Strongly Coupled Field Theories for Condensed Matter and Quantum Information Theory
IIP, Natal, August 19, 2015

Controllable Spin Entanglement Production in a Quantum Spin Hall Ring

Anders Ström (TU Braunschweig)
Henrik Johannesson (University of Gothenburg)
Patrik Recher (TU Braunschweig)

DFG Deutsche Forschungsgemeinschaft
Technische Universität Braunschweig
UNIVERSITY OF GOTHENBURG
Vetenskapsrådet

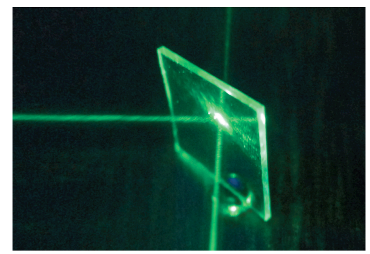
Electron quantum optics...

... conversion of *linear optics* into the solid state!

Background and motivation...

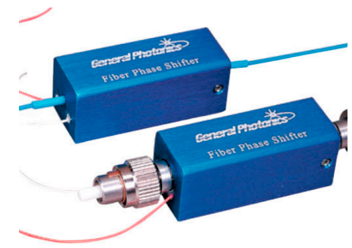
Linear optics:

noninteracting photons subject to ”linear optical elements”



beam splitters

mirrors

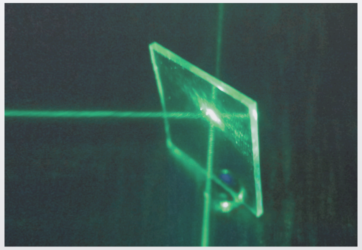


phase shifters

Background and motivation...

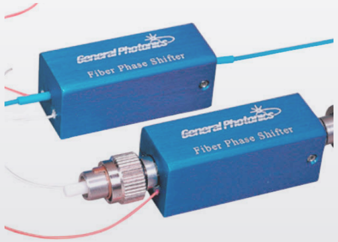
Linear optics:

noninteracting photons subject to ”linear optical elements”



beam splitters

mirrors



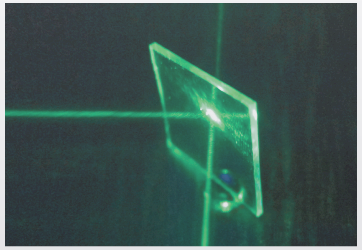
phase shifters

bilinear Hamiltonian (in photon creation and annihilation operators), conserves photon number

Background and motivation...

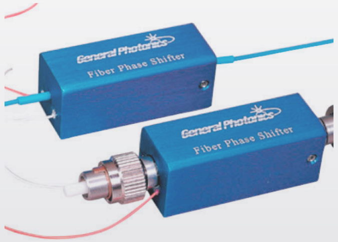
Linear optics:

noninteracting photons subject to ”linear optical elements”



beam splitters

mirrors



phase shifters

bilinear Hamiltonian (in photon creation and annihilation operators), conserves photon number

Boost in 2001: ”*Linear optics quantum computing*”

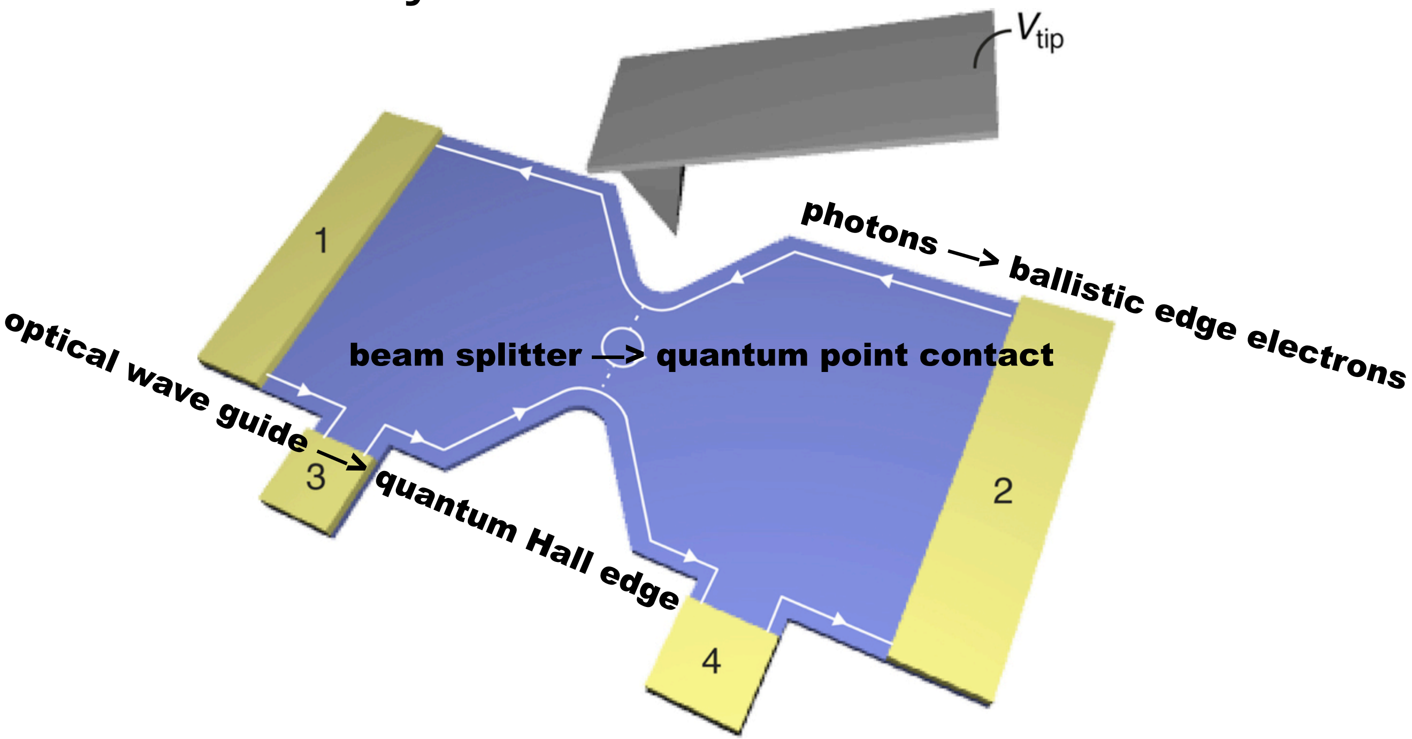
Knill, Laflamme & Milburn, Nature (2001)

Linear optics:

noninteracting photons subject to ”linear optical elements”

Analogue in the solid state:

Integer quantum Hall system



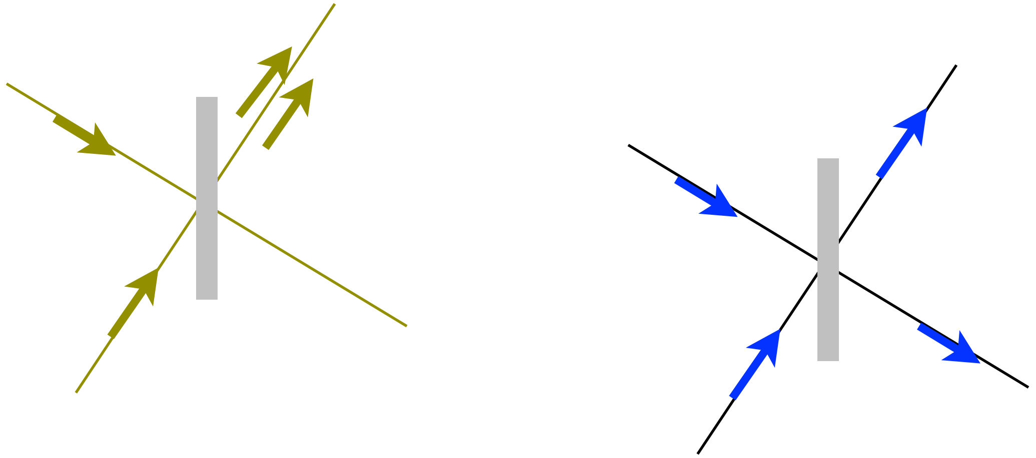
But... crucial differences:

Electrons carry charge and *interact*!

But... crucial differences:

Electrons carry charge and *interact*!

Statistics: photons **bunch**, electrons **antibunch**,...



Hong, Ou & Mandel, PRL (1987)
[experiment]

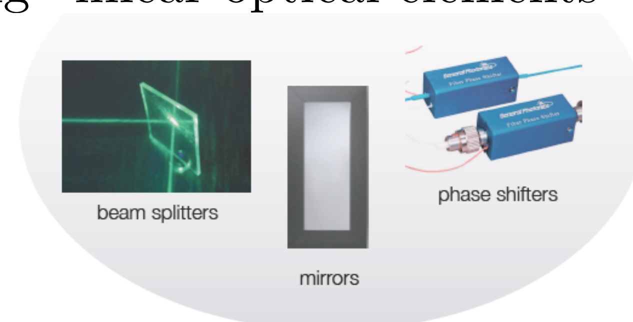
”Is there an electronic analogue to *linear optics quantum computing*?”

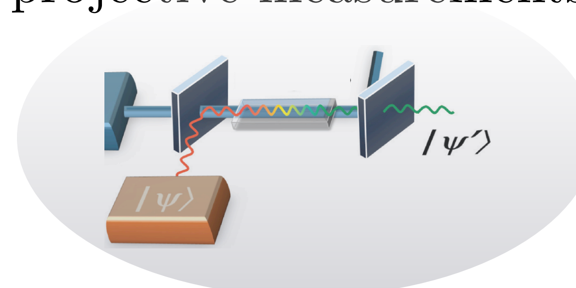
Quantum computing using linear optics (LOQC)

Knill, Laflamme & Milburn, Nature (2001)

$|\,\text{single-photon qubit}\,\rangle = \alpha\,|$  $\rangle + \beta\,|$  $\rangle$

quantum operations using "linear optical elements" and projective measurements



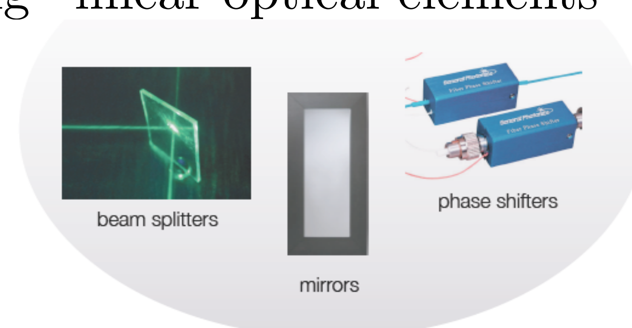


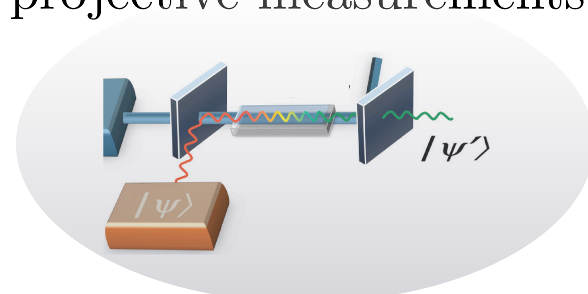
Quantum computing using linear optics (LOQC)

Knill, Laflamme & Milburn, Nature (2001)

$|\,\text{single-photon qubit}\,\rangle = \alpha\,|$  $\rangle + \beta\,|$  $\rangle$

quantum operations using "linear optical elements" and projective measurements





quantum gates

in computational basis

$|$  $\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|$  $\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

universal set of quantum gates

$$R_{[\pi/4]} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \qquad H = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

universal set of quantum gates

$$R_{[\pi/4]} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \qquad H = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

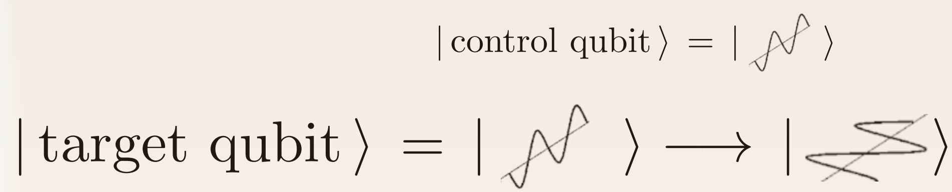
$|\,\text{control qubit}\,\rangle = |$  $\rangle$

$|\,\text{target qubit}\,\rangle = |$  $\rangle \longrightarrow |$  $\rangle$

universal set of quantum gates

$$R_{[\pi/4]} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \qquad H = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



universal set of quantum gates

$$R_{[\pi/4]} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \qquad H = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

But photons don't interact!

Linear optics quantum computing

universal set of quantum gates

$$R_{[\pi/4]} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \qquad H = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$|\,\text{control qubit}\,\rangle = |\quad\rangle$$

$$|\,\text{target qubit}\,\rangle = |\quad\rangle \longrightarrow |\quad\rangle$$

But photons don't interact!

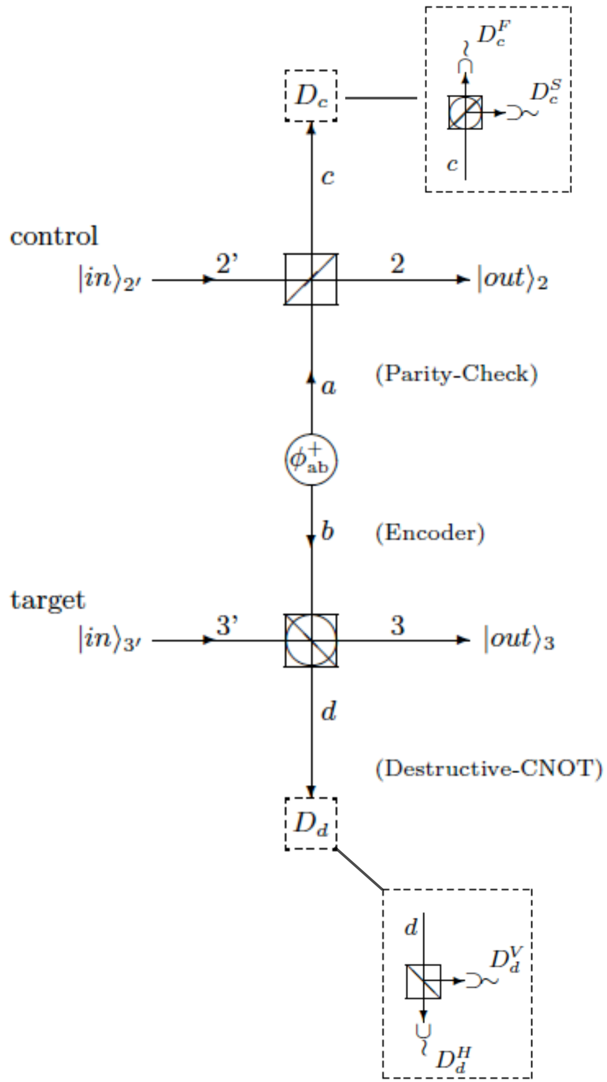
But photons do bunch!

"Probabilistic" 2-qubit gates are possible!

Knill, Laflamme & Milburn, Nature (2001)

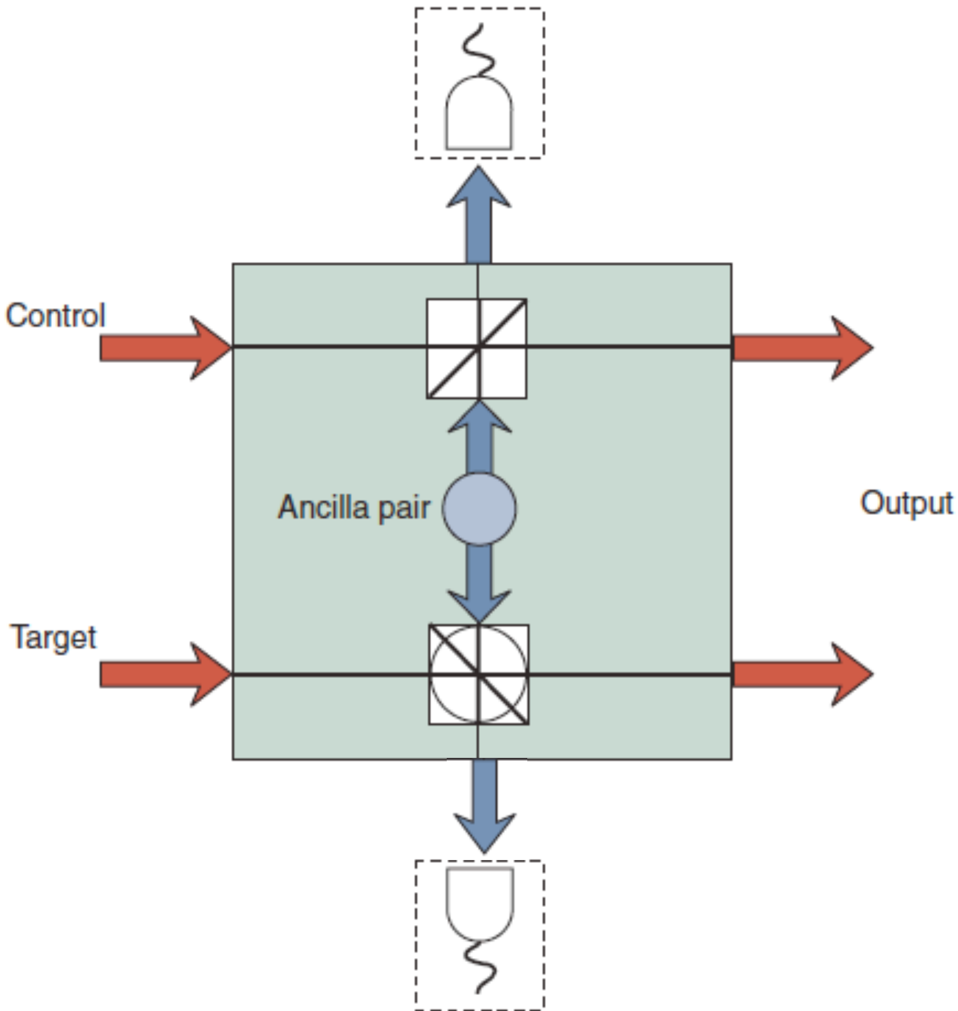
Koashi, Yamamoto & Imoto, PRA (2001)

