma_2 > \gamma_1$

Chiral symmetry-protection in the *harmonically driven SSH model*

$$H_0(t) = -\sum_{j=1}^{N}\Big((\gamma_1 - v(t))c^{\dagger}_{A,j}c_{B,j} + (\gamma_2 + v(t))c^{\dagger}_{B,j}c_{A,j+1} + \text{H.c.}\Big)\,, \quad v(t) \sim \cos(\Omega t)$$

$$\sigma_z U_0(k;0,T)\sigma_z = U_0^{-1}(k;0,T) \qquad U_0(k;t_0,t) = \mathcal{T}\exp\left(-i\int_{t_0}^{t} dt' H_0(k,t')\right)$$

J. K. Asbóth et al., Phys. Rev. B **90**, 125143 (2014)

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J. K. Asbóth et al., Phys. Rev. B **90**, 125143 (2014)

Sufficient condition: $\sigma_z H_0(k,t)\sigma_z = -H_0(k,-t)$

Proof:

Define $F \equiv U(0,\frac{T}{2})$ and $G \equiv U(\frac{T}{2},T)$.

$$\Gamma \equiv \sum_j (|j,A\rangle\langle j,A| - |j,B\rangle\langle j,B|)$$

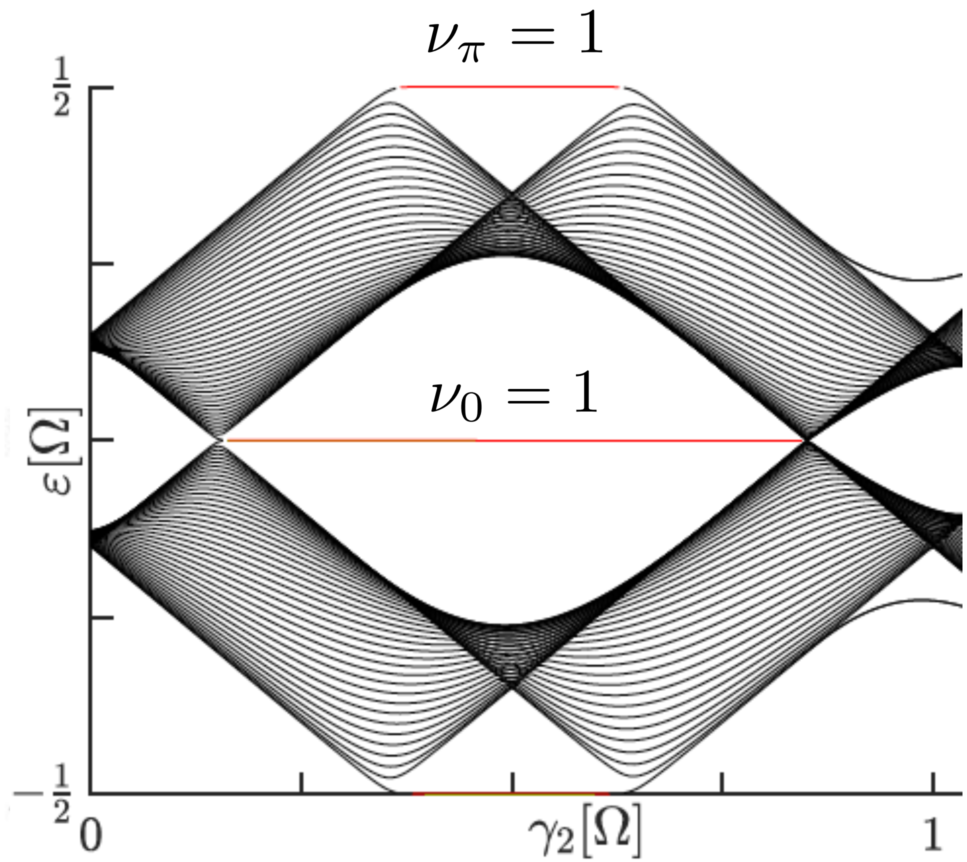
$$F = \sum_n (i)^n \int_0^{-\frac{T}{2}} d\tau_1 \ldots \int_0^{\tau_{n-1}} d\tau_n H(-\tau_1)\ldots H(-\tau_n)$$

$$= \sum_n (-i)^n \int_0^{-\frac{T}{2}} d\tau_1 \ldots \int_0^{\tau_{n-1}} d\tau_n \Gamma H(\tau_1)\Gamma \ldots \Gamma H(\tau_n)\Gamma$$

$$= \Gamma U(0,-\frac{T}{2})\Gamma = \Gamma G^\dagger \Gamma.$$

The chiral symmetry condition $\Gamma U(0,T)\Gamma = U^{-1}(0,T)$ then follows immediately from $U(0,T) = FG = \Gamma G^\dagger \Gamma G$.

Quasienergy spectrum of the harmonically driven SSH model



$\gamma_1 = 0.15\Omega, \; v(t) = 0.4\Omega\cos(\Omega t), \; \Omega = 2\pi/T$

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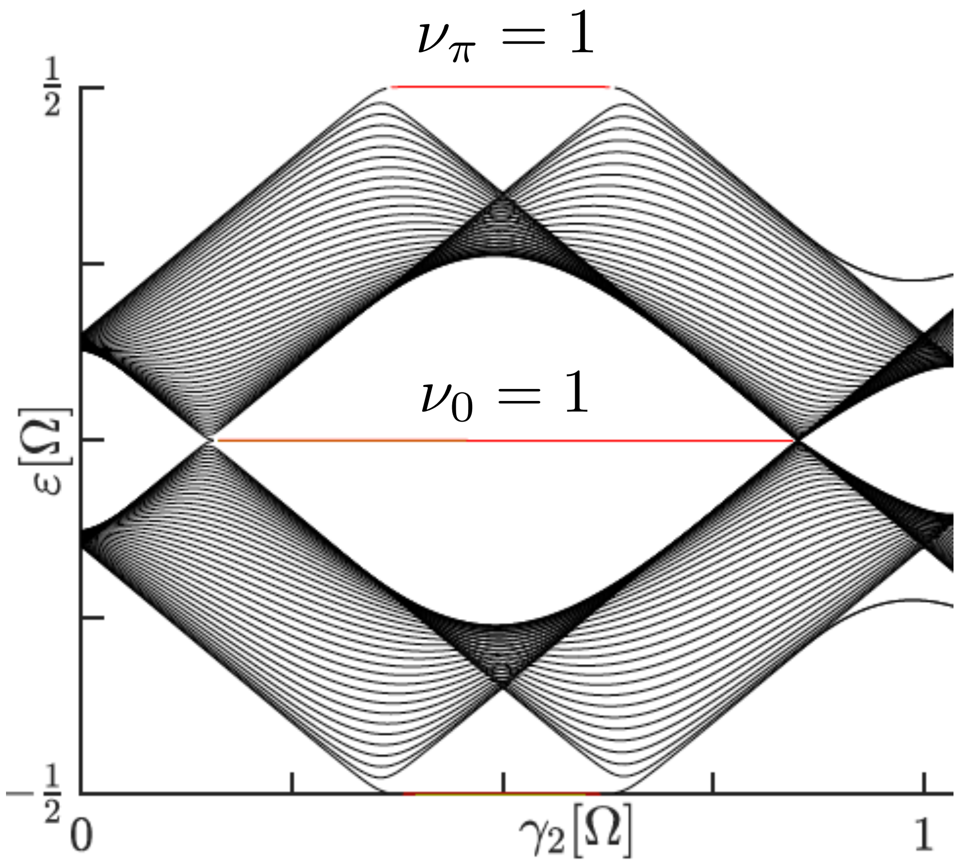