Probabilistic CNOT gate

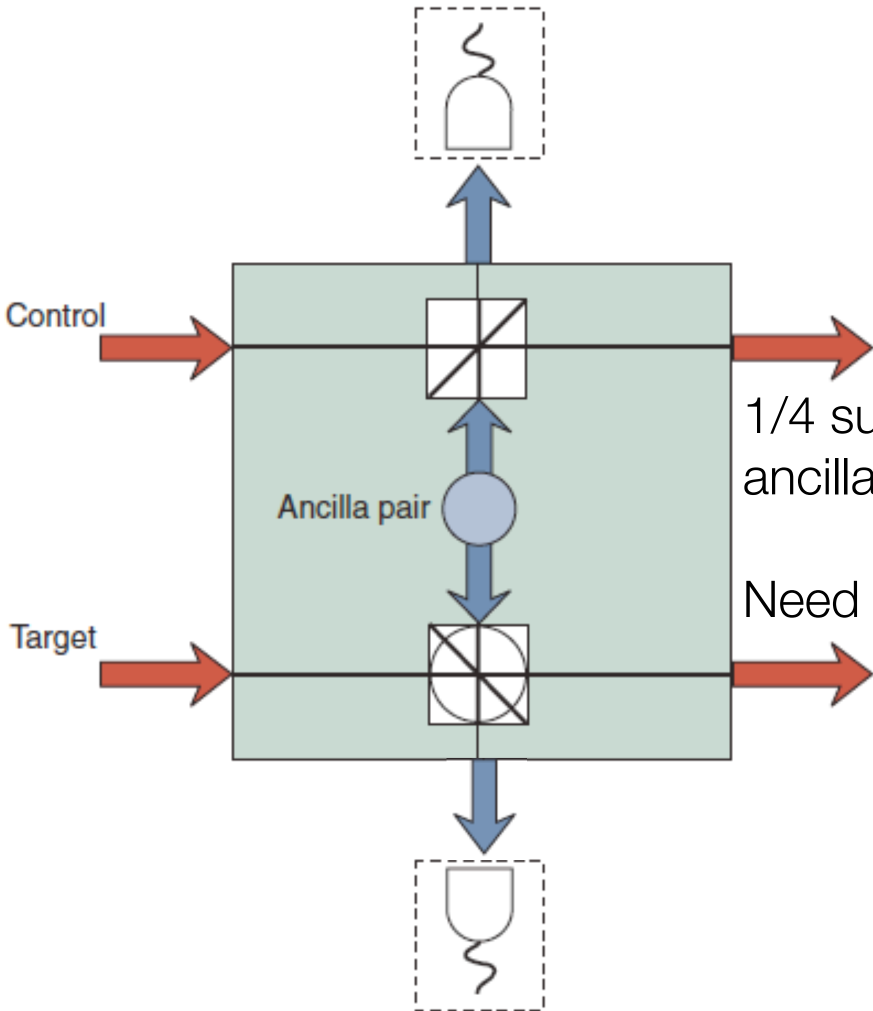


Pittman *et al.,* PRA (2001) [design]
Gasparoni *et al.,* PRL (2004) [experiment]

Probabilistic CNOT gate



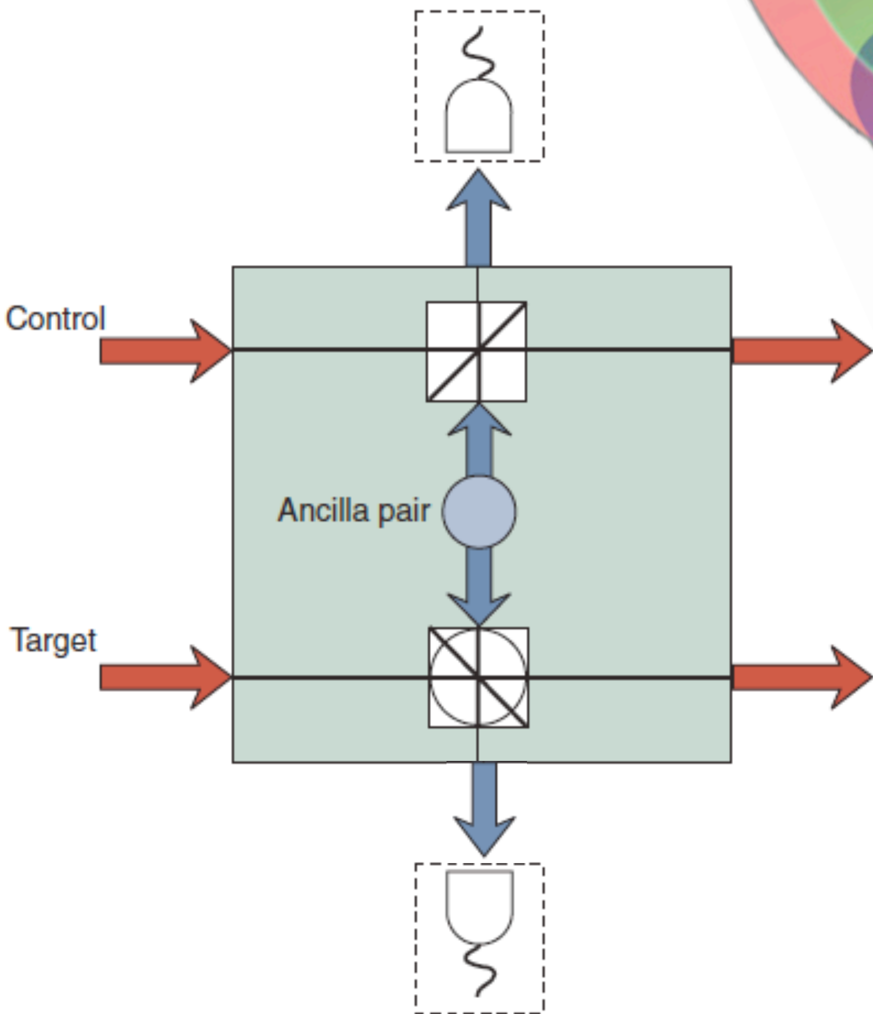
Probabilistic CNOT gate

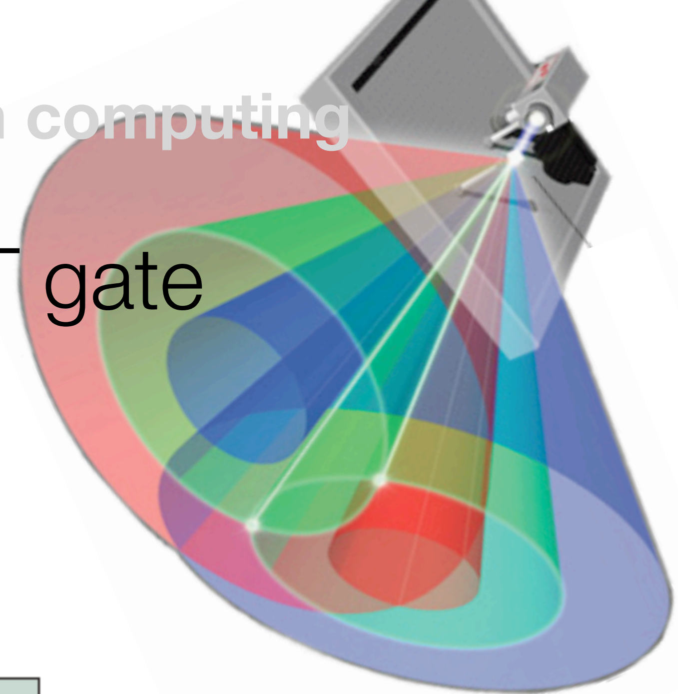


1/4 success rate with maximally entangled ancilla photons. Pittman *et al.*, PRA (2001)

Need good supply of photons!

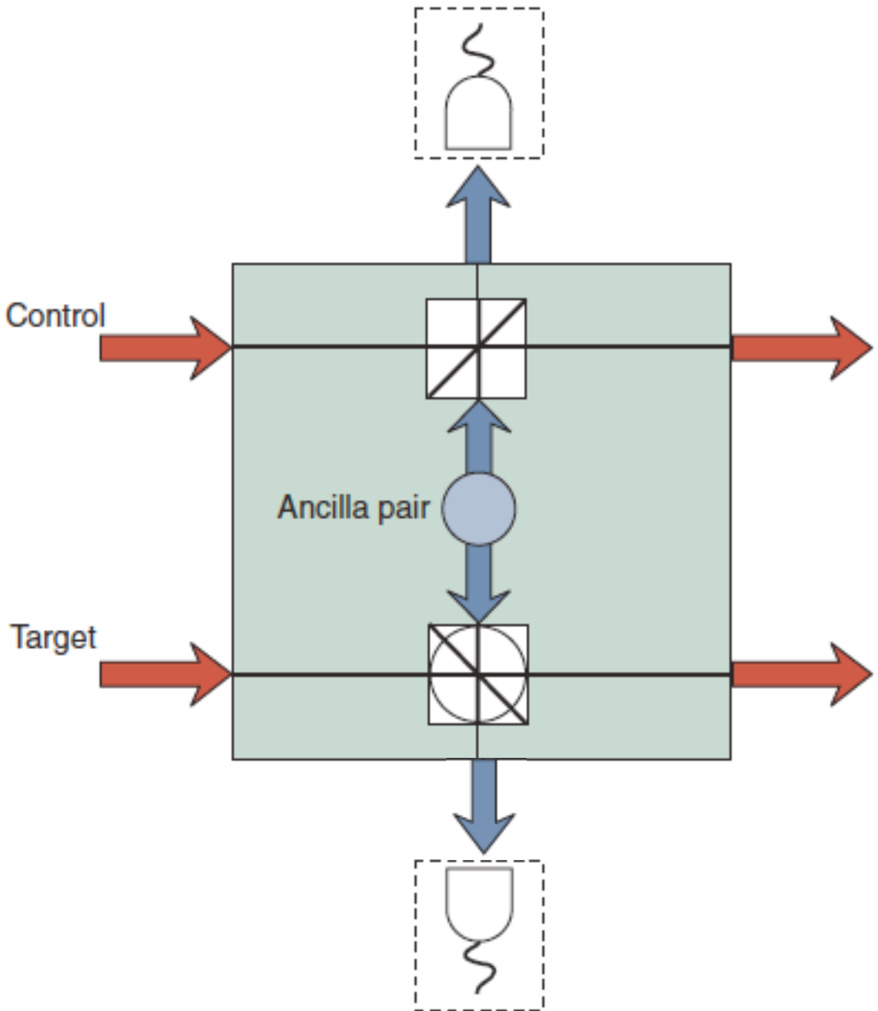
Probabilistic CNOT gate

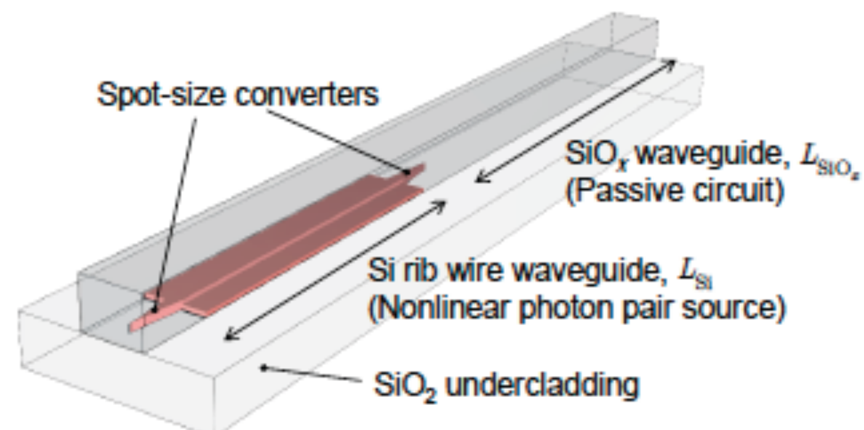




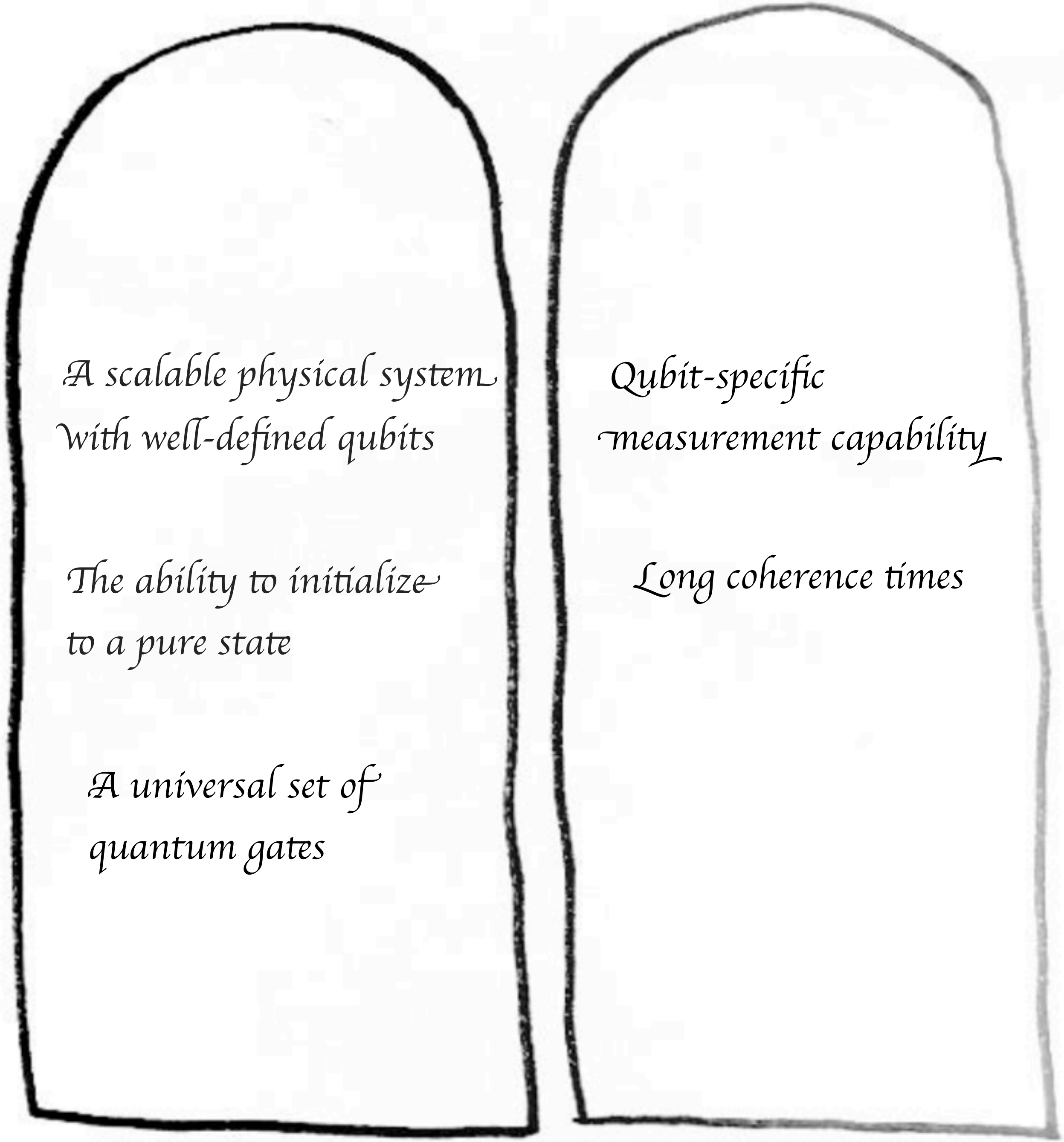
Spontaneous parametric down conversion:
rate of Bell pairs $\approx 10^5 s^{-1}$
Steinlechner *et al.*, Opt. Exp. (2012)
[experiment]

Probabilistic CNOT gate



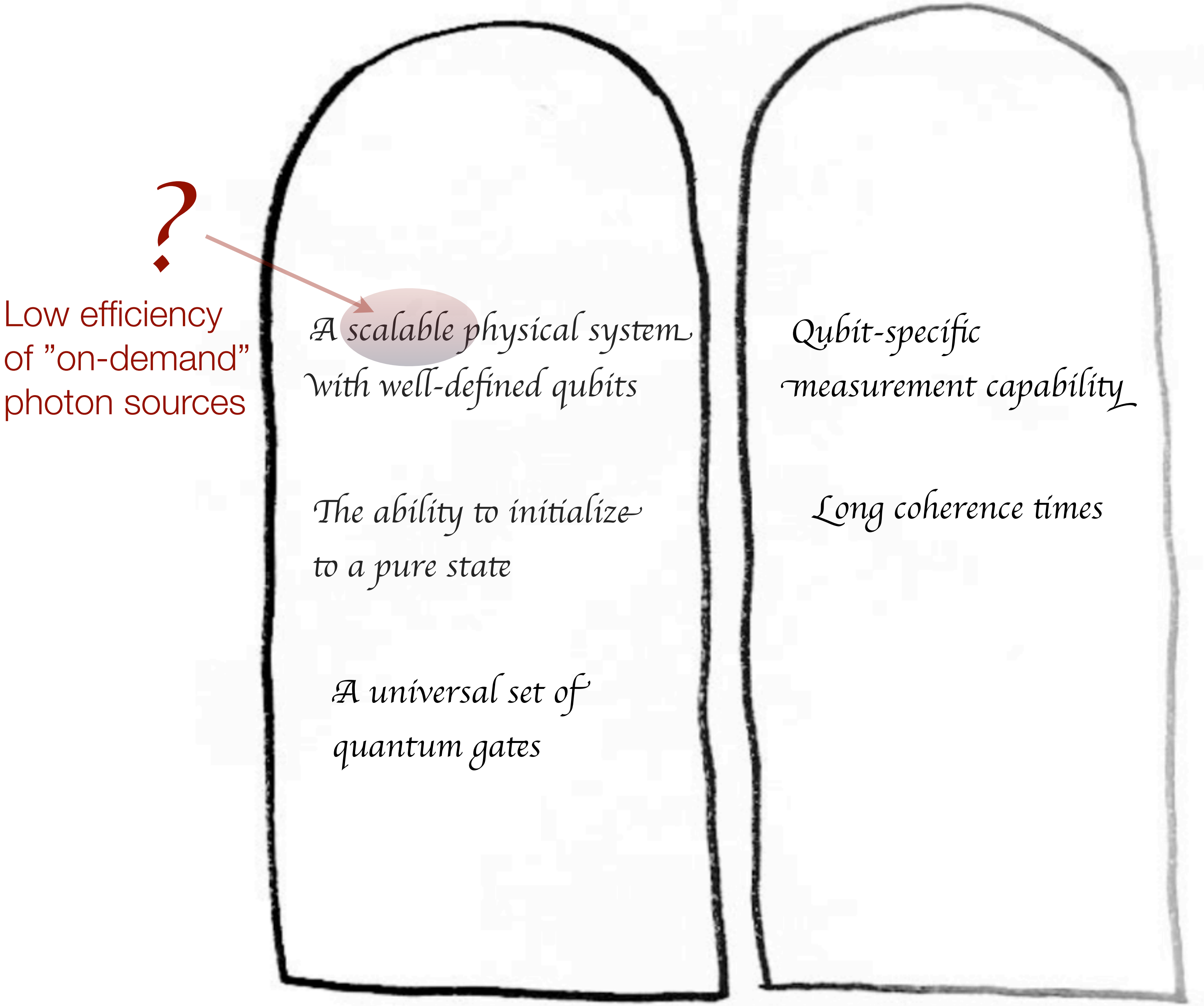


Silicon wave guide pair production:
rate of Bell pairs $\approx 10^3 s^{-1}$
Matsuda et *al.*, Opt. Exp. (2014)
[experiment]



DiVincenzo's criteria for quantum computation

DiVincenzo, Fortschr. Phys. (2000)



DiVincenzo’s criteria for quantum computation

DiVincenzo, Fortschr. Phys. (2000)

Electron quantum optics computing?

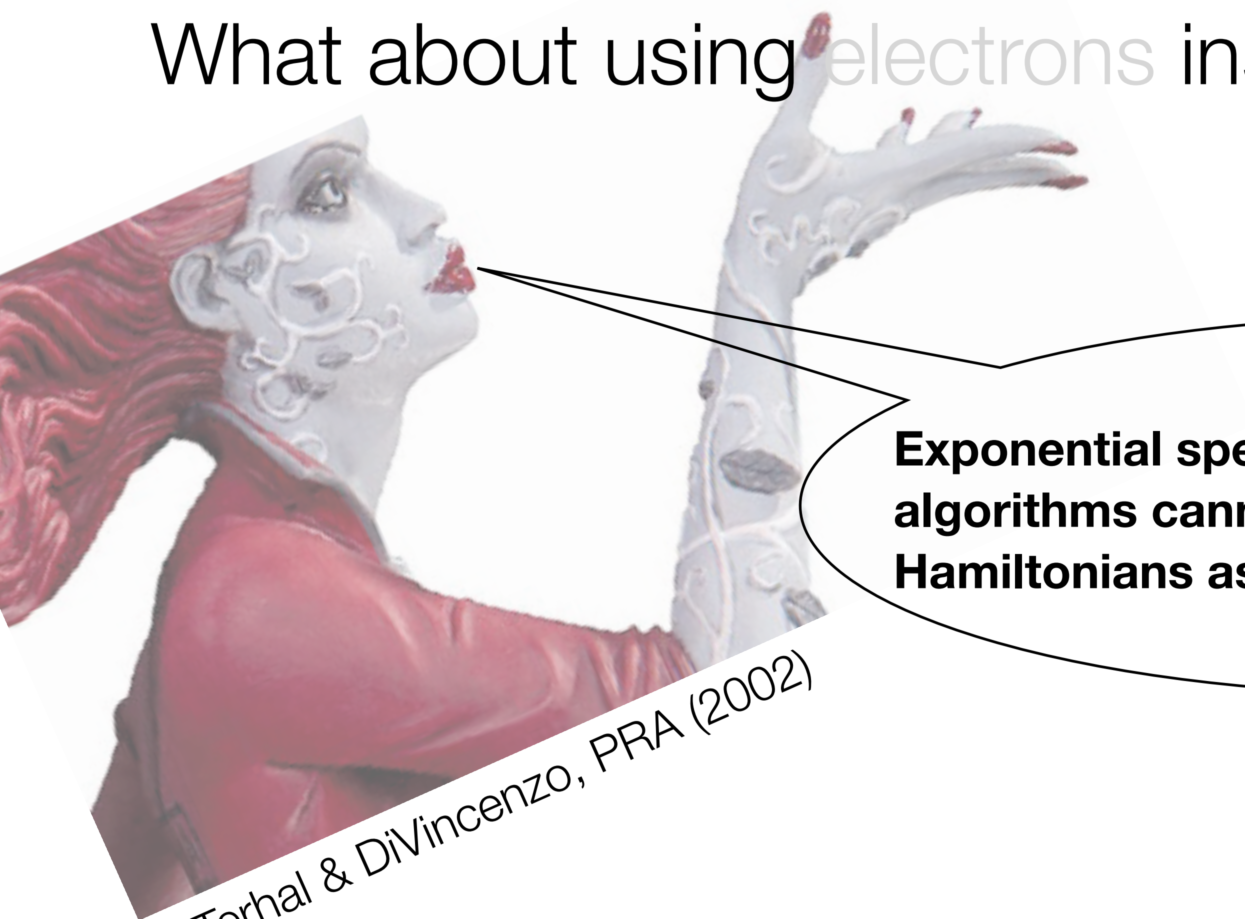
What about using electrons instead of photons?

Highly efficient ~ 1 THz single-electron sources are readily available!
Bocquillon *et al.*, Science (2013)

Spin-entanglement from 2-particle interferometry!
Bose & Home, PRL (2002)

Electron quantum optics computing?

What about using electrons instead of photons?

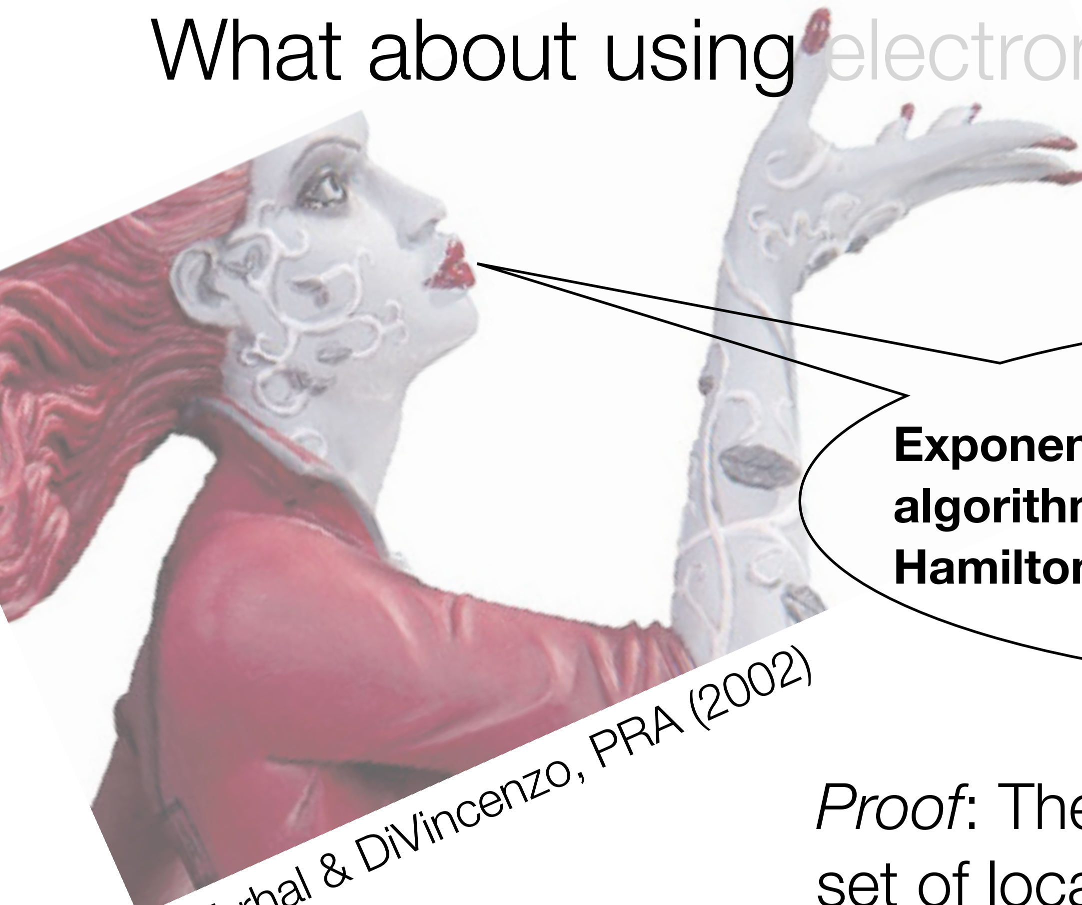


Exponential speed-up of quantum over classical algorithms cannot be reached with single-electron Hamiltonians assisted by spin measurements.

Terhal & DiVincenzo, PRA (2002)

Electron quantum optics computing?

What about using electrons instead of photons?



Terhal & DiVincenzo, PRA (2002)

Exponential speed-up of quantum over classical algorithms cannot be reached with single-electron Hamiltonians assisted by spin measurements.

Proof: The probability of the outcome of any set of local spin measurements $\sim\sqrt{\text{determinant}}$
Thus, computable in polynomial time!

Getting around the *No-go theorem* of Terhal & DiVincenzo:

Add charge measurements!

Beenakker *et al.,* PRL (2004)

Proof: The charge operator $Q_i = n_{i\uparrow} + n_{i\downarrow}$ is the sum of two local spin projection operators **—>** the probability that *M* sites are, say, singly occupied consists of 2^M determinants **—>** *not* computable in polynomial time!

Getting around the *No-go theorem* of Terhal & DiVincenzo:

Add charge measurements!

Beenakker *et al.*, PRL (2004)

Proof: The charge operator $Q_i = n_{i\uparrow} + n_{i\downarrow}$ is the sum of two local spin projection operators **—>** the probability that *M* sites are, say, singly occupied consists of 2^M determinants **—>** *not* computable in polynomial time!

Electron quantum optics OK for quantum computation!

Getting around the *No-go theorem* of Terhal & DiVincenzo:

Add charge measurements!

Beenakker *et al.*, PRL (2004)

Proof: The charge operator $Q_i = n_{i\uparrow} + n_{i\downarrow}$ is the sum of two local spin projection operators **—>** the probability that *M* sites are, say, singly occupied consists of 2^M determinants **—>** *not* computable in polynomial time!

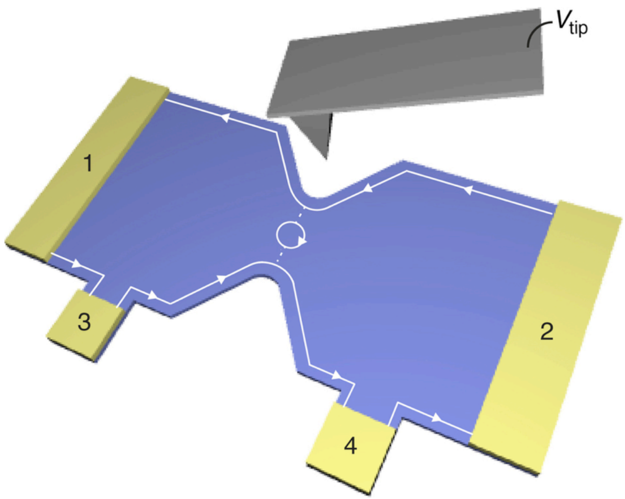
in principle

Electron quantum optics OK for quantum computation!

Implementations?

Implementations?

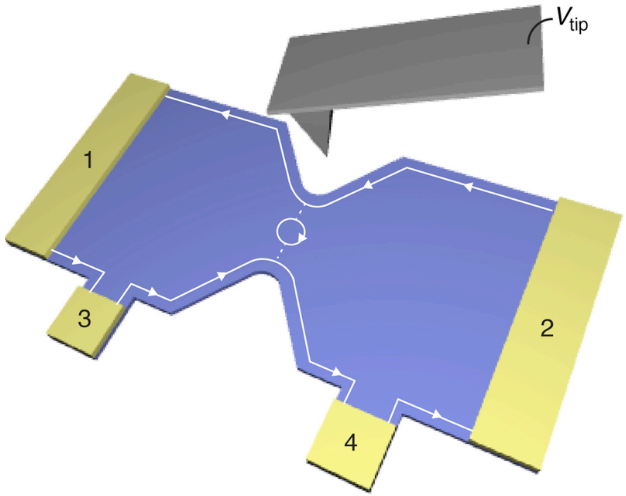
IQHE edge states?



Implementations?

IQHE edge states?

no spin!

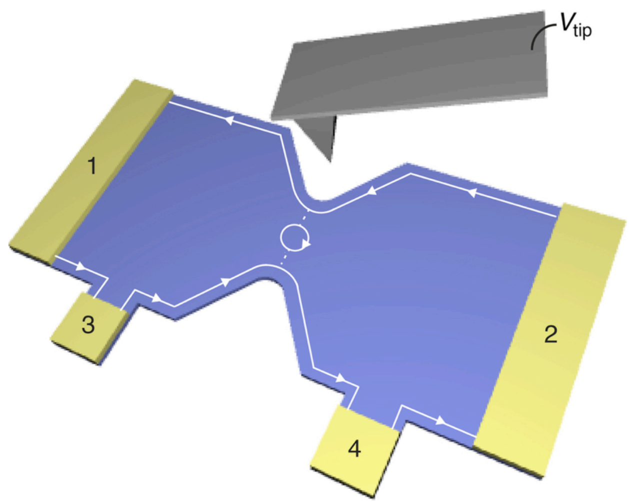


Electron quantum optics computing?

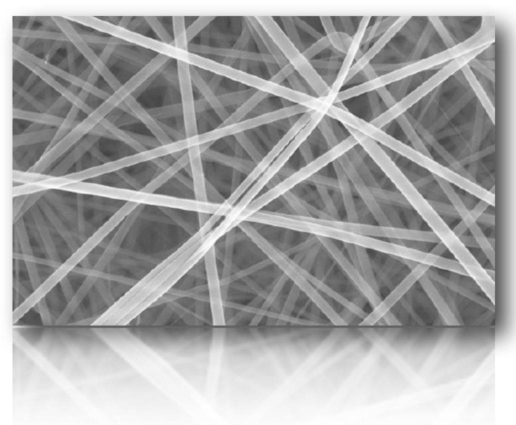
Implementations?

IQHE ~~edge states?~~

no spin!



Quantum wires?

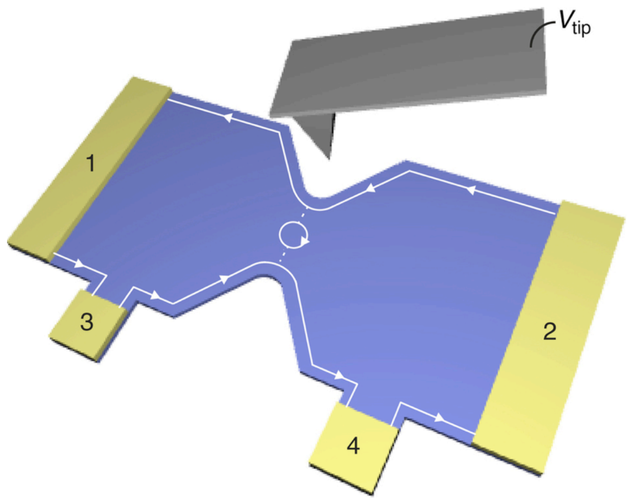


Electron quantum optics computing?

Implementations?

IQHE edge states?

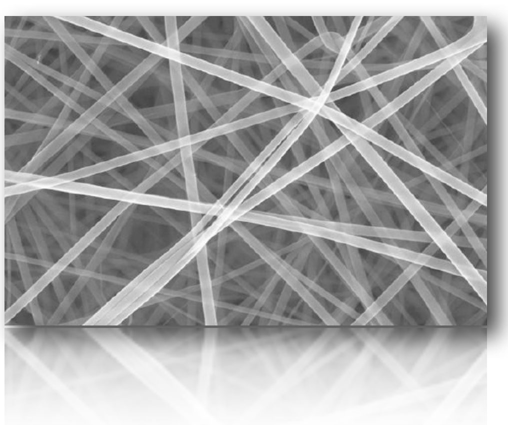
no spin!