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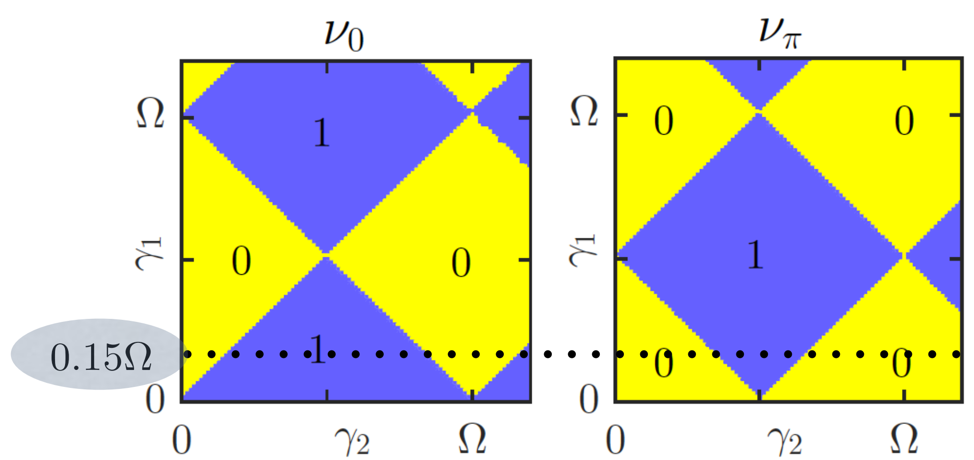
topological invariants



numerical computation, using a method by
J. K. Asbóth *et al.*, Phys. Rev. B **90**, 125143 (2014)

The boundary states are expected to be robust under local time-periodic perturbations $V(t) = V(t+T)$ satisfying

$$\Gamma V(t)\Gamma = -V(-t)$$

$$\Gamma \equiv \sum_j (|j,A\rangle\langle j,A| - |j,B\rangle\langle j,B|)$$

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$$\Gamma V(t)\Gamma = -V(-t)$$

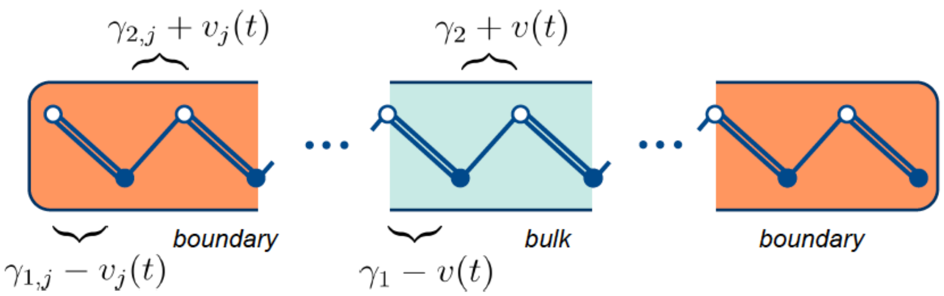
$$\Gamma \equiv \sum_j (|j, A\rangle\langle j, A| - |j, B\rangle\langle j, B|)$$

examples

- *time-periodic spatial **disordering** of the SSH hopping amplitudes close to a boundary:*

$$\gamma_i \to \gamma_{i,j} f(t),\ \ i = 1, 2;\ \ j = 1, ..., n \ll N$$

$$f(t+T) = f(t), \quad f(-t) = f(t)$$



The boundary states are expected to be robust under local time-periodic perturbations $V(t) = V(t+T)$ satisfying

$$\Gamma V(t)\Gamma = -V(-t)$$

$$\Gamma \equiv \sum_j (|j,A\rangle\langle j,A| - |j,B\rangle\langle j,B|)$$

examples

- *time-periodic spatial **disordering** of the SSH hopping amplitudes close to a boundary:*

$$\gamma_i \to \gamma_{i,j} f(t), \ i = 1,2; \ j = 1, ..., n \ll N$$

$$f(t+T) = f(t), \quad f(-t) = f(t)$$

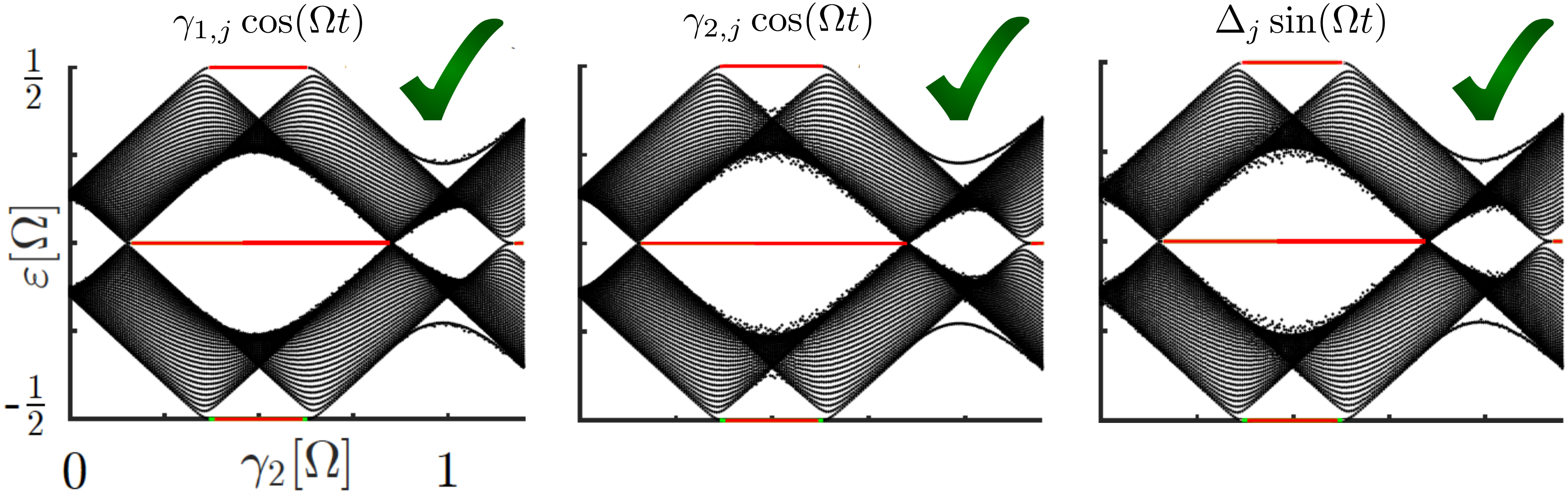
- ***adding** a time-periodic disordered staggered chemical potential close to a boundary*

$$+ g(t) \sum_{j=1}^{n} \Delta_j (c^\dagger_{A,j} c_{A,j} - c^\dagger_{B,j} c_{B,j}), \ n \ll N$$

$$g(t+T) = g(t), \quad g(-t) = -g(t)$$

Numerical diagonalization (truncated Hamiltonian in frequency domain):

The disordered $\gamma_{1/2}$ and Δ amplitudes vary randomly $\in[-0.2\Omega, 0.2\Omega]$ in the "boundary region" $j=1,\ldots,10$, bulk intracell hopping $\gamma_1 = 0.15\Omega$, bulk driving $v(t) = 0.2\cos(\Omega t)$, chain length $N = 40$