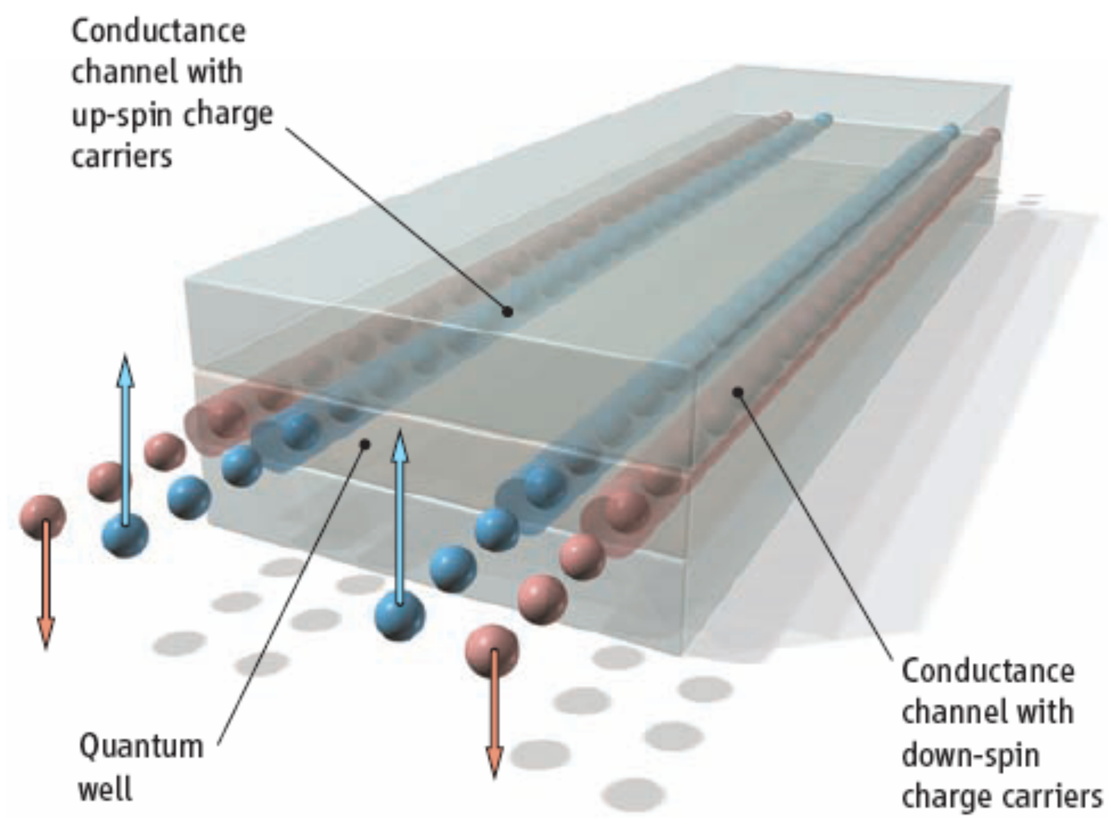
Quantum wires?

?

fragile electron transport ...



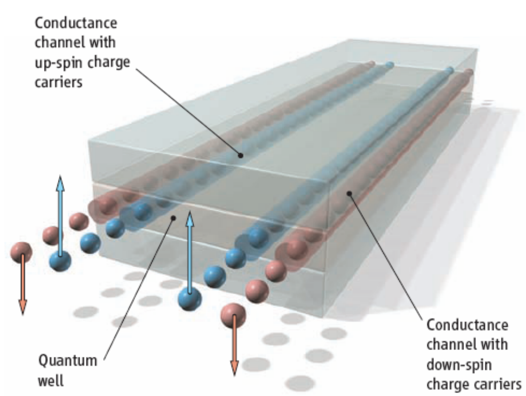
Quantum spin Hall edge states



from M. König *et al.*, Science **318**, 766 (2007)

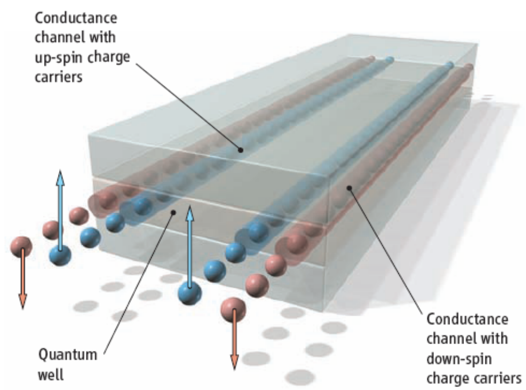
Quantum spin Hall physics: some basics

”Proof-of-concept”: Kane & Mele, two papers in PRL (2005)
Bernevig & Zhang, PRL (2006)
Prediction: Bernevig, Hughes & Zhang, Science (2006)
Experiment: König *et al.,* Science (2007)



Quantum spin Hall physics: some basics

"Proof-of-concept": Kane & Mele, two papers in PRL (2005)
Bernevig & Zhang, PRL (2006)
Prediction: Bernevig, Hughes & Zhang, Science (2006)
Experiment: König *et al.*, Science (2007)



Strong atomic spin-orbit interactions

↓

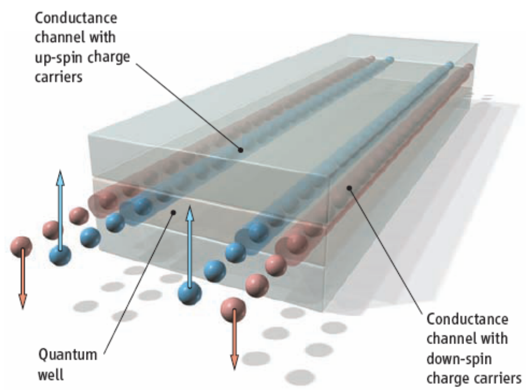
topologically nontrivial band structure

↓

insulating bulk, conducting edge

Quantum spin Hall physics: some basics

"Proof-of-concept": Kane & Mele, two papers in PRL (2005)
Bernevig & Zhang, PRL (2006)
Prediction: Bernevig, Hughes & Zhang, Science (2006)
Experiment: König *et al.*, Science (2007)



Strong atomic spin-orbit interactions

topologically nontrivial band structure

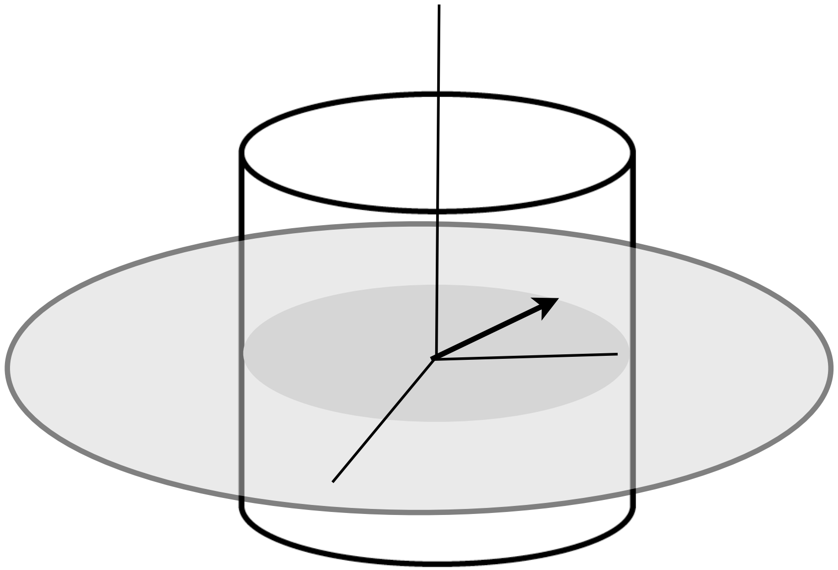
insulating bulk, perfectly spin-filtered **conducting edge states**

Quantum spin Hall physics: some basics

To develop some intuition, consider a Gedanken experiment...

Bernevig & Zhang, PRL (2006)

To develop some intuition, consider a Gedanken experiment...



uniformly charged cylinder with electric field

$\boldsymbol{E} = E(x, y, 0)$

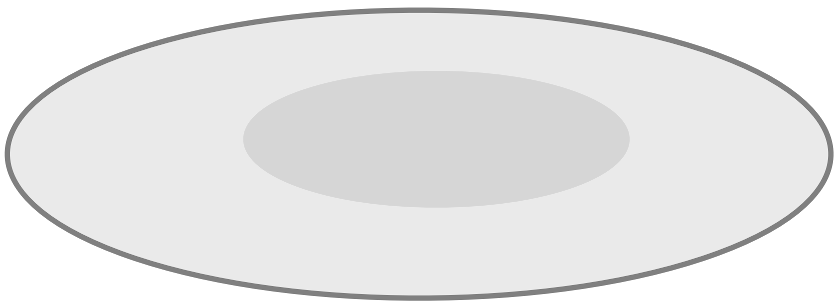
spin-orbit interaction

time-reversal invariant

$$(\boldsymbol{E} \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} = E\sigma^z (k_y x - k_x y)$$

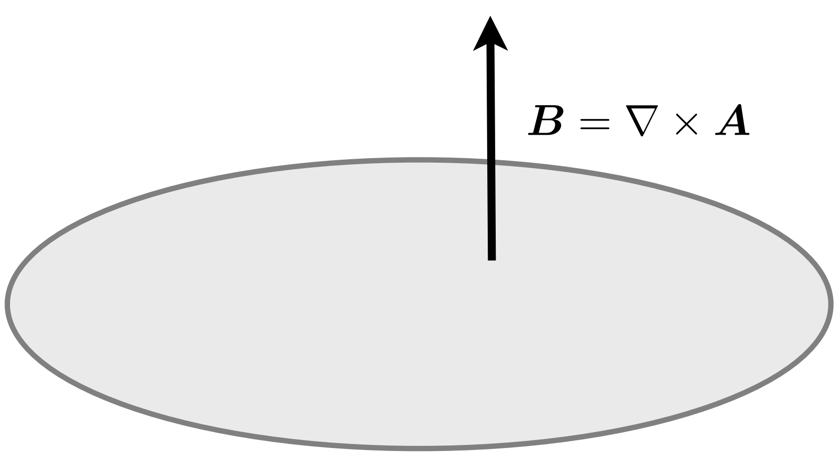
Quantum spin Hall physics: some basics

To develop some intuition, consider a Gedanken experiment...



spin-orbit interaction

$$(\boldsymbol{E} \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} = E\sigma^z (k_y x - k_x y)$$



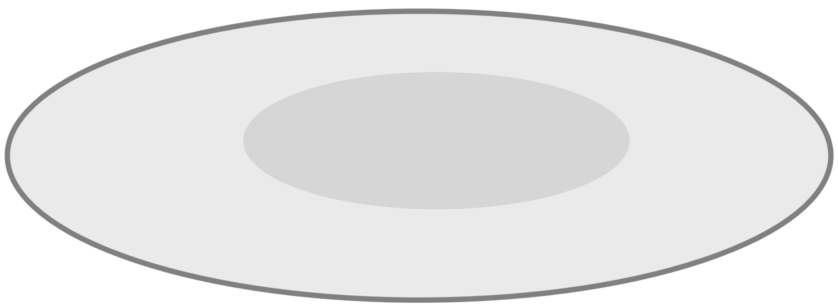
compare with an integer quantum Hall system

Lorentz force

$$\boldsymbol{A} \cdot \boldsymbol{k} \sim eB(k_y x - k_x y)$$

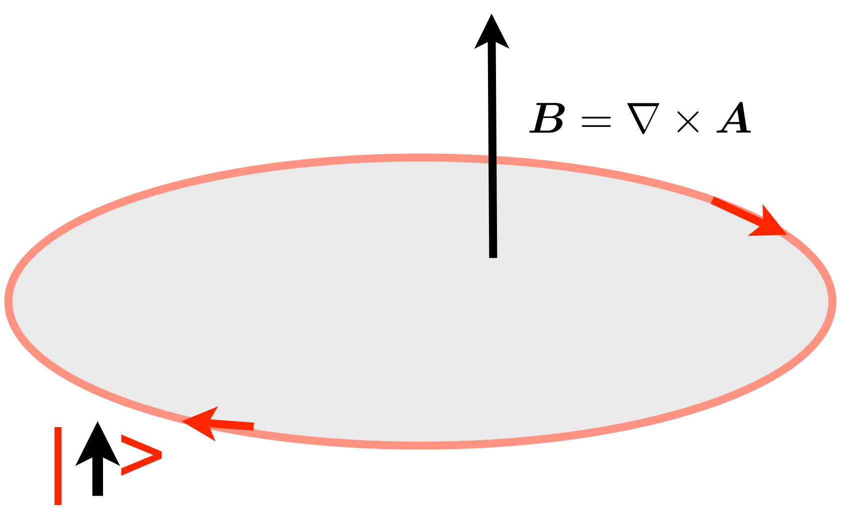
Quantum spin Hall physics: some basics

To develop some intuition, consider a Gedanken experiment...



spin-orbit interaction

$$(\boldsymbol{E} \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} = E\sigma^z (k_y x - k_x y)$$



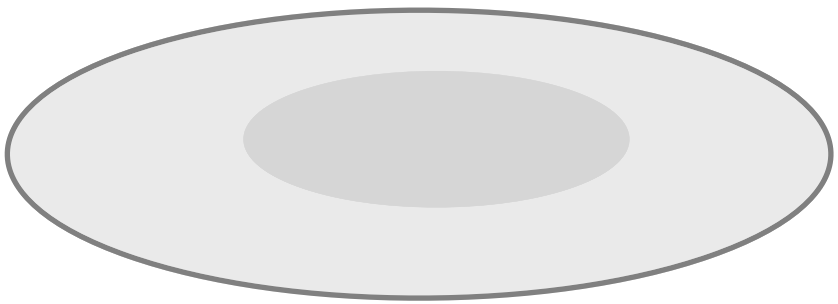
compare with an integer quantum Hall system

Lorentz force

$$\boldsymbol{A} \cdot \boldsymbol{k} \sim eB(k_y x - k_x y)$$

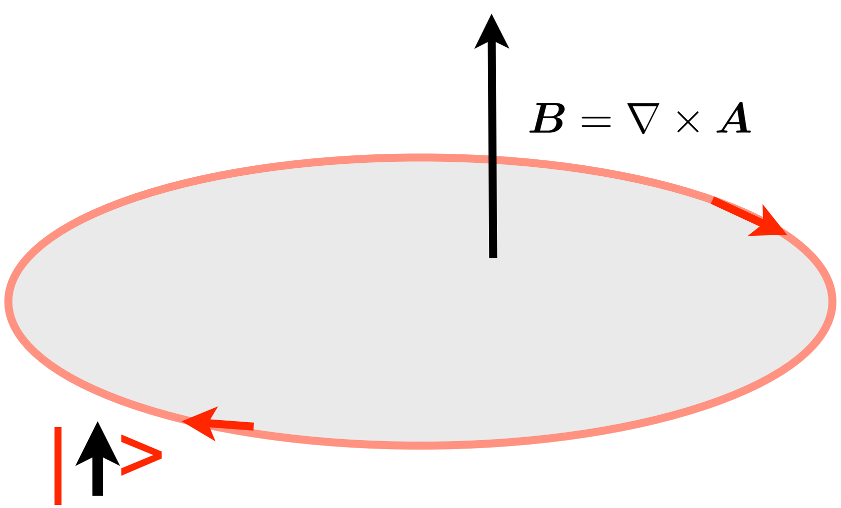
Quantum spin Hall physics: some basics

To develop some intuition, consider a Gedanken experiment...



spin-orbit interaction

$$(\boldsymbol{E} \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} = E\sigma^z(k_y x - k_x y)$$



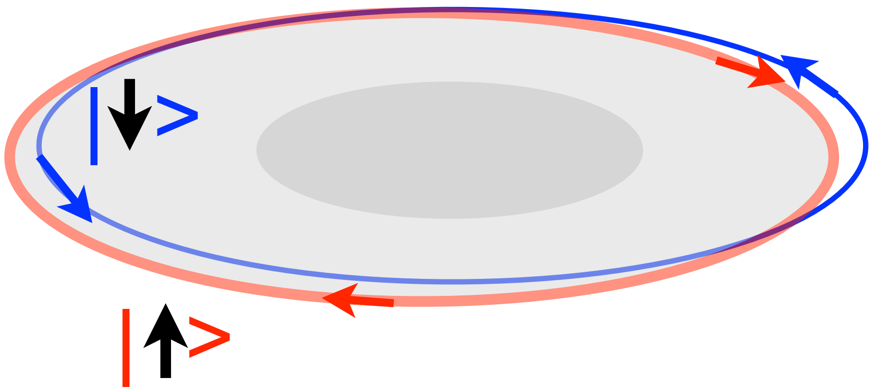
compare with an integer quantum Hall system

Lorentz force

$$\boldsymbol{A} \cdot \boldsymbol{k} \sim eB(k_y x - k_x y)$$

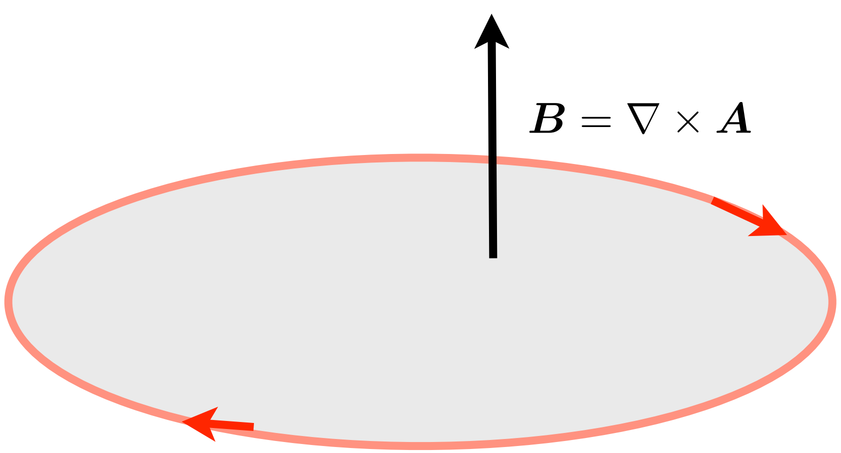
Quantum spin Hall physics: some basics

To develop some intuition, consider a Gedanken experiment...



spin-orbit interaction

$$(\boldsymbol{E} \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} = E\sigma^z(k_y x - k_x y)$$



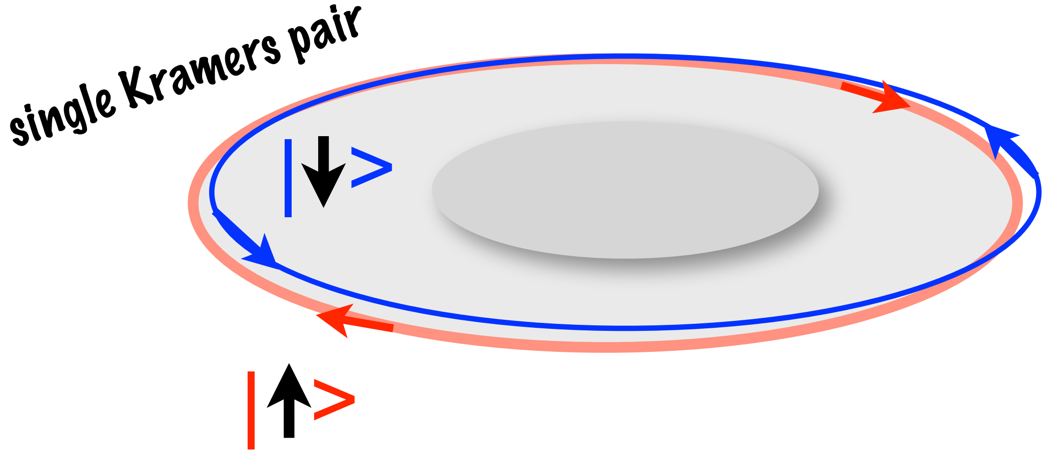
compare with an integer quantum Hall system

Lorentz force

$$\boldsymbol{A} \cdot \boldsymbol{k} \sim eB(k_y x - k_x y)$$

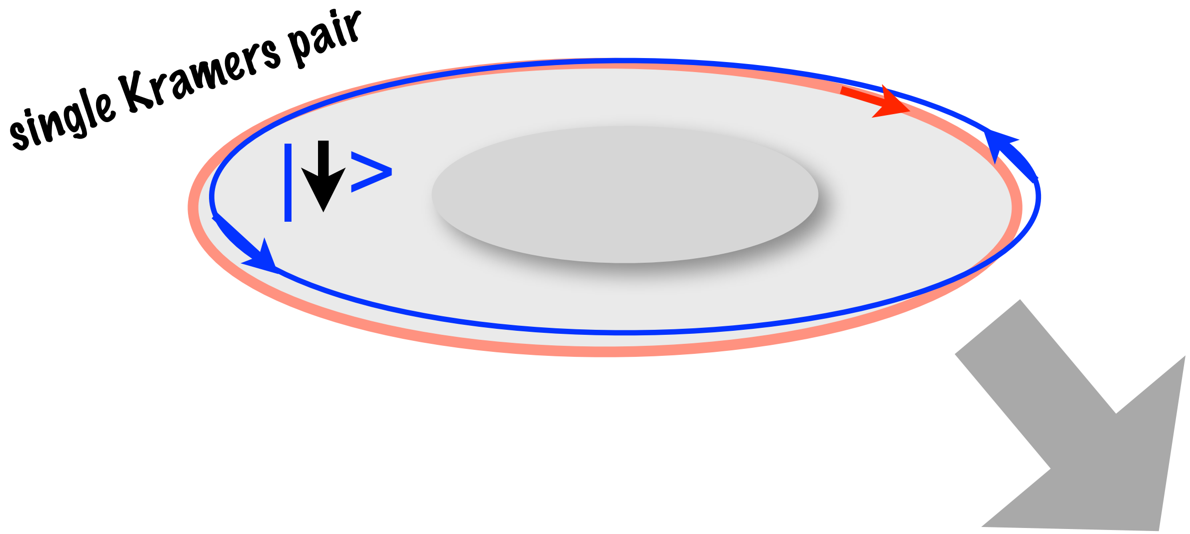
Quantum spin Hall (QSH) insulator

Two copies of a quantum Hall system, bulk insulator with **helical edge states**



Quantum spin Hall (QSH) insulator

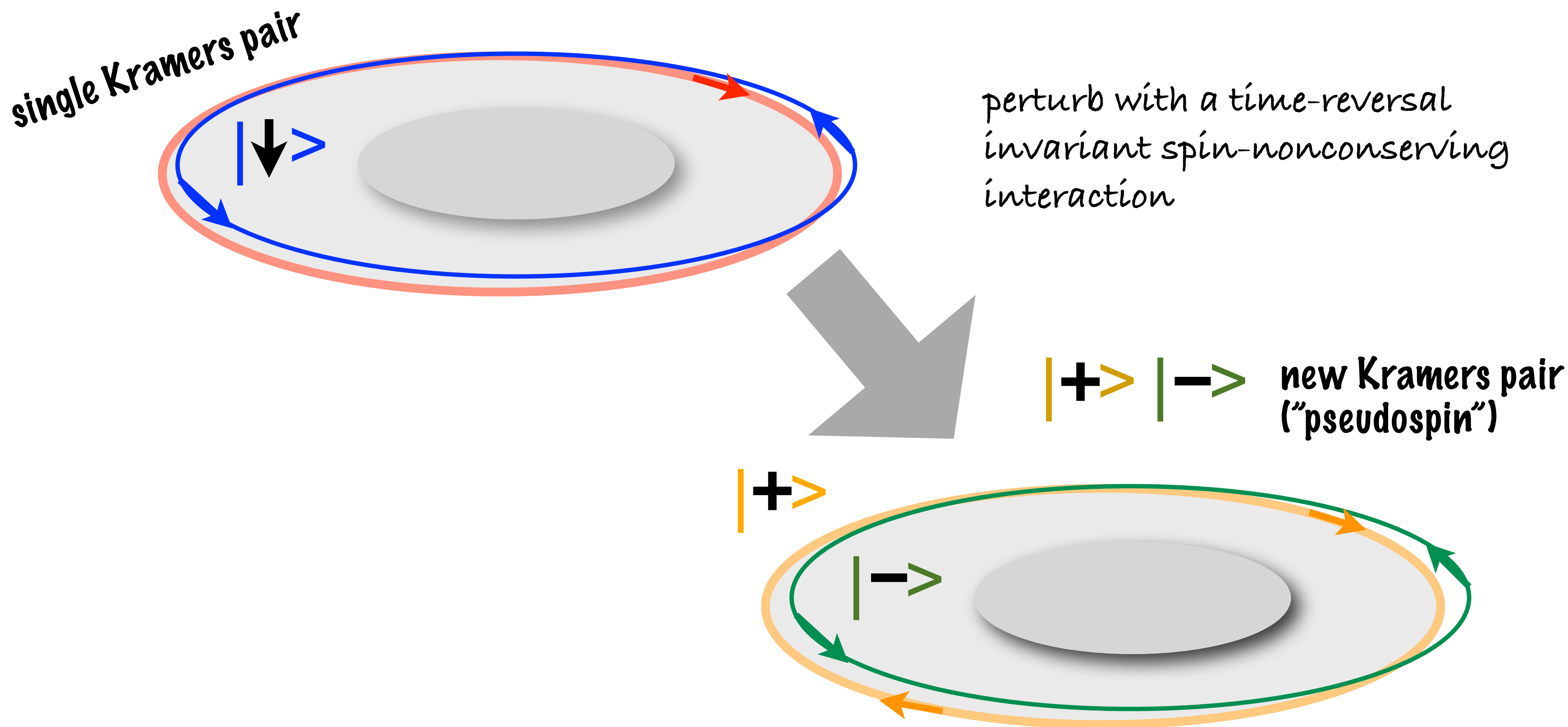
Two copies of a quantum Hall system, bulk insulator with **helical edge states**



perturb with a time-reversal invariant spin-nonconserving interaction

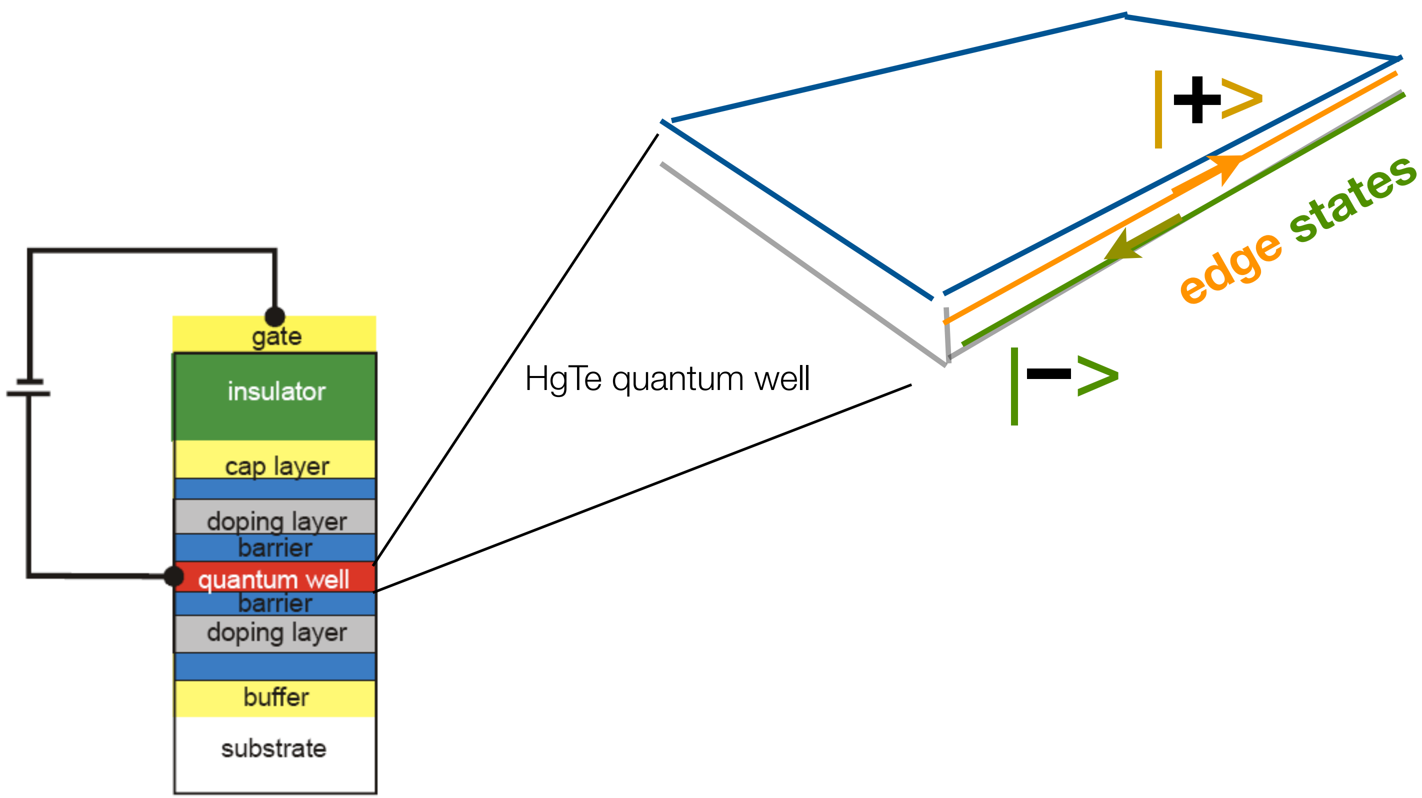
Quantum spin Hall (QSH) insulator

Two copies of a quantum Hall system, bulk insulator with **helical edge states**

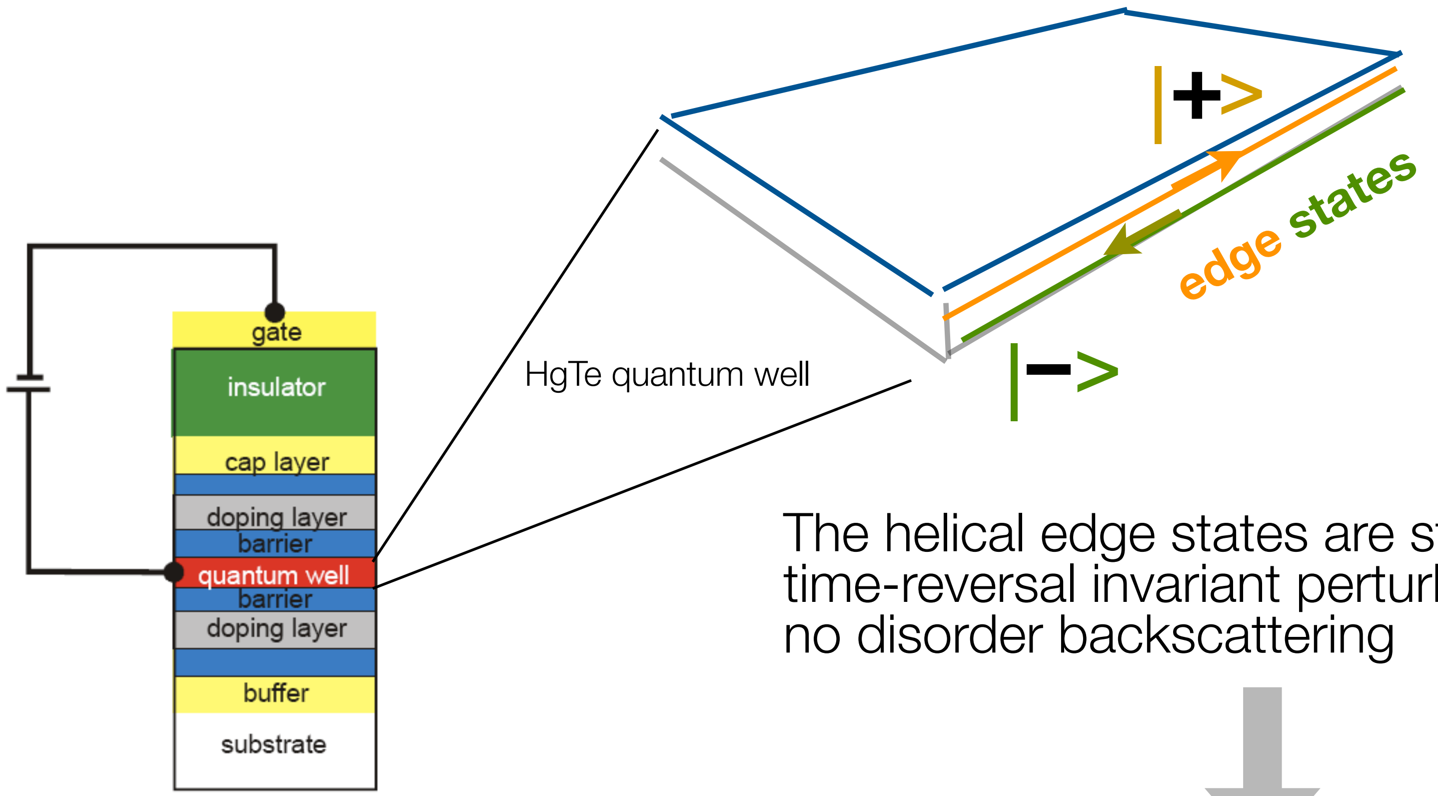


2D topological insulator

Experimental observation by König *et al*., Science, 2007



Experimental observation by König *et al*., Science, 2007



The helical edge states are stable against time-reversal invariant perturbations, no disorder backscattering

ballistic edge electrons carrying ”pseudospin”

ballistic edge electrons
carrying ~~pseudo~~spin **spin !**

ballistic edge electrons carrying ~~pseudo~~spin !

"BHZ" model for HgTe quantum wells

Bernevig, Hughes & Zhang, Science (2006)

$$|+\rangle = \Psi_1 |\, E_1 + \rangle + \Psi_2 |\, H_1 + \rangle$$

$$|-\rangle = \hat{T}|+\rangle = -\Psi_1^* |\, E_1 - \rangle - \Psi_2^* |\, H_1 - \rangle$$

$$|E_1 \pm\rangle = \alpha |\Gamma_6, \pm\frac{1}{2}\rangle + \beta |\Gamma_8, \pm\frac{1}{2}\rangle$$

$$|H_1 \pm\rangle = \alpha |\Gamma_8, \pm\frac{3}{2}\rangle$$

$$\begin{aligned} \langle + | \boldsymbol{S} | + \rangle &= \frac{\beta}{\sqrt{3}}(\Psi_2^*\Psi_1 + \Psi_1^*\Psi_2)\hat{X} \\ &+ \frac{i\beta}{\sqrt{3}}(-\Psi_2^*\Psi_1 + \Psi_1^*\Psi_2)\hat{Y} \\ &+ (|\Psi_2|^2\,[1 + |\alpha|^2] + \frac{|\Psi_1\beta|^2}{3})\hat{Z} \\ \langle - | \boldsymbol{S} | - \rangle &= -\langle + | \boldsymbol{S} | + \rangle \end{aligned}$$

choose spin quantization axis along $\langle + \,|\, \boldsymbol{S} \,|\, + \rangle$

spin-filtered helical edge states for constant $\Psi_{1,2}$

OK in a small energy range!

How to take advantage of the spin-filtered edge electrons?

How to take advantage of the spin-filtered edge electrons?

Add a superconductor
and a magnetic field

Majorana fermions!
Fu & Kane, PRL (2008)

Topological quantum computing?
Das Sarma, Freedman & Nayak, NJP (2015)
[review]

How to take advantage of the spin-filtered edge electrons?

Add a superconductor
and a magnetic field

Majorana fermions!

Fu & Kane, PRL (2008)

Topological quantum computing?

Das Sarma, Freedman & Nayak, NJP (2015)
[review]

How to take advantage of the spin-filtered edge electrons?

Add a superconductor
and a magnetic field

Majorana fermions!
Fu & Kane, PRL (2008)

Topological quantum computing?
Das Sarma, Freedman & Nayak, NJP (2015)
[review]

Electron quantum optics computing?