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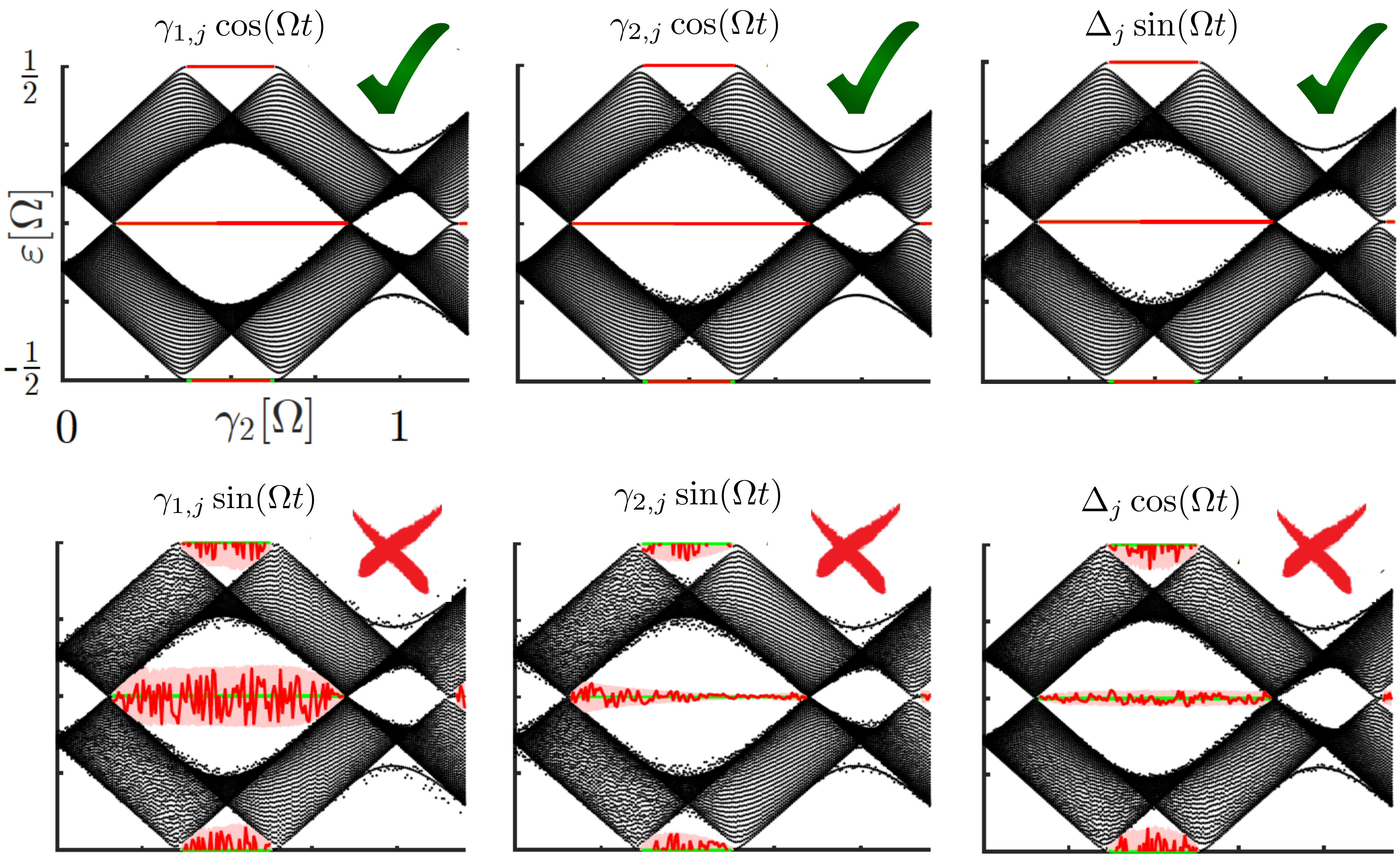
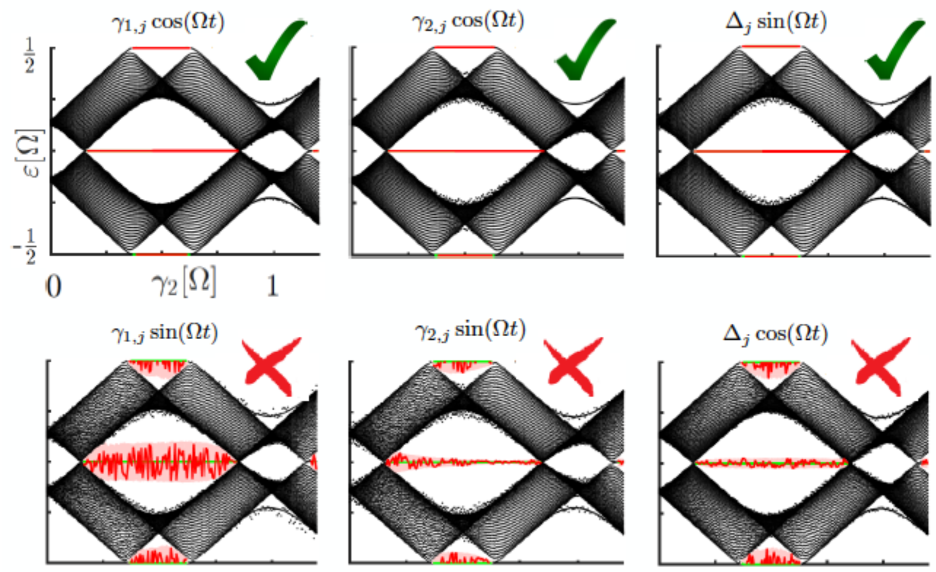
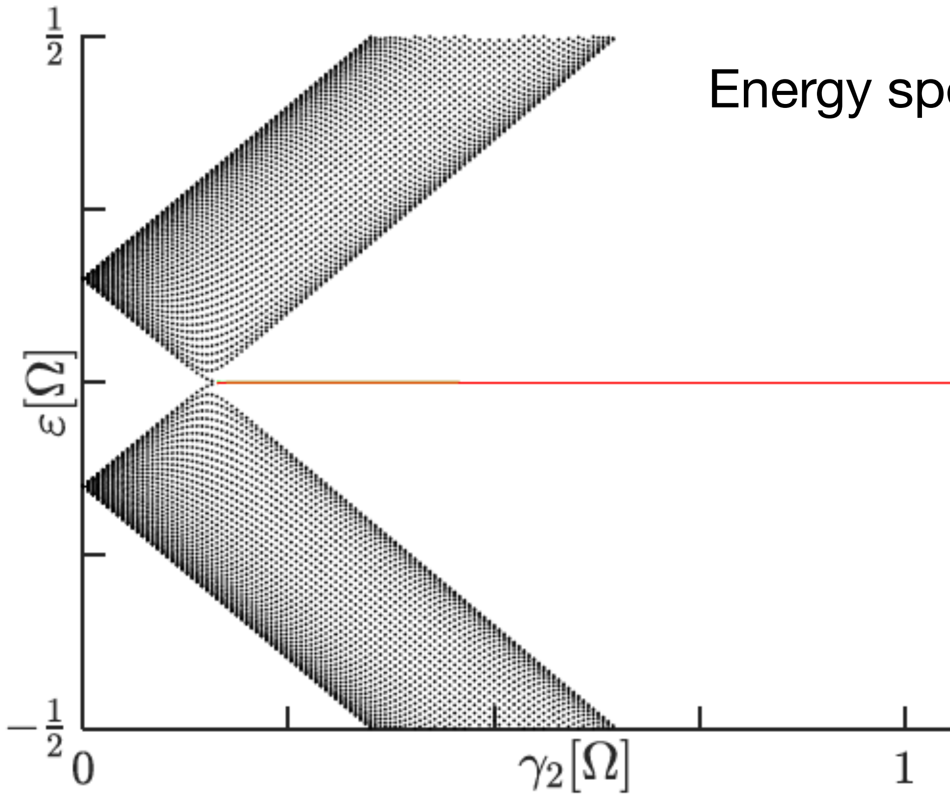




The robustness of the boundary states depends critically on the relative phase between bulk driving and perturbation.