Basic resource: spin entanglement

Basic resource: spin entanglement

Solid-state proposals:

SN junctions: Recher, Sukhorukov & Loss, PRB (2001)
Lesovik, Martin & Blatter, EJP (2001)
Bena *et al.*, PRL (2002)
Schönberger *et al.,* Nature *(2009)* [experiment]
Hartmann *et al.,* PRL (2010) [experiment]
Wei & Chandrasekhar, Nat. Phys. (2010) [experiment]

3-terminal quantum dot device: Oliver, Yamaguchi & Yamamoto, PRL (2002)

Quantum-wire interferometry: Signal & Zülicke, APL (2005)

Kondo scattering: Costa & Bose, PRL (2001)
Sodano, Bayat & Bose, PRB (2010)

Majorana entangler: Sodano & Bose NJP (2011)

QSH edge interferometry: Inhofer & Bercioux, PRB (2013)
Sato, Trif & Tserkovnyak, PRB (2014)

Basic resource: spin entanglement

Solid-state proposals:

SN junctions: Recher, Sukhorukov & Loss, PRB (2001)
Lesovik, Martin & Blatter, EJP (2001)
Bena *et al.*, PRL (2002)
Schönberger *et al.*, Nature *(2009)* [experiment]
Hartmann *et al.*, PRL (2010) [experiment]
Wei & Chandrasekhar, Nat. Phys. (2010) [experiment]

3-terminal quantum dot device: Oliver, Yamaguchi & Yamamoto, PRL (2002)

Quantum-wire interferometry: Signal & Zülicke, APL (2005)

Kondo scattering: Costa & Bose, PRL (2001)
Sodano, Bayat & Bose, PRB (2010)

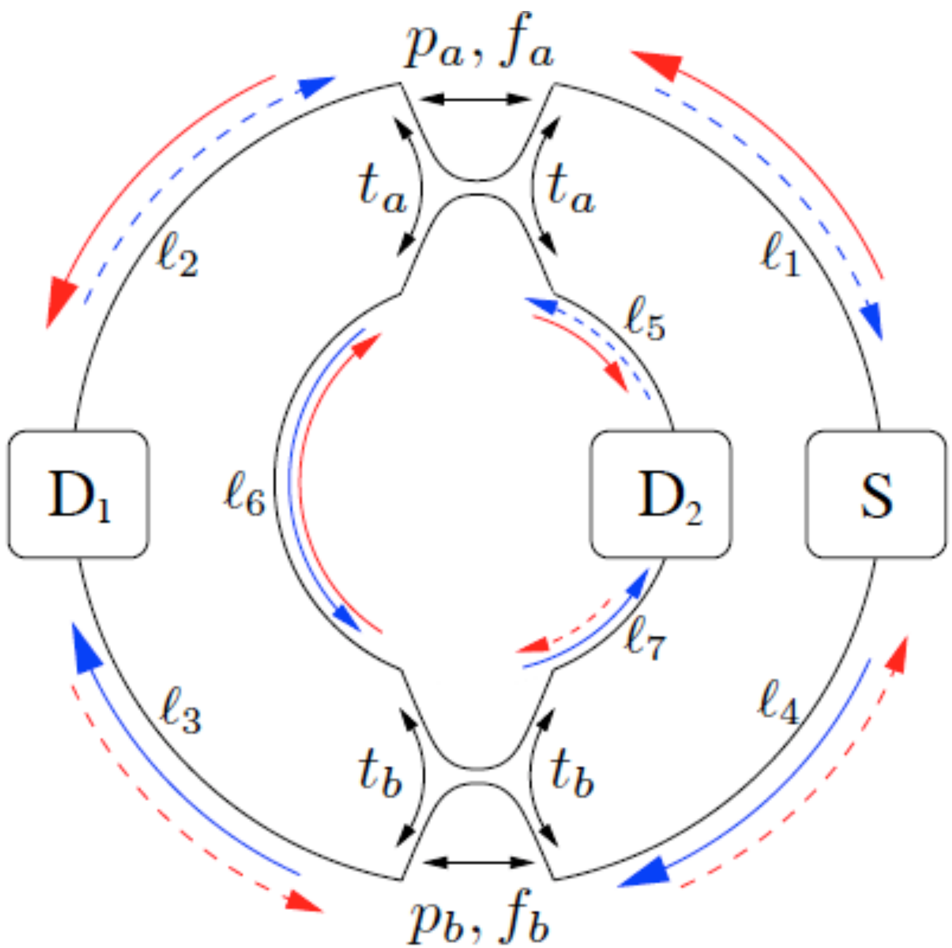
Majorana entangler: Sodano & Bose NJP (2011)

QSH edge interferometry: Inhofer & Bercioux, PRB (2013)
Sato, Trif & Tserkovnyak, PRB (2014)

***"Optimally controlled" QSH edge interferometry:* our work, PRB (2015)**

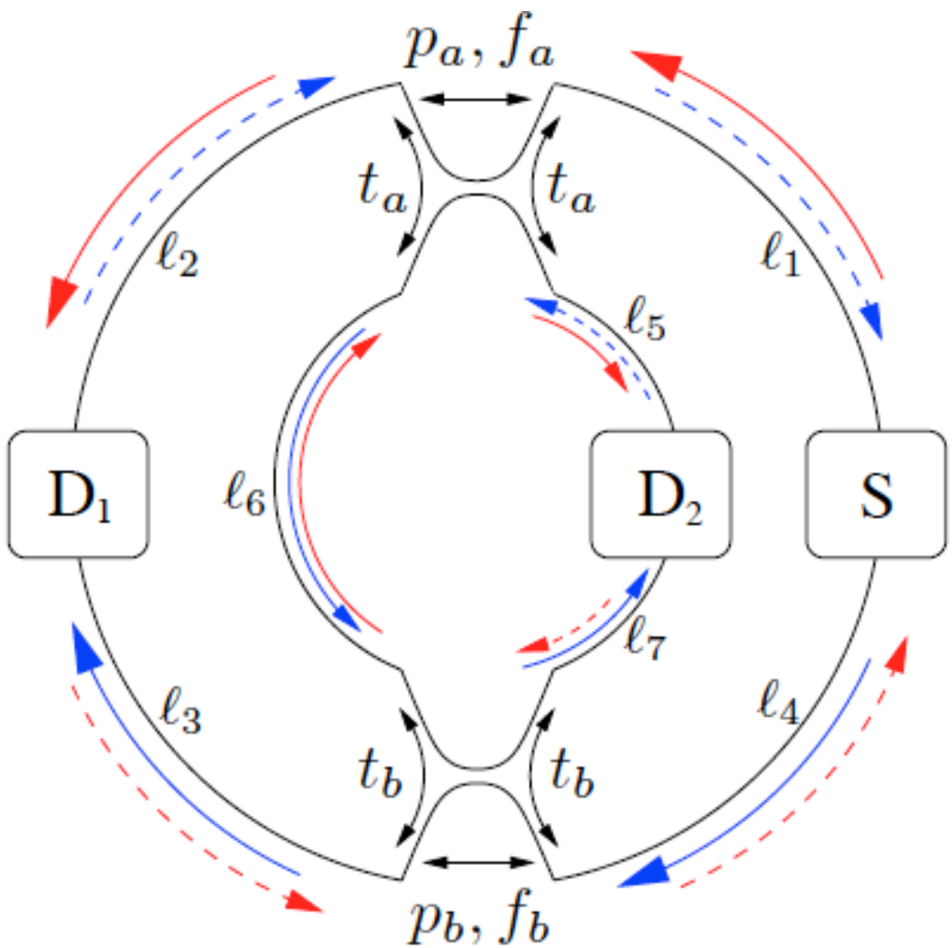
"On-demand" electronic spin entangler

A. Ström, H. J. & P. Recher, PRB **91**, 245406 (2015)



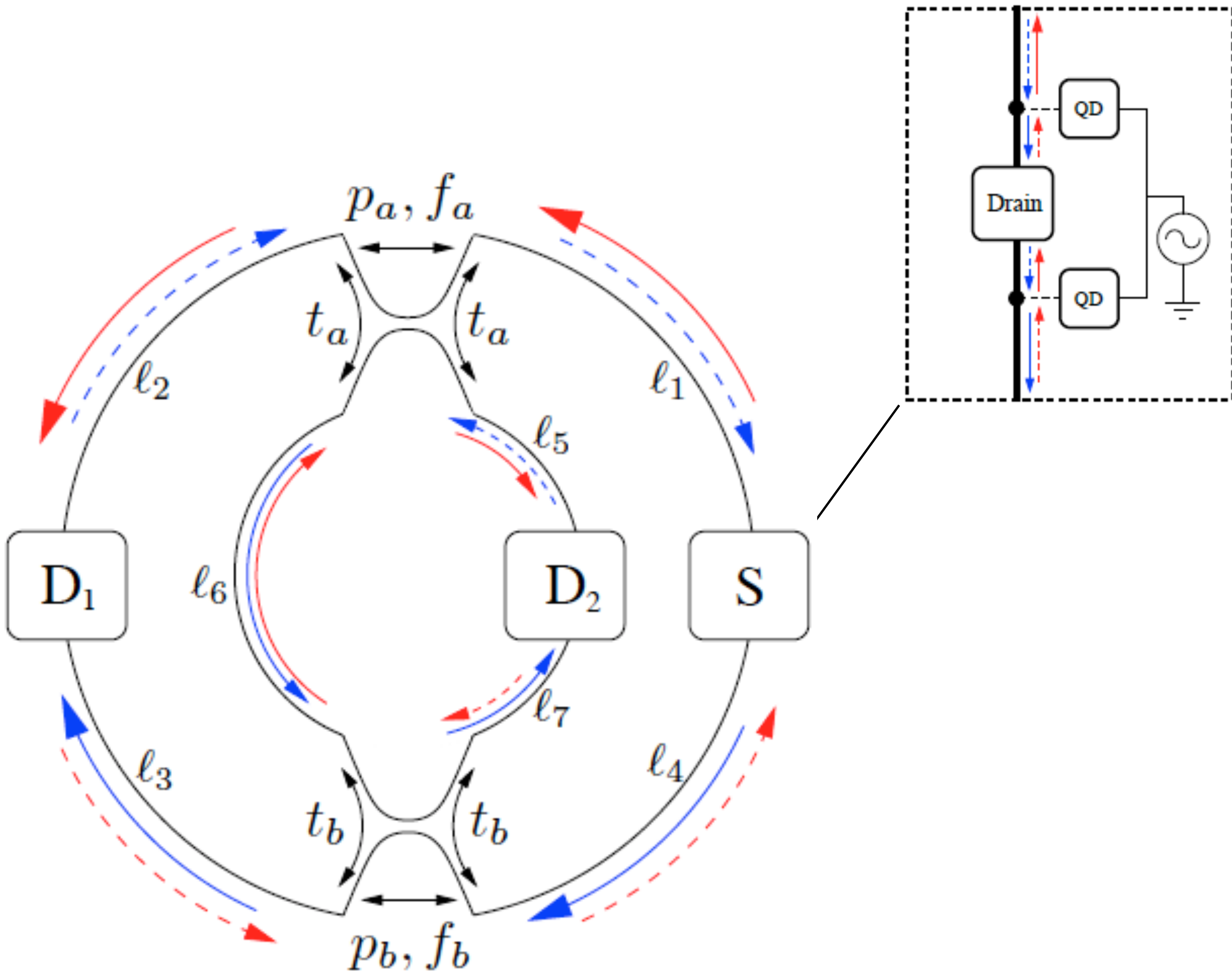
''On-demand'' electronic spin entangler

A. Ström, H. J. & P. Recher, PRB **91**, 245406 (2015)

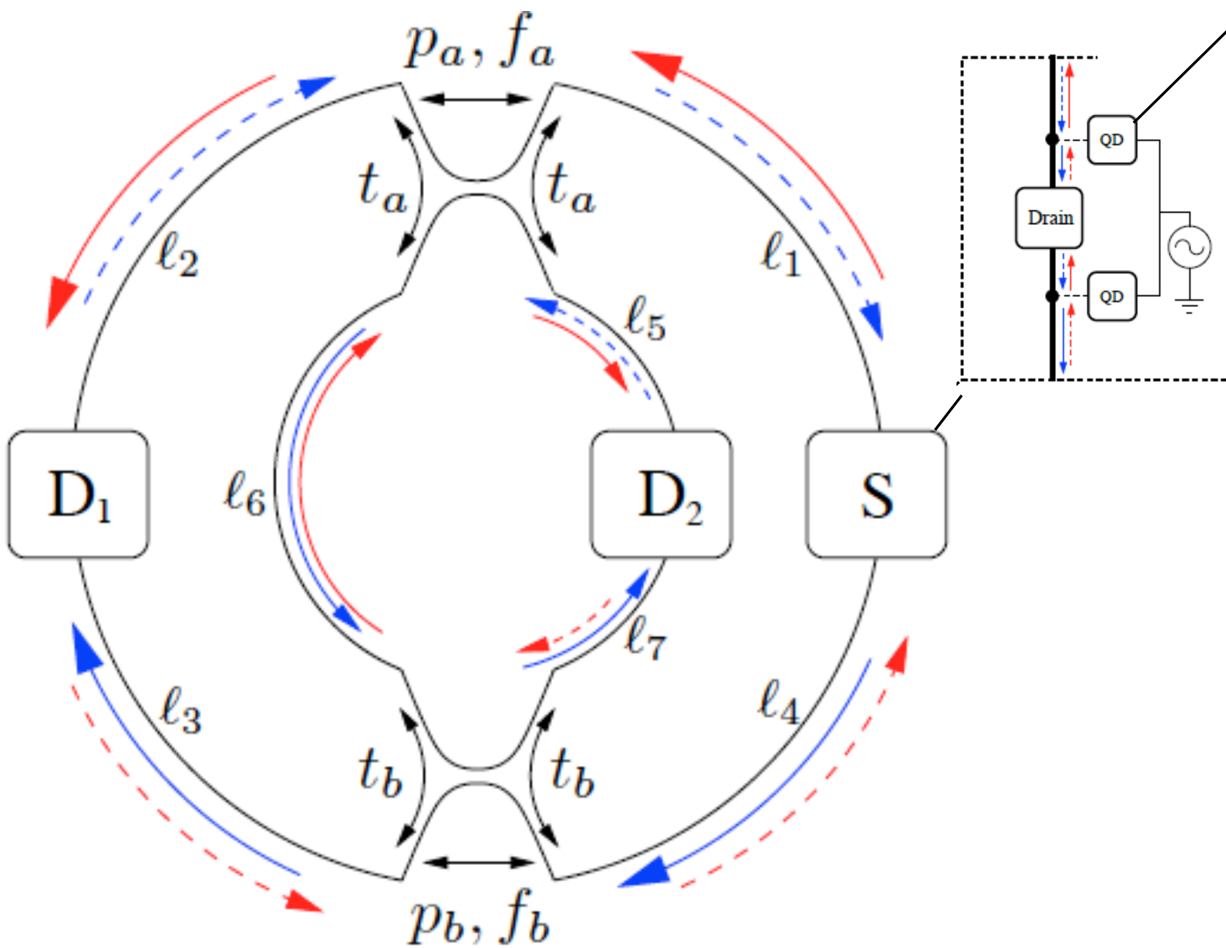


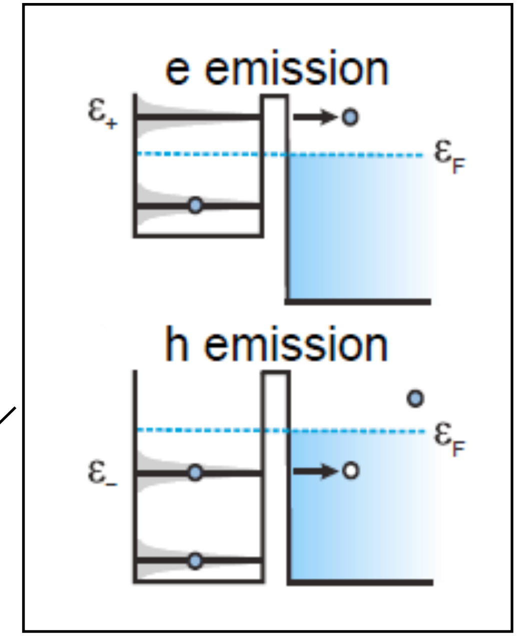
Experiments on rings in HgTe quantum wells:
König *et al.*, PRL (2005)

QSH spin entangler



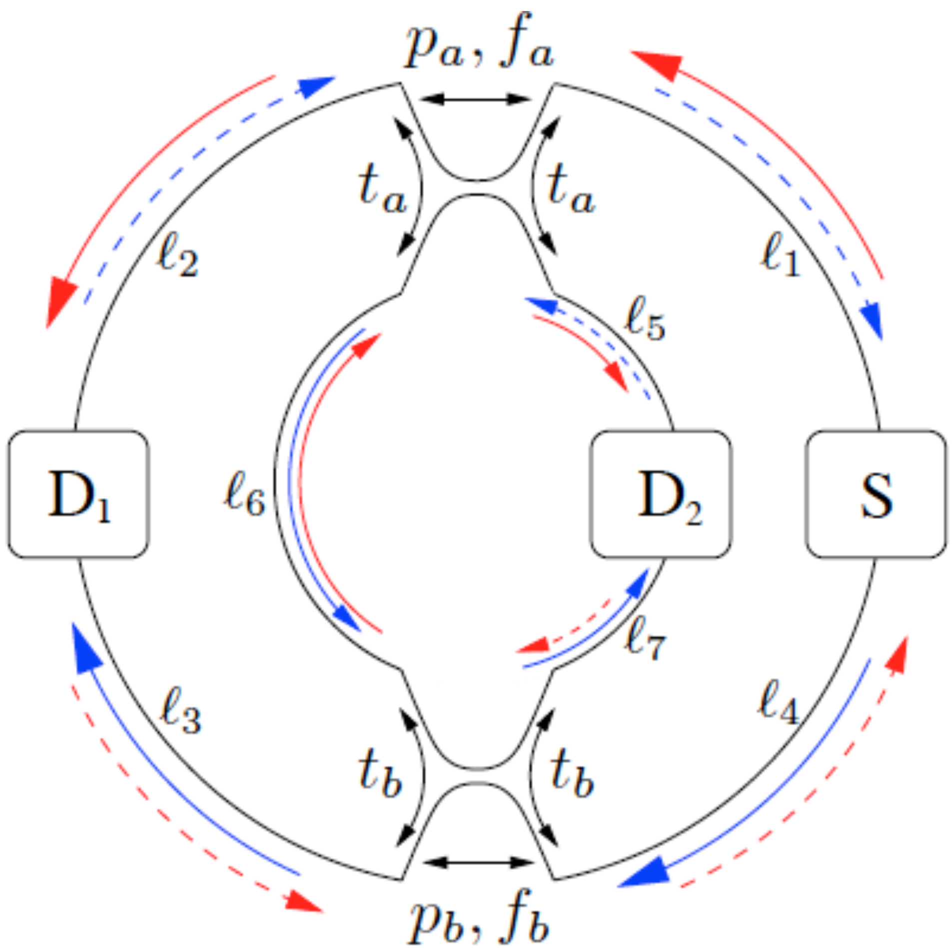
QSH spin entangler

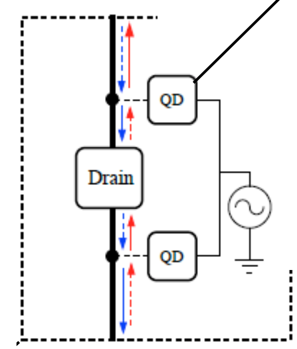


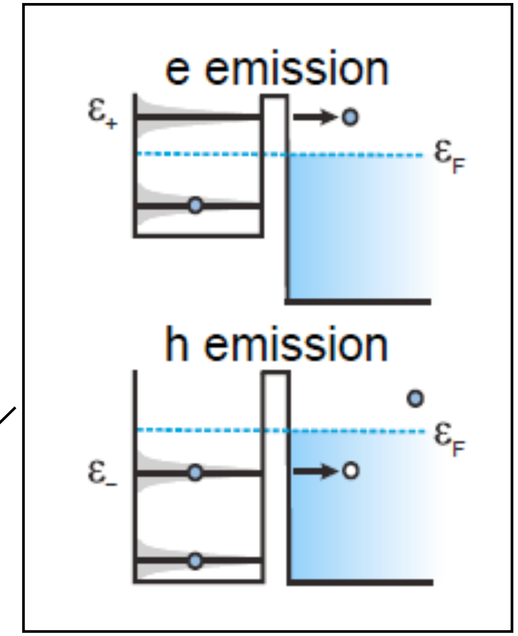


Bocquillon *et al.*, Science (2013)

QSH spin entangler

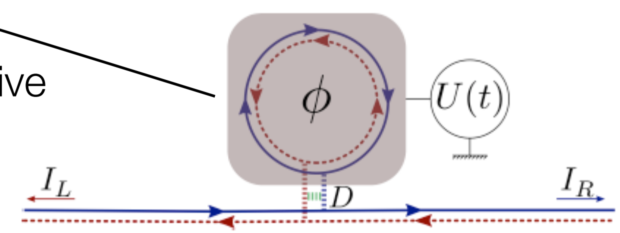






Bocquillon *et al.*, Science (2013)
[experiment]

alternative

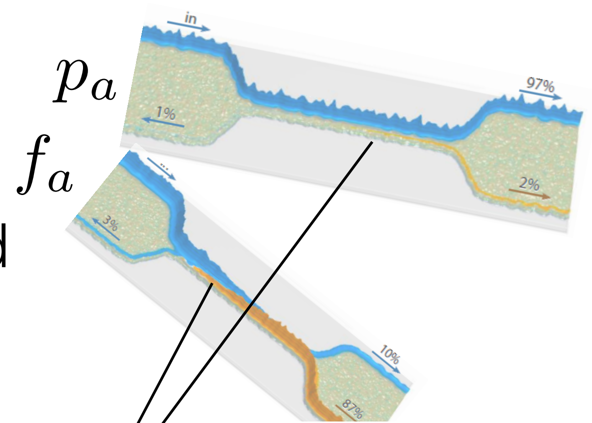


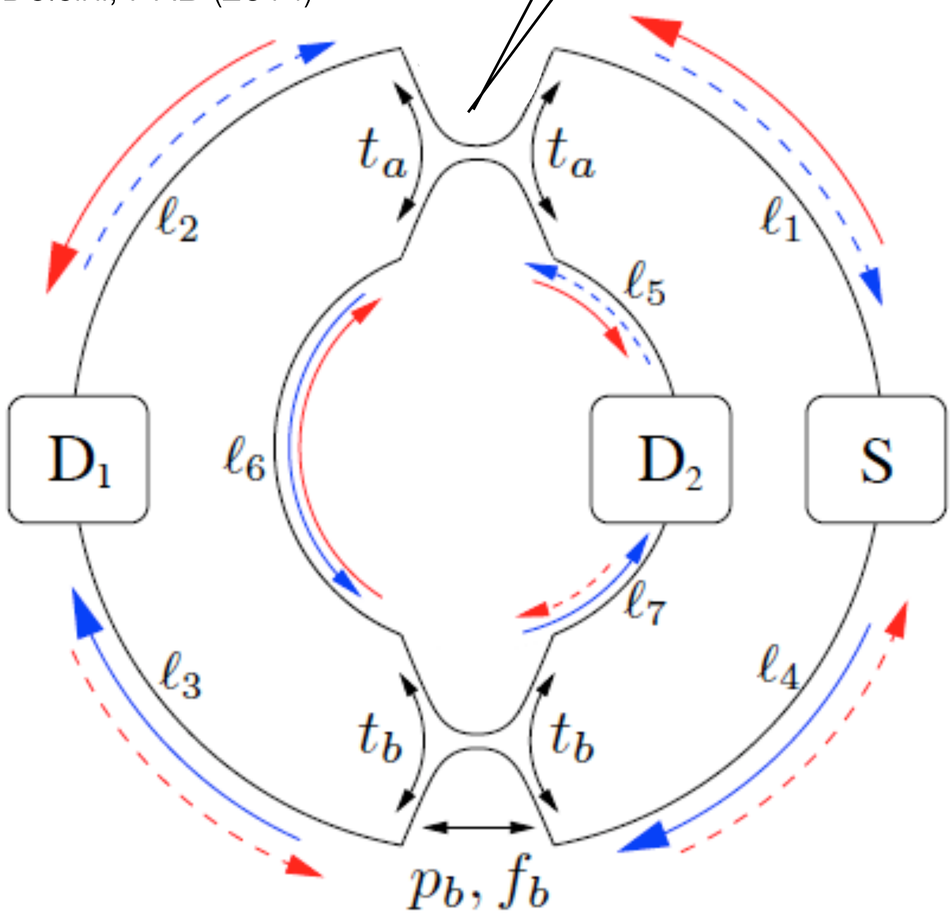
Hofer & Büttiker, PRB (2013)
Inhofer & Bercioux, PRB (2013)
[theory]

QSH spin entangler

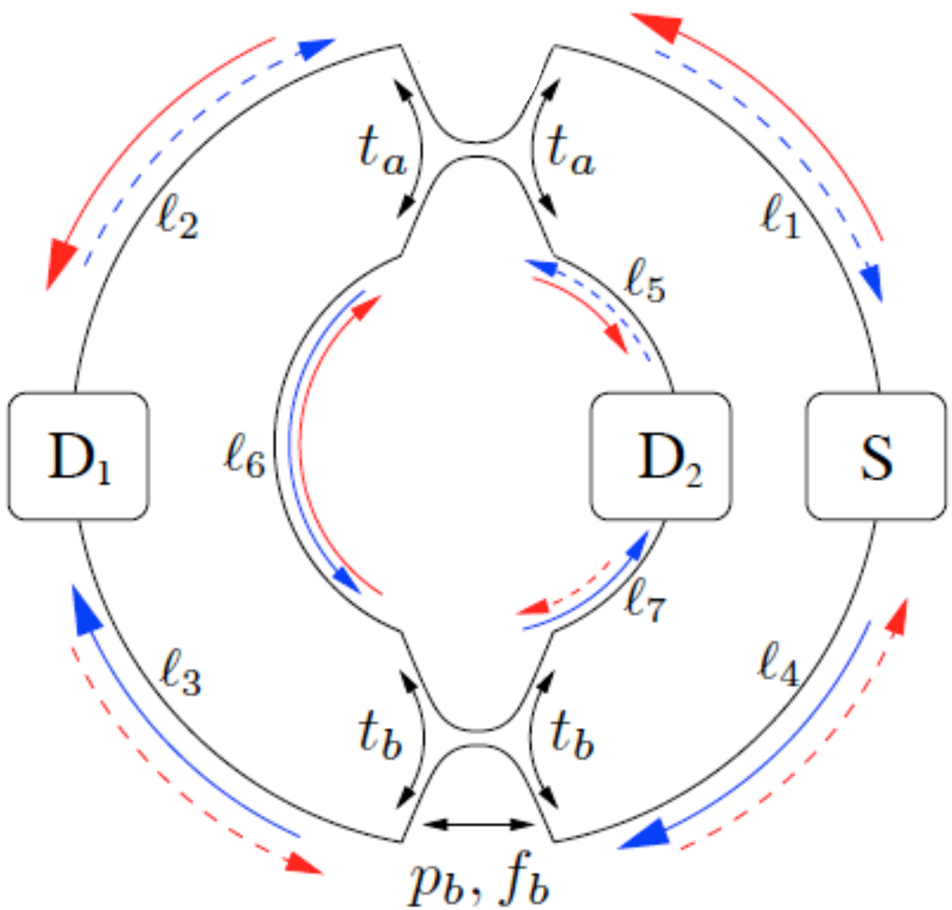
Rashba gate-controlled tunneling amplitudes

Krueckl & Richter, PRL (2011)
Sternativo & Dolcini, PRB (2014)
[theory]

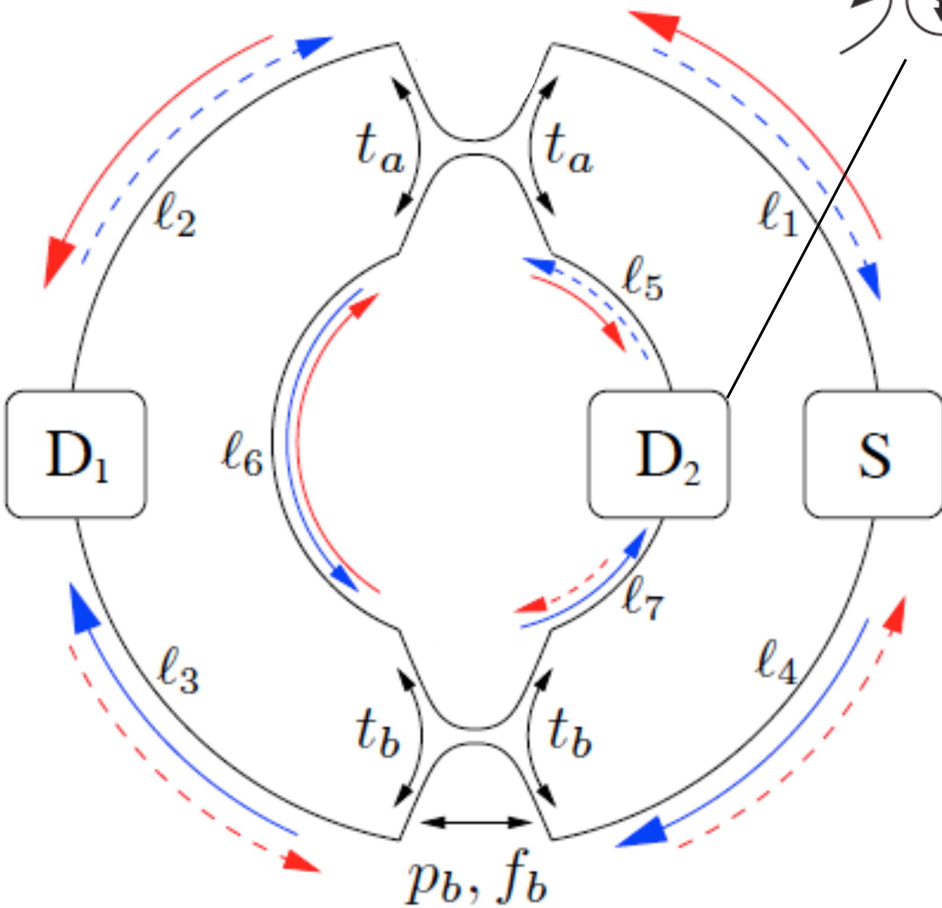


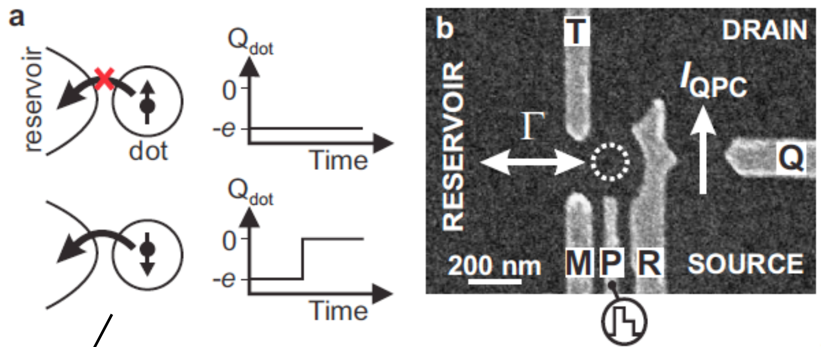


QSH spin entangler



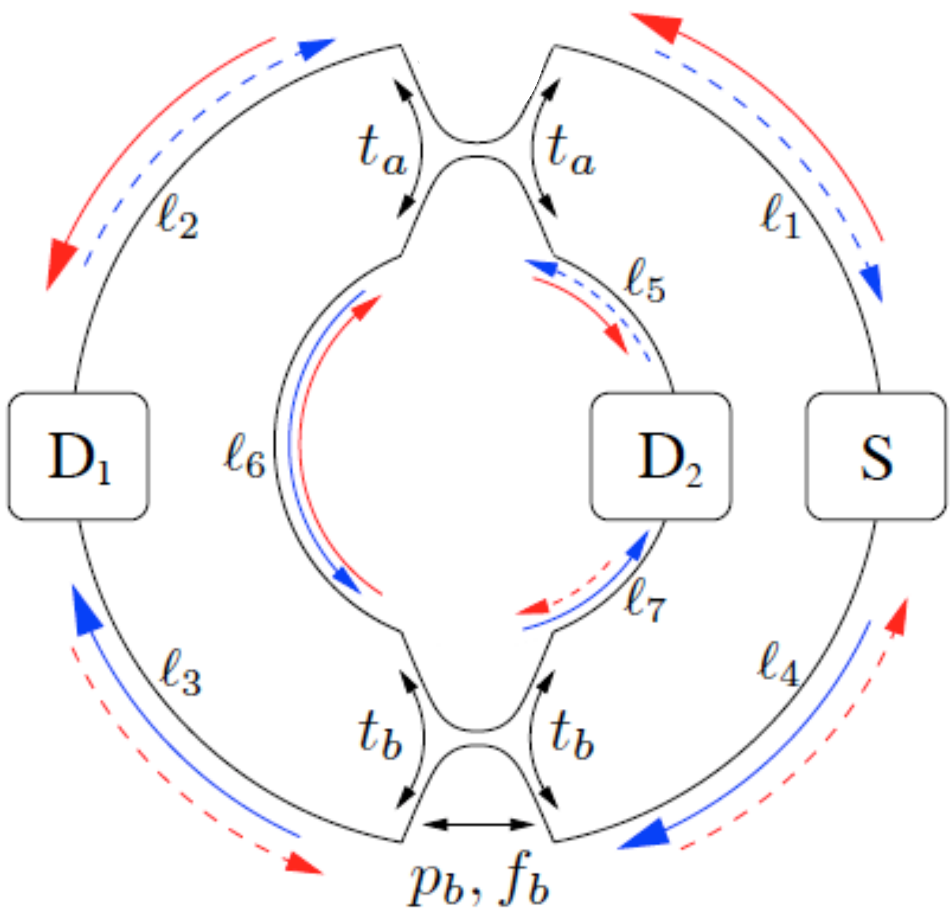
QSH spin entangler



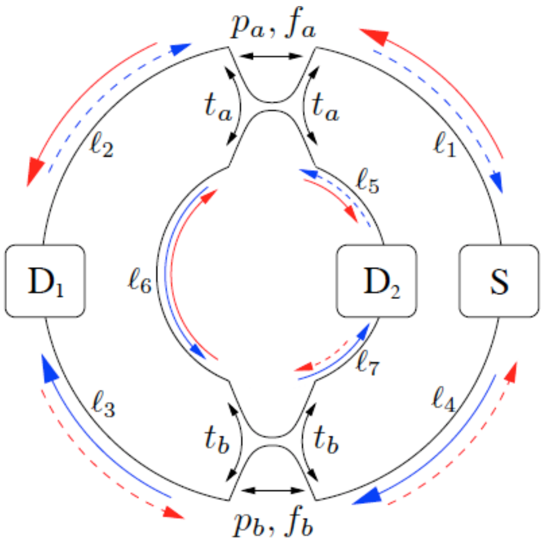


Elzerman *et al.*, Nature (2004) [experiment]

QSH spin entangler

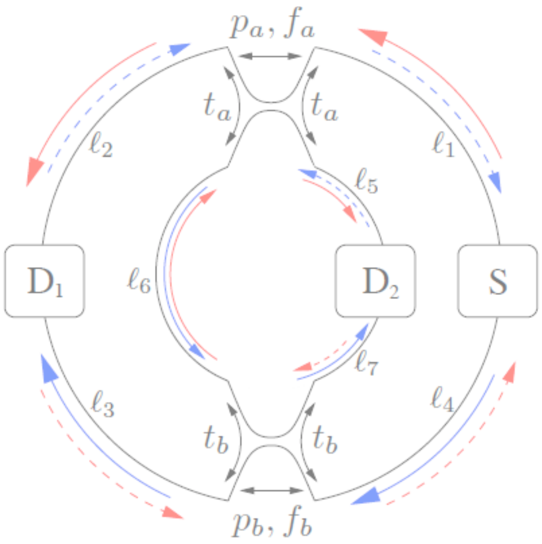


QSH spin entangler



$$|\Psi_{\text{in}}\rangle \xrightarrow{S} |\Psi_{\text{out}}\rangle$$

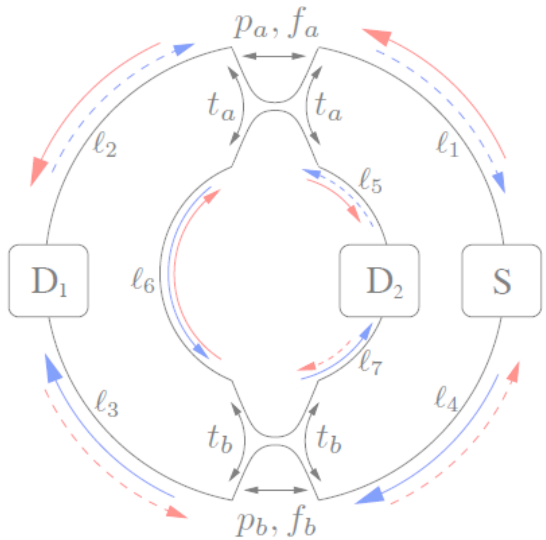
QSH spin entangler



$$|\Psi_{\text{in}}\rangle \xrightarrow{S} |\Psi_{\text{out}}\rangle$$

$$\begin{pmatrix} b_{1\uparrow} \\ b_{1\downarrow} \\ b_{2\uparrow} \\ b_{2\downarrow} \\ b_{S\uparrow} \\ b_{S\downarrow} \end{pmatrix} = S \begin{pmatrix} a_{1\downarrow} \\ a_{1\uparrow} \\ a_{2\downarrow} \\ a_{2\uparrow} \\ a_{S\downarrow} \\ a_{S\uparrow} \end{pmatrix}$$