Time-independent SSH model within Floquet theory

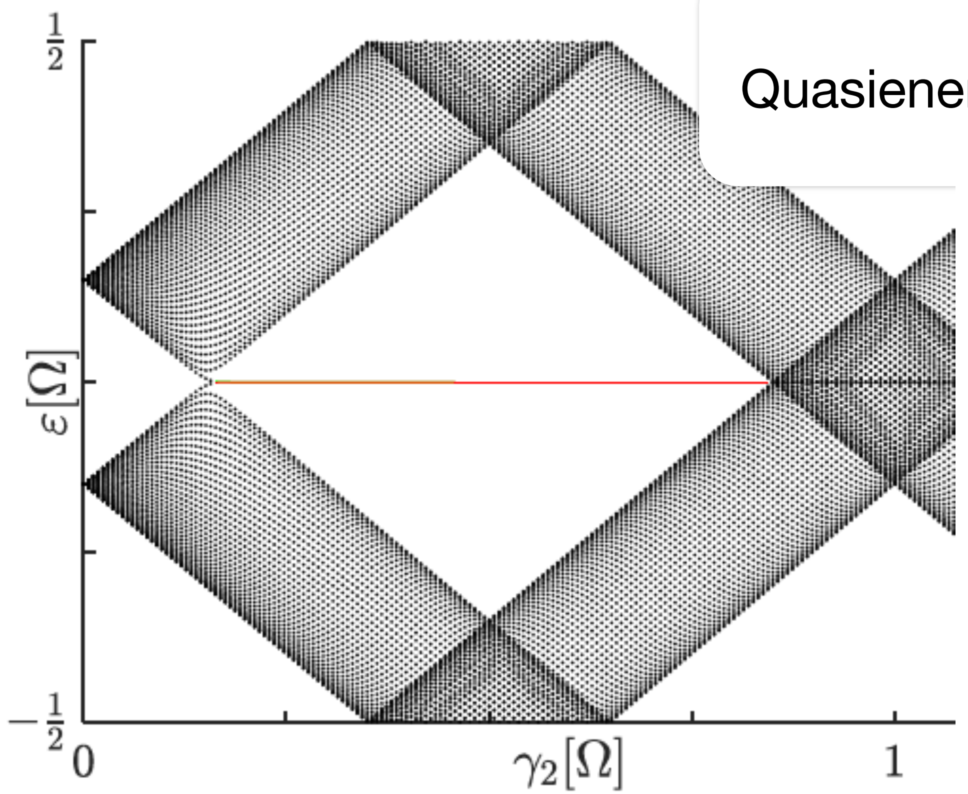


Energy spectrum

$\gamma_1 = 0.15\Omega$

Time-independent SSH model within Floquet theory

Quasienergy spectrum



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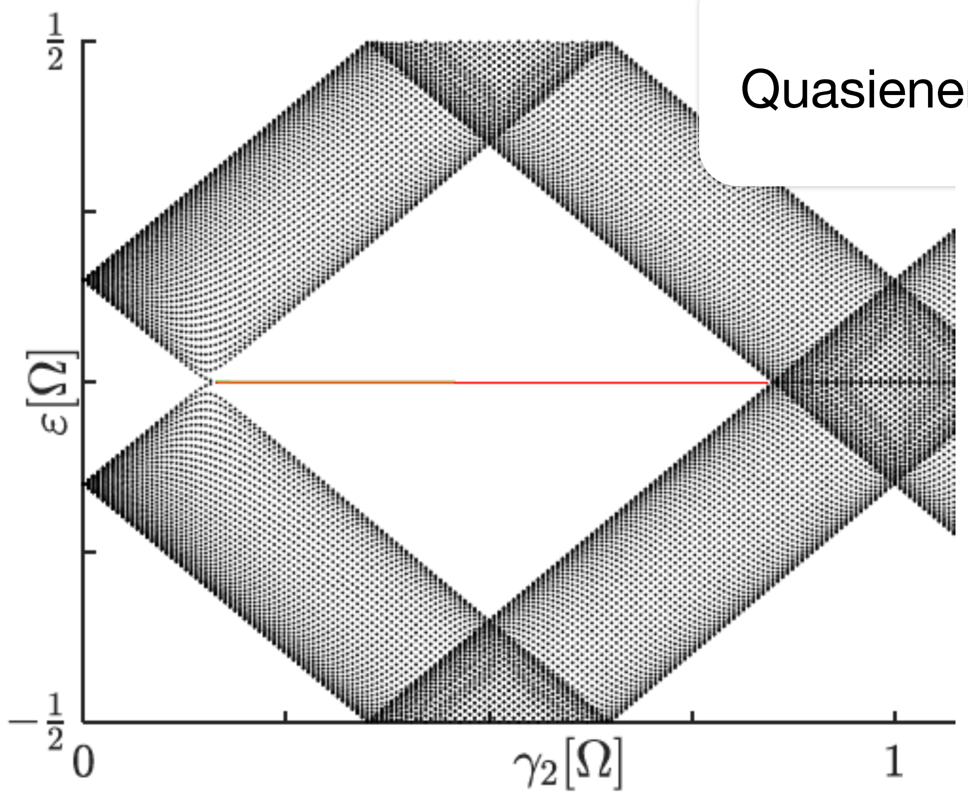
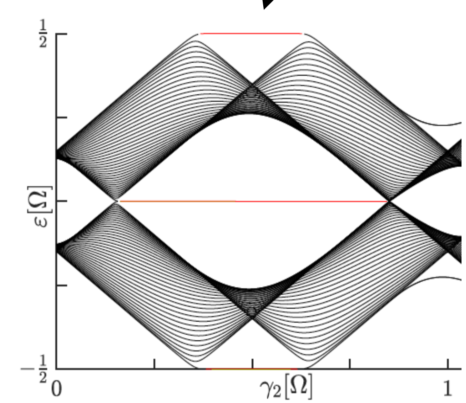


Quasienergy spectrum

harmonically driven SSH model



$$\gamma_1 = 0.15\Omega$$

What about robustness of boundary states in the ordinary *time-independent SSH model* subject to local time-periodic perturbations?

What about robustness of boundary states in the ordinary *time-independent SSH model* subject to local time-periodic perturbations?

Since there's no bulk driving and hence no constraint from a relative phase, one expects that the boundary states are robust against a much larger class of perturbations*!

* provided there is at least one reference time t_0 for which the perturbation is chiral symmetric

Numerical diagonalization (truncated Hamiltonian in frequency domain):

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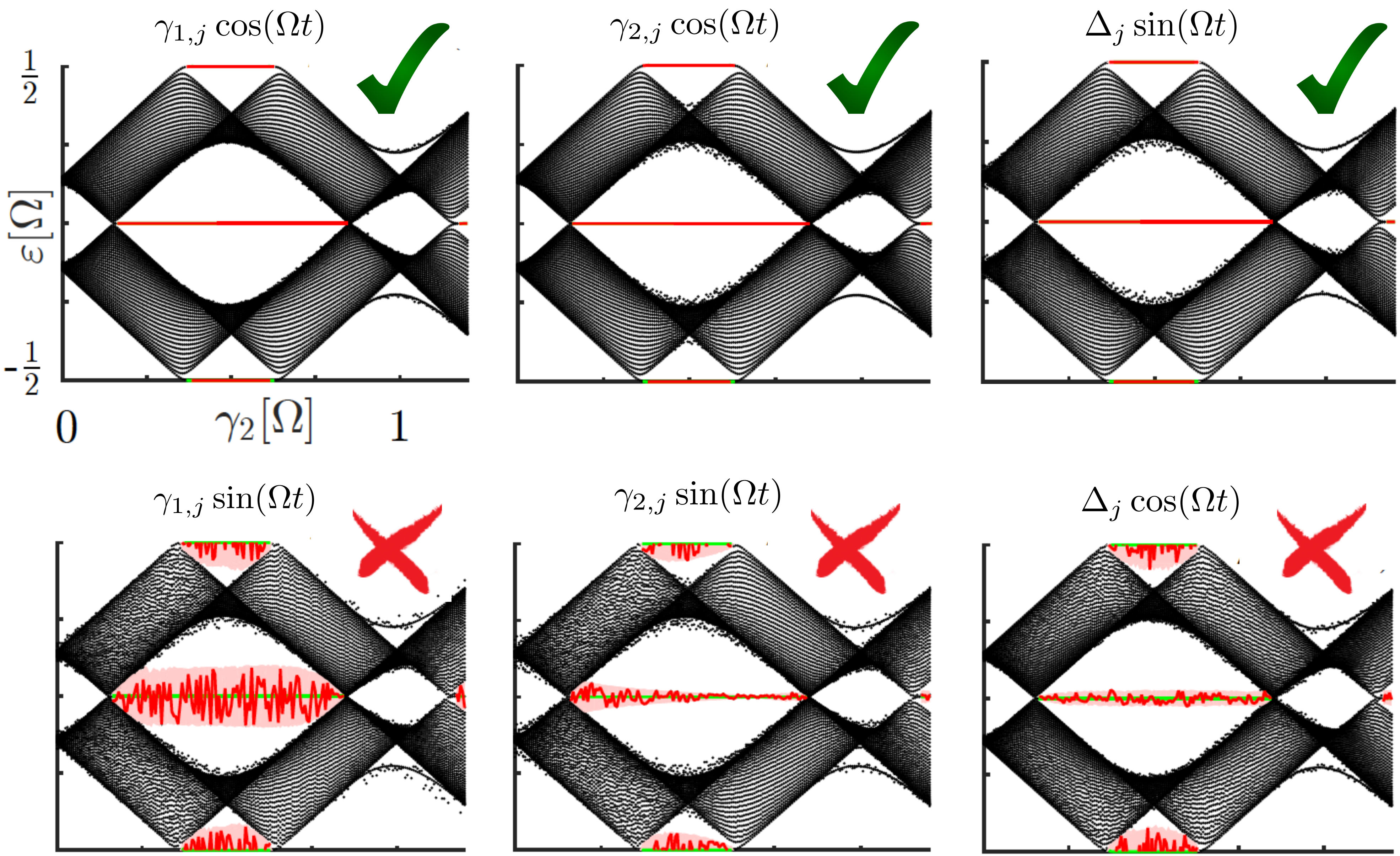

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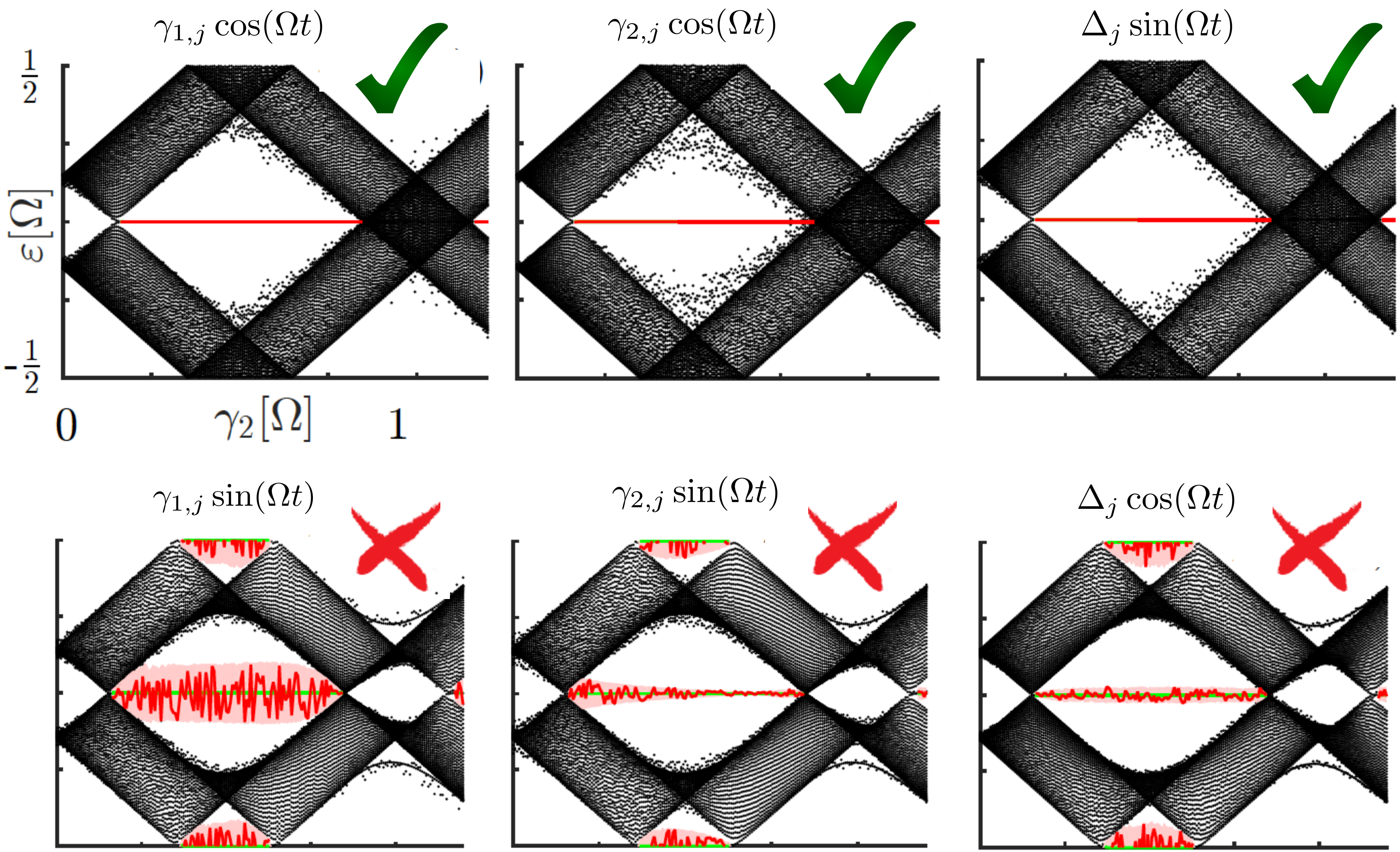

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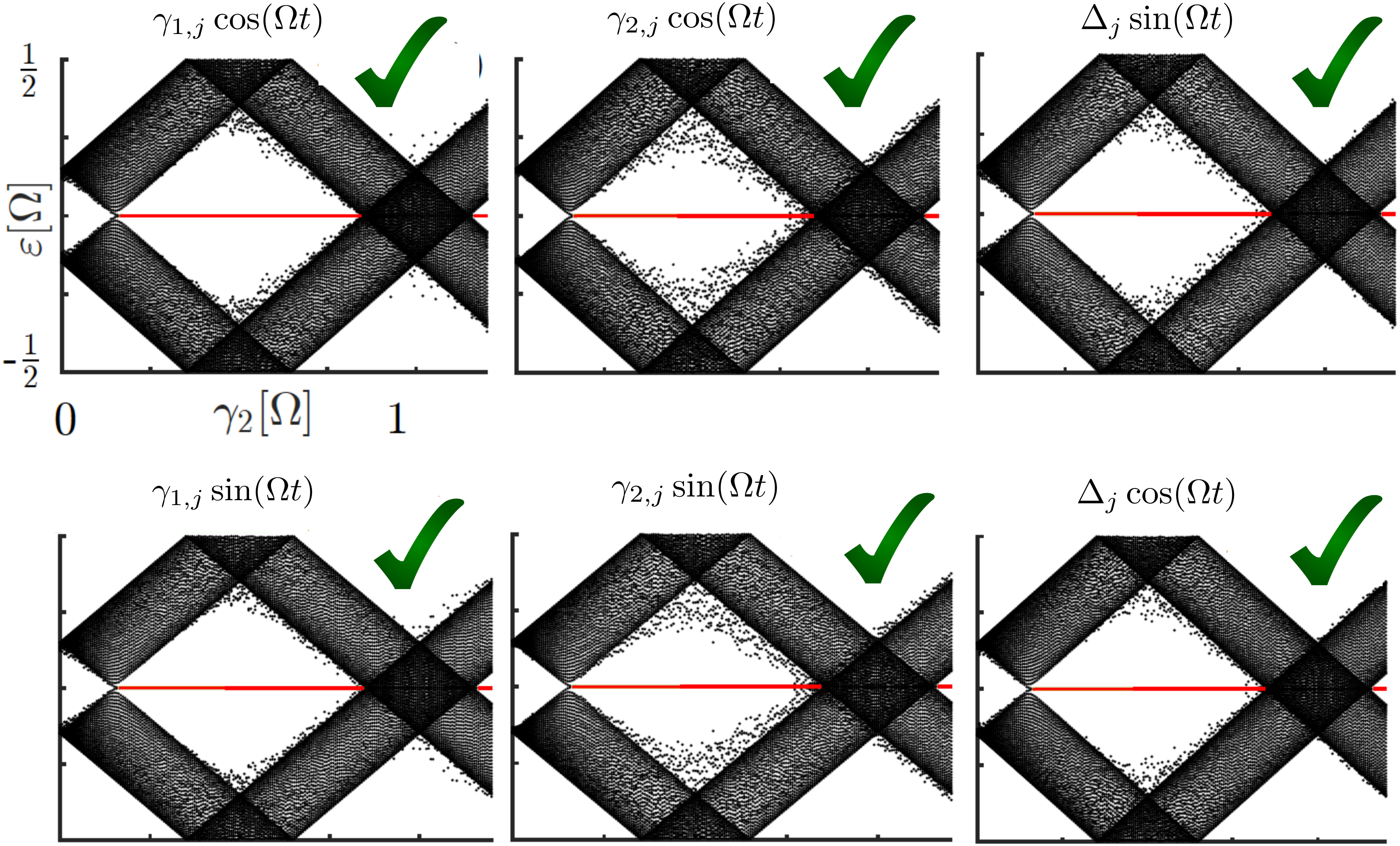

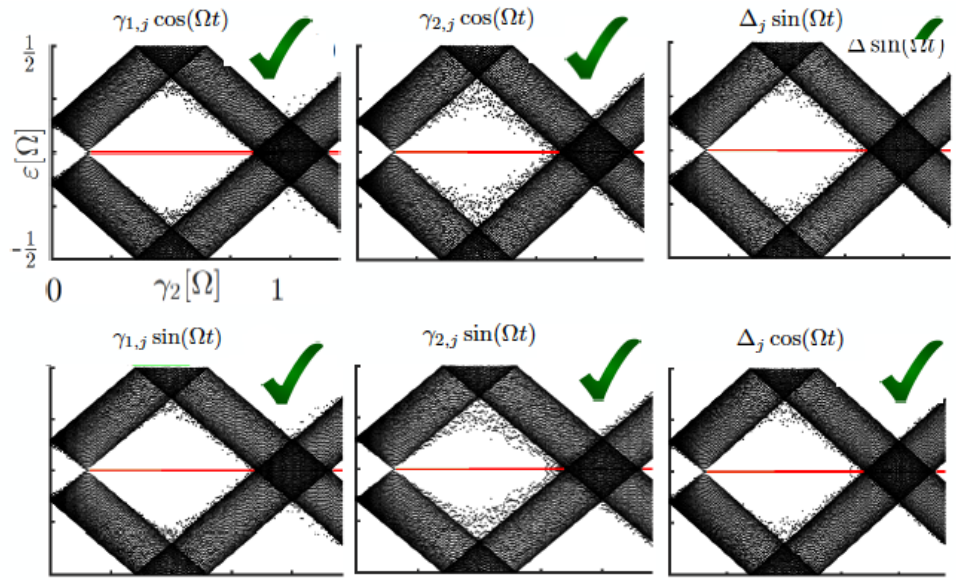

Boundary states of chiral-invariant static systems are robust against *all* time-periodic perturbations which respect chiral symmetry for *some* reference time.

Time-independent SSH model:
resilience of boundary states against
symmetry-breaking time-periodic perturbations

Time-independent SSH model:
resilience of boundary states against *symmetry-breaking* time-periodic perturbations

Consider perturbations of the form

$$V(t) = \sum_n V_n \cos(n\Omega t + \phi_n)$$

local perturbation

where $\forall n \in \mathbb{N} : \Gamma V_n \Gamma = \pm V_n$ and $\phi_n \in \mathbb{R}$

$$\Gamma \equiv \sum_j (|j,A\rangle\langle j,A| - |j,B\rangle\langle j,B|)$$

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$$\Gamma \equiv \sum_j (|j,A\rangle\langle j,A| - |j,B\rangle\langle j,B|)$$

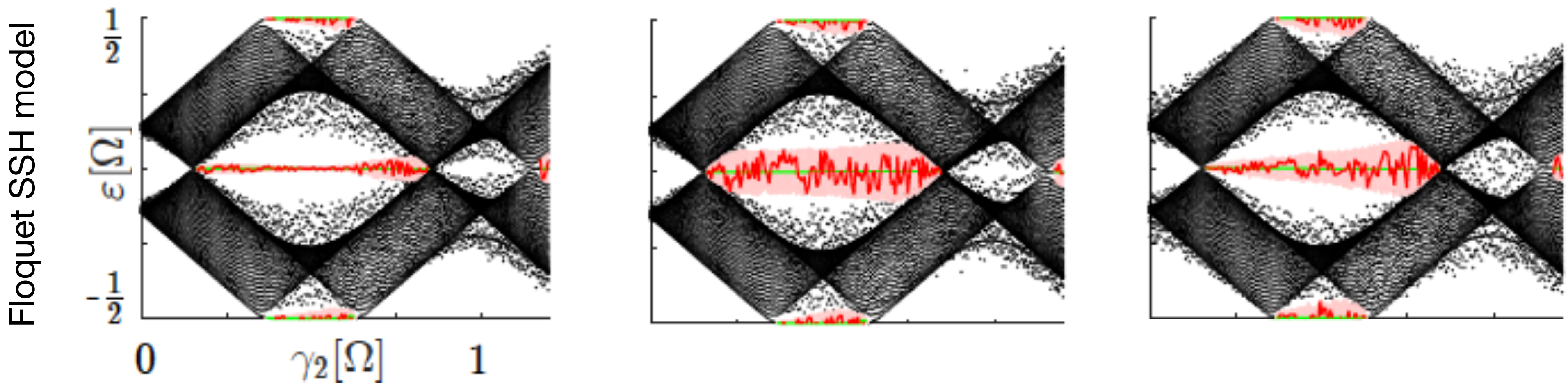
This class of perturbations generically break chiral symmetry for all choices of reference times t_0:
$\Gamma U(t_0, t_0+T)\Gamma \neq U^{-1}(t_0, t_0+T), \ \forall t_0$

Numerical diagonalization (truncated Hamiltonian in frequency domain):

The disordered $\gamma_{1/2}$ and Δ amplitudes vary randomly $\in[-0.5\Omega, 0.5\Omega]$ in the "boundary region" $j=1,...,10$, bulk intracell hopping $\gamma_1 = 0.15\Omega$, bulk driving $v(t) = 0.4\cos(\Omega t)$, chain length $N = 40$

Perturbations

$\gamma_2\cos(\Omega t) + \gamma_1\sin(2\Omega t)$ | $\gamma_2\cos(\Omega t) + \Delta\cos(2\Omega t)$ | $\Delta\sin(\Omega t) + \gamma_2\sin(2\Omega t)$



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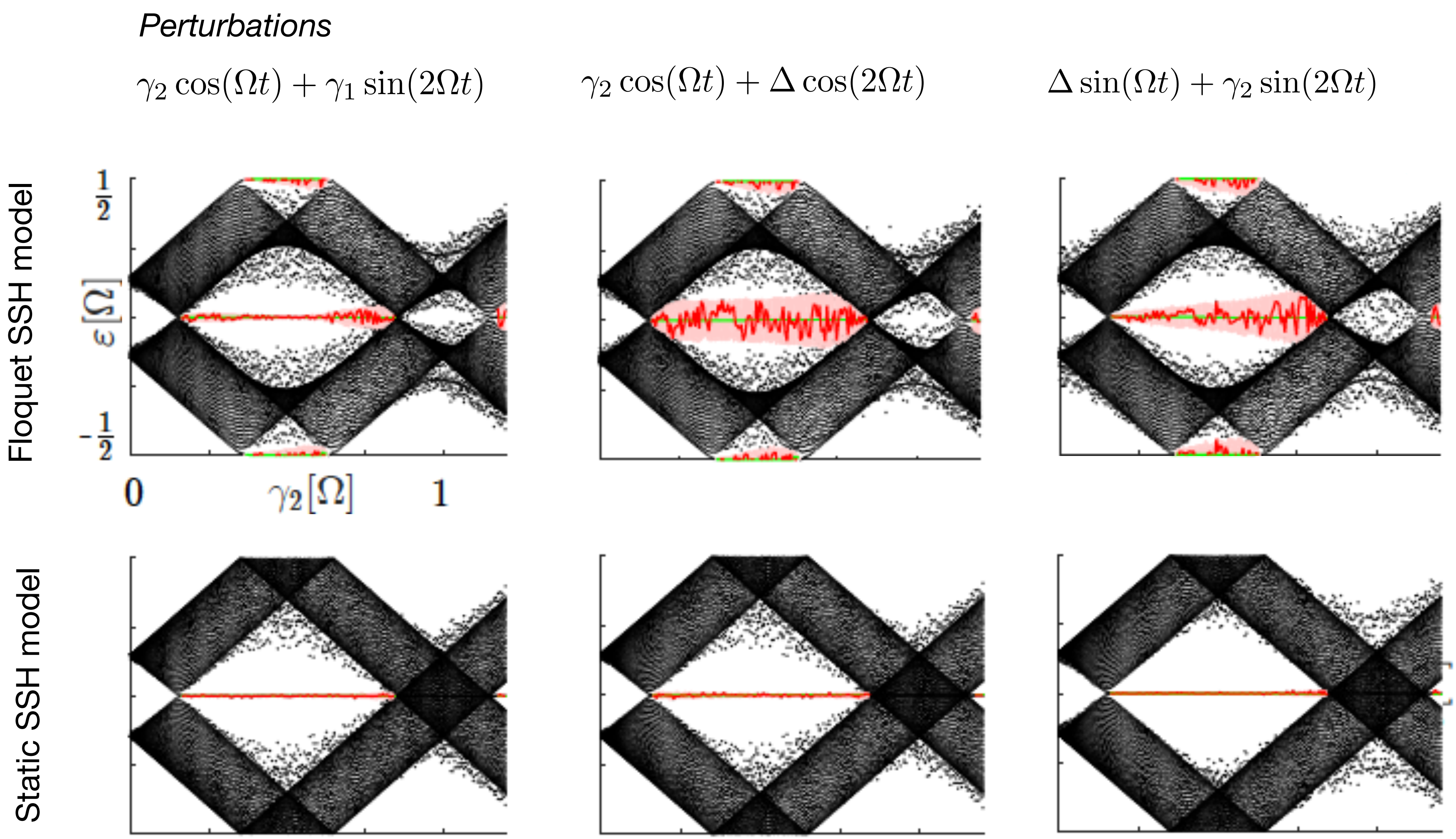


Floquet perturbation theory for a static chiral-invariant system

First- and second-order quasienergy corrections to
any nondegenerate level due to a time-periodic perturbation $V(t)$:

$$\varepsilon_\psi^{(1)} = \frac{1}{T}\int_0^T \langle\psi^0(t)|V(t)|\psi^0(t)\rangle dt, \quad \varepsilon_\psi^{(2)} = \sum_{\beta\neq\psi} \frac{\left|\frac{1}{T}\int_0^T \langle\beta^0(t)|V(t)|\psi^0(t)\rangle dt\right|^2}{\varepsilon_\psi^0 - \varepsilon_\beta^0}$$

$|\psi^0(t)\rangle, |\beta^0(t)\rangle$ unperturbed Floquet states associated with quasienergies ε_ψ^0 and ε_β^0

$$V(t) = \sum_n V_n \cos(n\Omega t + \phi_n) \qquad \forall n \in \mathbb{N} : \Gamma V_n \Gamma = \pm V_n$$

the first- and second-order contributions to the
unperturbed zero-quasienergy level $\varepsilon_\psi^0 = 0$ vanish identically!

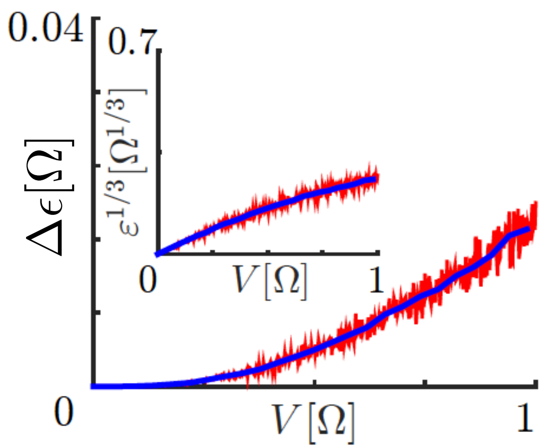
Numerical test of leading order cubic scaling of quasienergy shifts
(static SHH model)

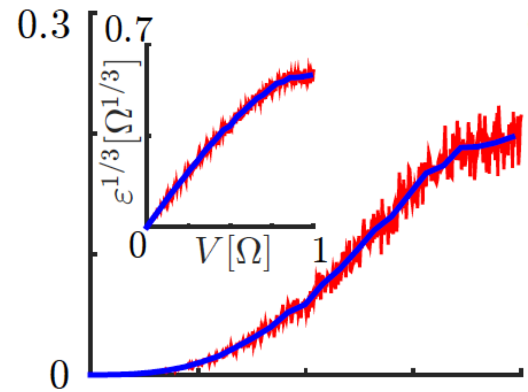
Perturbations

$\Delta_j \sin(\Omega t) + \gamma_{2,j} \sin(2\Omega t)$

$\gamma_{1,j} \cos(\Omega t) + \Delta_j \cos(2\Omega t)$

Static SSH model





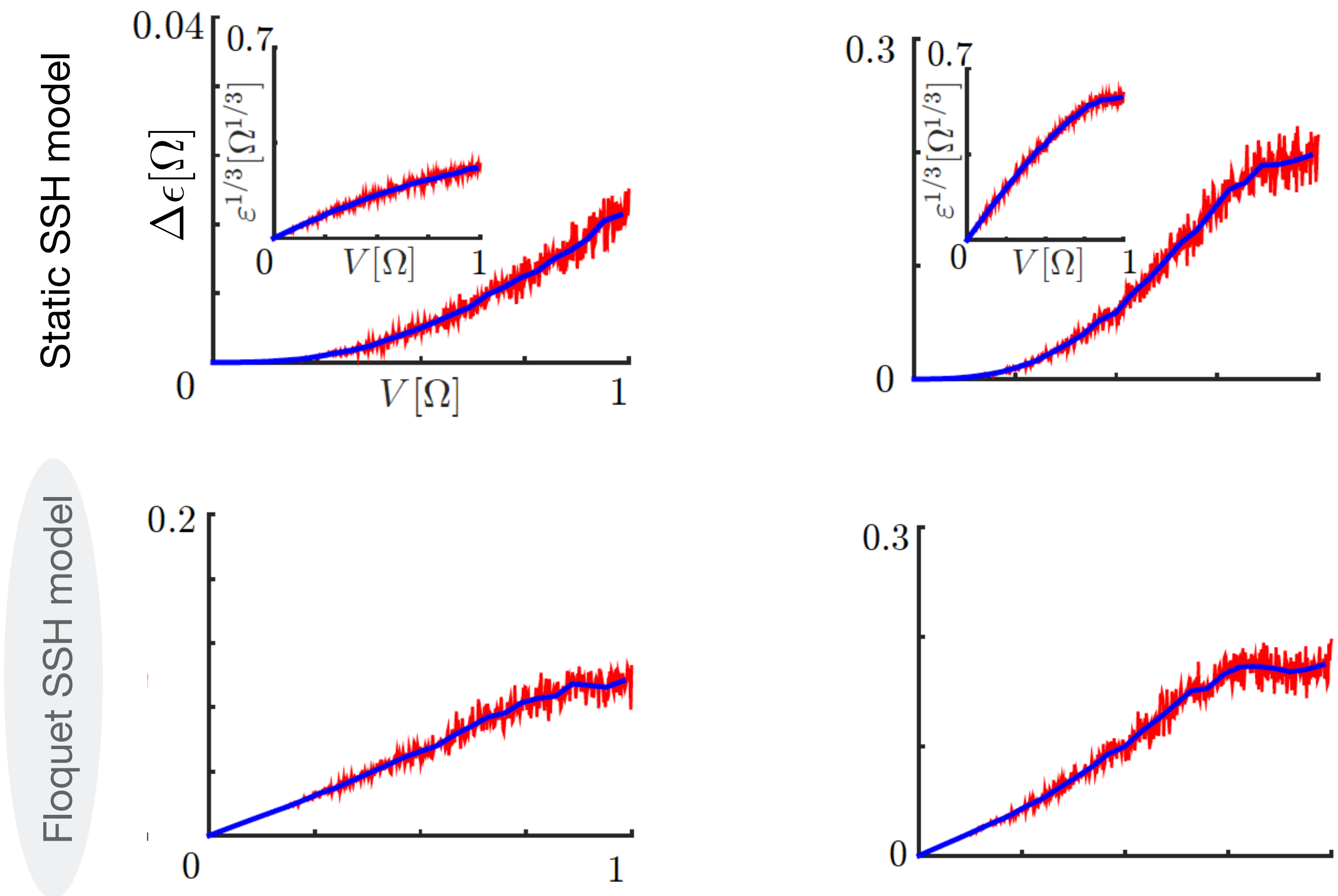
linear

Numerical test of leading order cubic scaling of quasienergy shifts (static SHH model)

Floquet

Perturbations

$\Delta_j \sin(\Omega t) + \gamma_{2,j} \sin(2\Omega t)$ $\gamma_{1,j} \cos(\Omega t) + \Delta_j \cos(2\Omega t)$



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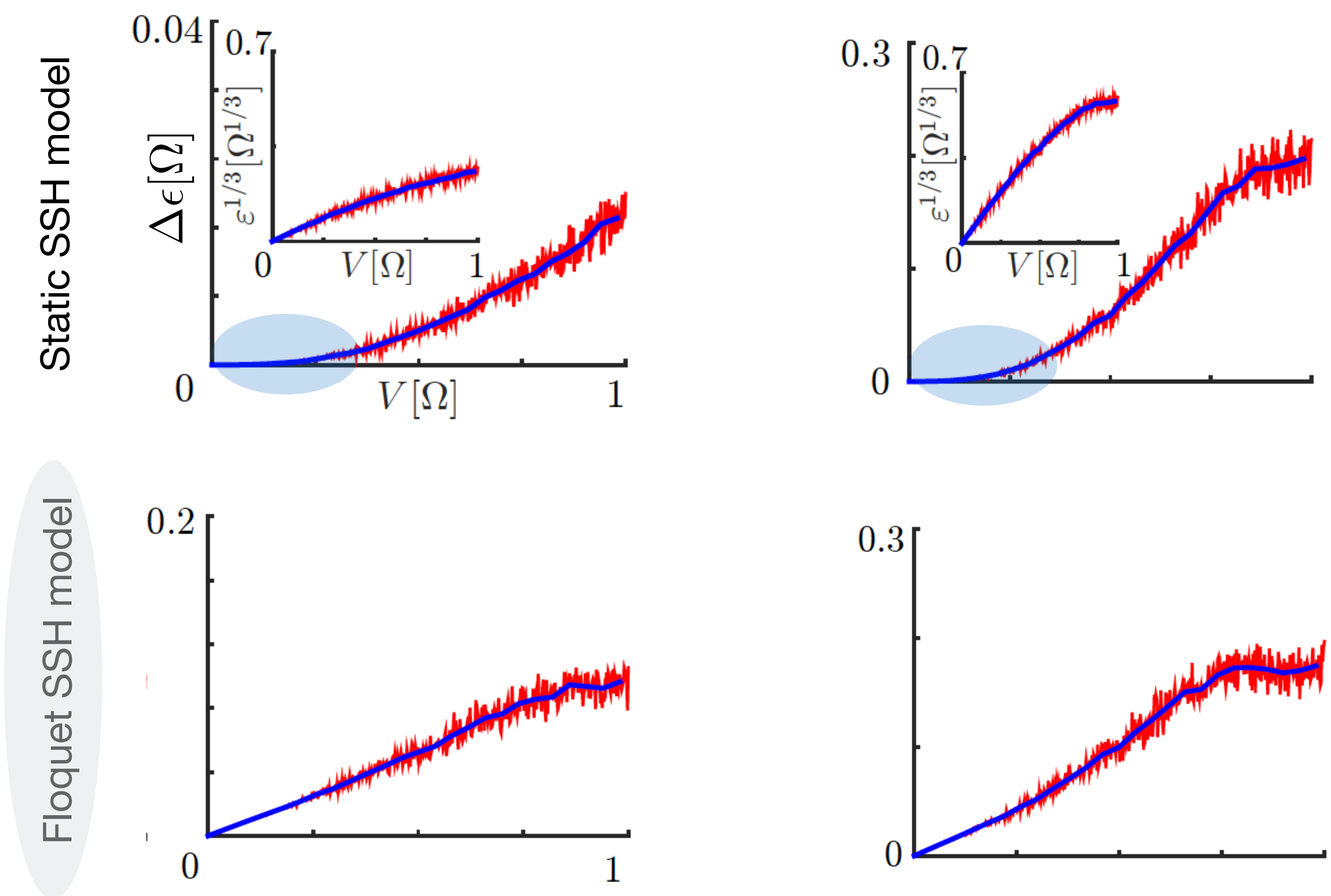

The perturbative results are not the full story…

… the resilience against chiral symmetry-breaking perturbations frequently hold also in the nonperturbative regime, and also for many other types of perturbations!

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How is this possible?

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… the resilience against chiral symmetry-breaking perturbations frequently hold also in the nonperturbative regime, and also for many other types of perturbations!

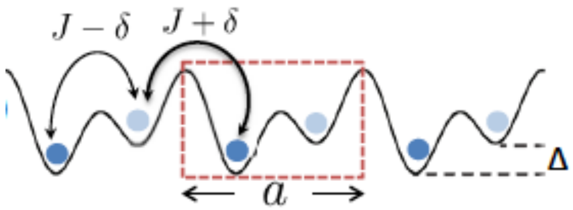
How is this possible?

Because of the freedom of choice of reference time t_0 for a time-independent model!

Isolate the manifestly symmetry-breaking part of the perturbation! Choose t_0 so that this part gets minimized! This choice of t_0 puts a bound on the effect of the perturbation.

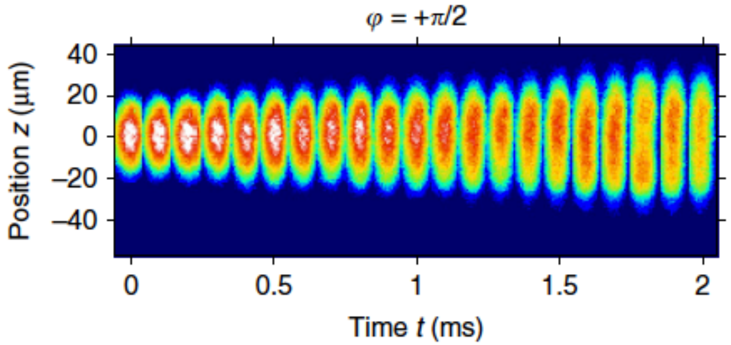
Experimental tests?

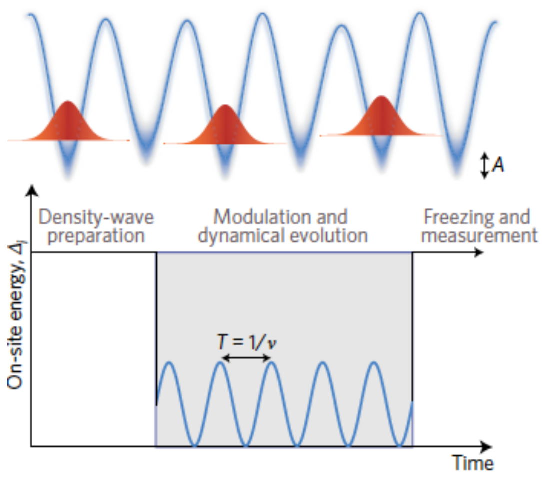
Proposal for simulating SSH Hamiltonian in 1D optical lattices
D.-W. Zhang *et al.*, Phys. Rev. A **92**, 013612 (2015)



Realized for bosonic SSH model with ^{87}Rb atoms
M. Leder *et al.*, Nat. Commun. **7**,13112 (2016)

optical real-space imaging of edge states





Experimental realization of periodically driven random potential in 1D optical lattice with ^{40}K atoms
P. Bordia *et al.*, Nat. Phys. **13**, 460 (2017)

Summary

O. Balabanov & H. J., Phys. Rev. B **96**, 035149 (2017) [arXiv:1704.00782]

The effect of time-periodic symmetry-preserving local perturbations of a chiral-invariant topological *Floquet system* depends critically on the relative phase between the drive of the system and that of the perturbation.

No such constraint for chiral-invariant *time-independent* unperturbed systems.

Enhanced resilience of boundary states against time-periodic *symmetry-breaking* perturbations!