QSH spin entangler

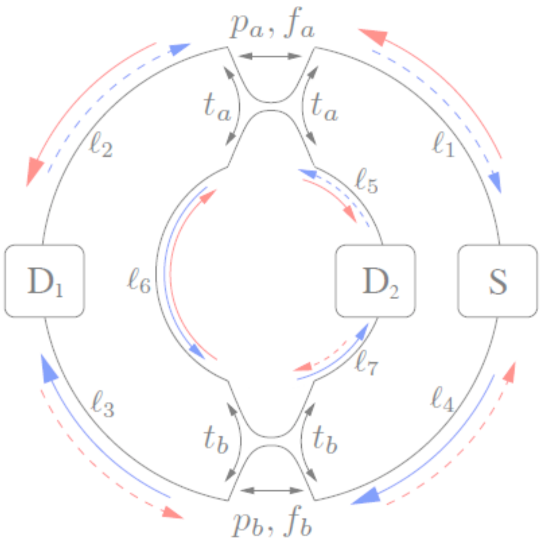


$$|\Psi_{\text{in}}\rangle \xrightarrow{S} |\Psi_{\text{out}}\rangle$$

$$\begin{pmatrix} b_{1\uparrow} \\ b_{1\downarrow} \\ b_{2\uparrow} \\ b_{2\downarrow} \\ b_{S\uparrow} \\ b_{S\downarrow} \end{pmatrix} = S \begin{pmatrix} a_{1\downarrow} \\ a_{1\uparrow} \\ a_{2\downarrow} \\ a_{2\uparrow} \\ a_{S\downarrow} \\ a_{S\uparrow} \end{pmatrix}$$

low T: assume no leakage from detectors

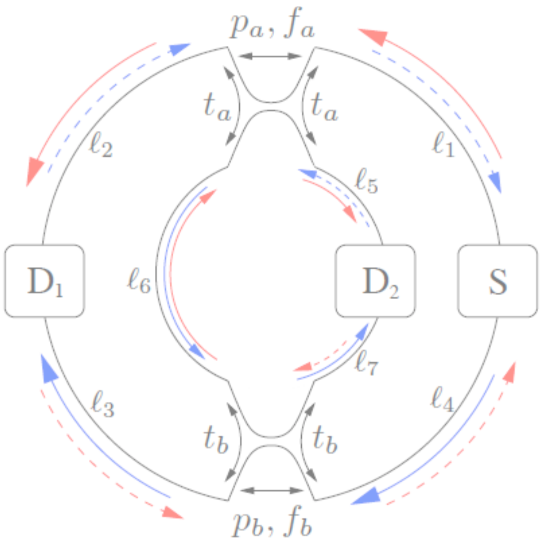
QSH spin entangler



$$|\Psi_{\text{in}}\rangle \xrightarrow{\tilde{S}} |\Psi_{\text{out}}\rangle$$

$$\begin{pmatrix} b_{1\uparrow} \\ b_{1\downarrow} \\ b_{2\uparrow} \\ b_{2\downarrow} \\ b_{S\uparrow} \\ b_{S\downarrow} \end{pmatrix} = \tilde{S} \begin{pmatrix} a_{S\uparrow} \\ a_{S\downarrow} \end{pmatrix}$$

QSH spin entangler

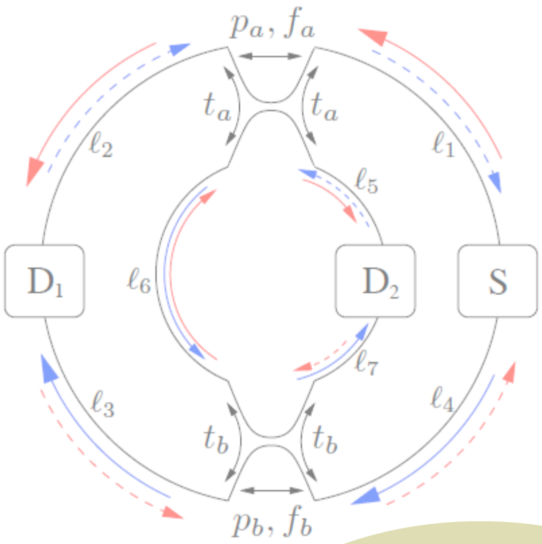


$$|\Psi_{\text{in}}\rangle \xrightarrow{\tilde{S}} |\Psi_{\text{out}}\rangle$$

$$|\Psi_{\text{in}}\rangle = a^\dagger_{S\uparrow} a^\dagger_{S\downarrow}|0\rangle$$

$$\xrightarrow{\tilde{S}} |\Psi_{\text{out}}\rangle$$

QSH spin entangler



$$|\Psi_{\rm in}\rangle = a^\dagger_{S\uparrow} a^\dagger_{S\downarrow}|0\rangle$$

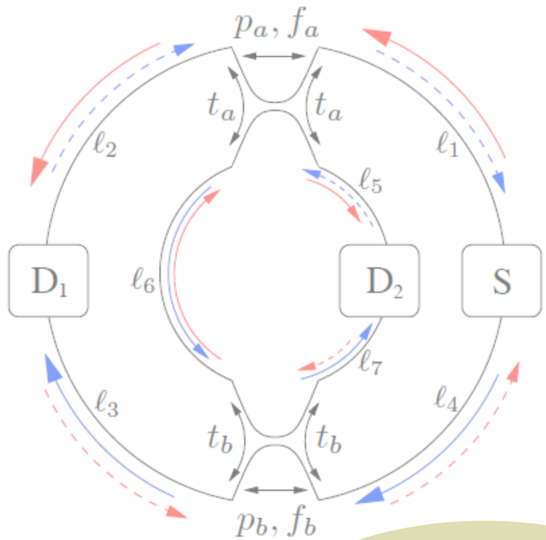
$$\xrightarrow{\tilde{S}} |\Psi_{\rm out}\rangle$$

$$\longrightarrow |\Psi'_{\rm out}\rangle$$

"postselection":

keep only terms in $|\Psi_{\rm out}\rangle$ where one electron gets detected in D_1 and the other in D_2

QSH spin entangler



$$|\Psi_{\text{in}}\rangle = a^\dagger_{S\uparrow} a^\dagger_{S\downarrow}|0\rangle$$

$$\xrightarrow{\tilde{S}} |\Psi_{\text{out}}\rangle$$

"postselection":

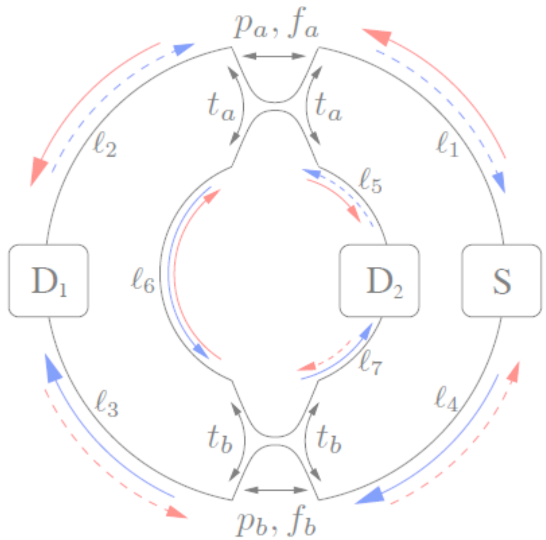
keep only terms in $|\Psi_{\text{out}}\rangle$ where one electron gets detected in D_1 and the other in D_2

$$\longrightarrow |\Psi'_{\text{out}}\rangle$$

$$= N\Big(f_b\left(|p_a|^2 + |t_a|^2\right)e^{-iK(l_{\uparrow\uparrow}+l_6)}\,|\uparrow\uparrow\rangle$$
$$+ f_a^*\left(|p_b|^2 + |t_b|^2\right)e^{-iK(l_{\downarrow\downarrow}+l_6)}\,|\downarrow\downarrow\rangle$$
$$+ \left[p_a t_b^* e^{-iKl_{\uparrow\downarrow}} + f_a^* f_b p_b t_a^* e^{-iK(l_{\uparrow\downarrow}+2l_6)}\right]|\uparrow\downarrow\rangle$$
$$+ \left[p_b^* t_a e^{-iKl_{\downarrow\uparrow}} + f_a^* f_b p_a^* t_b e^{-iK(l_{\downarrow\uparrow}+2l_6)}\right]|\downarrow\uparrow\rangle\Big)$$

$$|\sigma\sigma'\rangle = b^\dagger_{1\sigma} b^\dagger_{2\sigma'}|0\rangle$$

QSH spin entangler



$$|\Psi_{\rm in}\rangle = a^\dagger_{S\uparrow} a^\dagger_{S\downarrow}|0\rangle$$

$$\xrightarrow{\tilde{S}} |\Psi_{\rm out}\rangle$$

$$\longrightarrow |\Psi'_{\rm out}\rangle$$

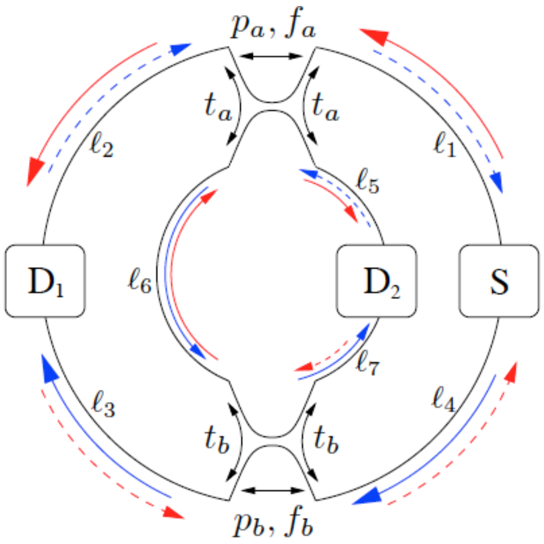
By tuning gate voltages, $|\Psi'_{\rm out}\rangle$ can be chosen as *any* of the four maximally entangled Bell states

$$|\Psi_1^\pm\rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$$

$$|\Psi_2^\pm\rangle = (|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)/\sqrt{2}$$

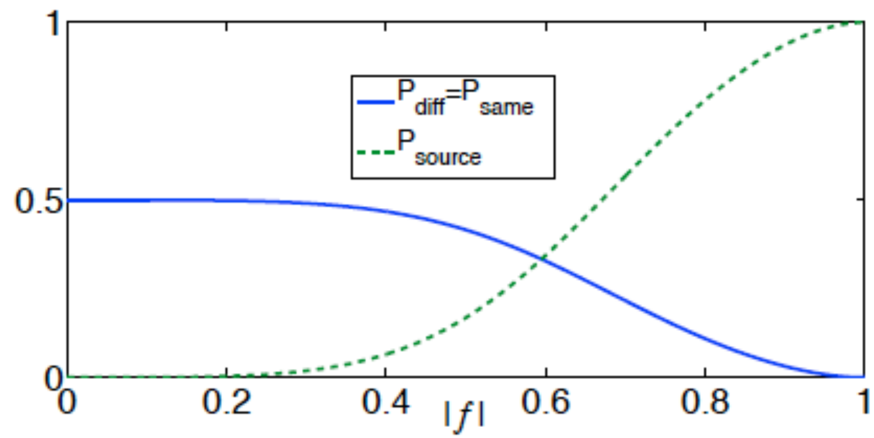
$$\left\{ \begin{aligned} &= N\Big(f_b\left(|p_a|^2 + |t_a|^2\right) e^{-iK(l_{\uparrow\uparrow}+l_6)}\,|\uparrow\uparrow\rangle \\ &+ f_a^*\left(|p_b|^2 + |t_b|^2\right) e^{-iK(l_{\downarrow\downarrow}+l_6)}\,|\downarrow\downarrow\rangle \\ &+ \left[p_a t_b^* e^{-iKl_{\uparrow\downarrow}} + f_a^* f_b p_b t_a^* e^{-iK(l_{\uparrow\downarrow}+2l_6)}\right]|\uparrow\downarrow\rangle \\ &+ \left[p_b^* t_a e^{-iKl_{\downarrow\uparrow}} + f_a^* f_b p_a^* t_b e^{-iK(l_{\downarrow\uparrow}+2l_6)}\right]|\downarrow\uparrow\rangle\Big) \end{aligned} \right.$$

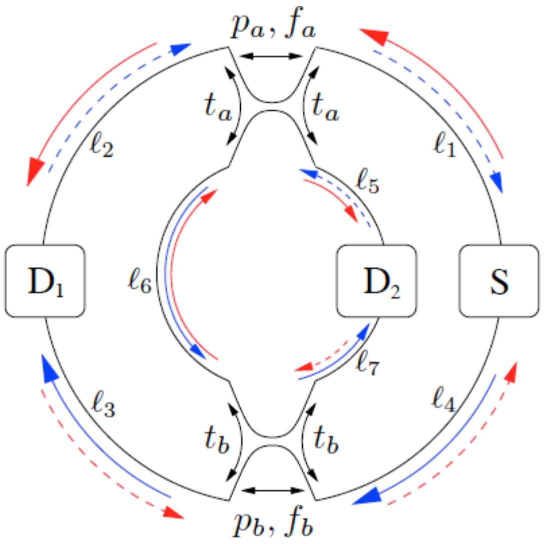
$$|\sigma\sigma'\rangle = b^\dagger_{1\sigma} b^\dagger_{2\sigma'}|0\rangle$$



Efficiency?

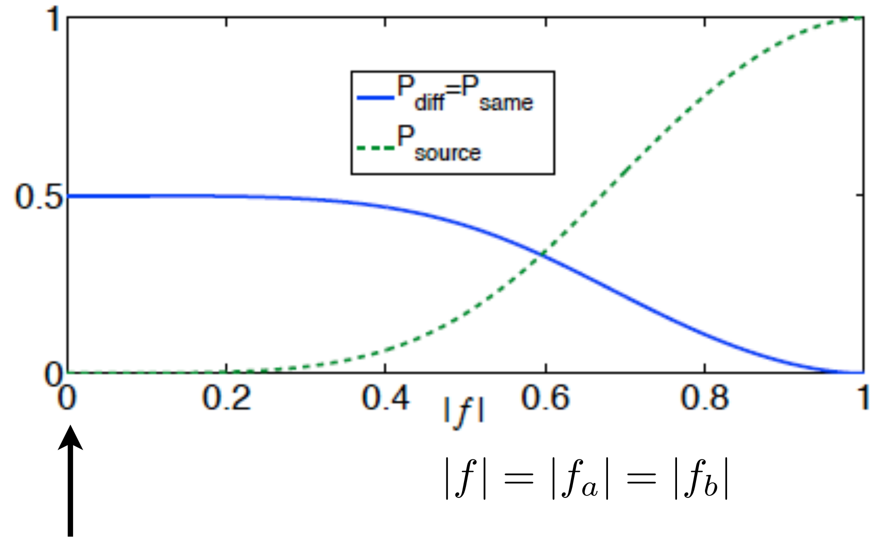
Choose $|p_a| = |p_b| = |t_a| = |t_b|$ by fine tuning the local gates at the junctions





Efficiency?

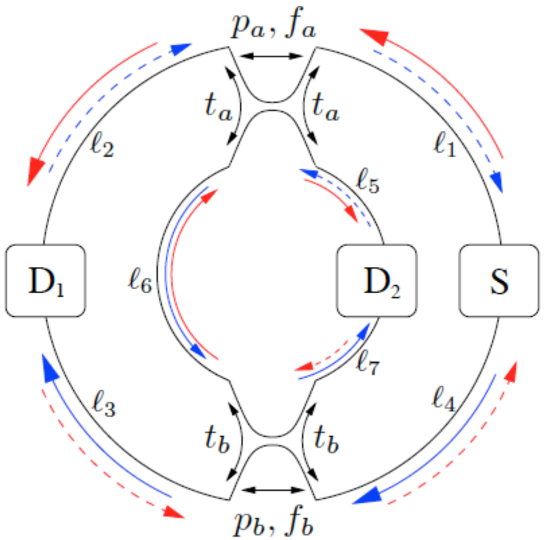
Choose $|p_a| = |p_b| = |t_a| = |t_b|$ by fine tuning the local gates at the junctions



$|f| = |f_a| = |f_b|$

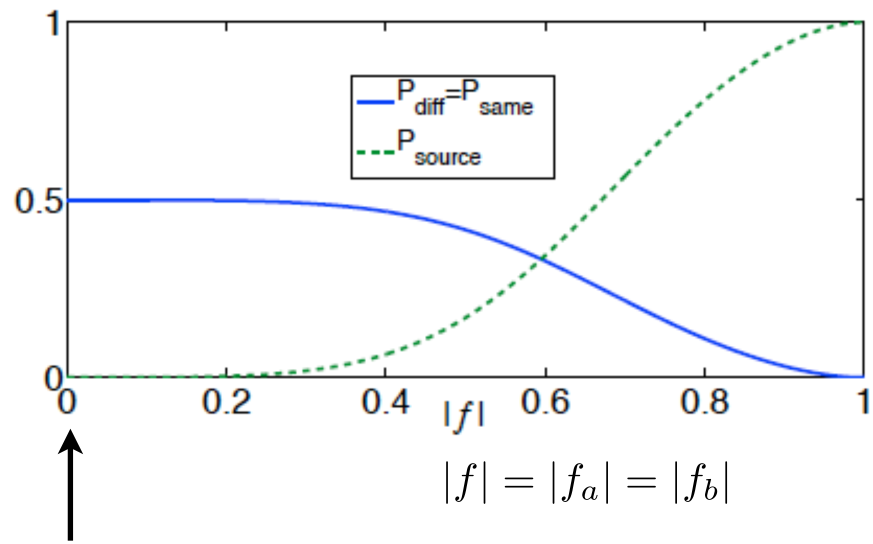
$$|\Psi_1^{[\phi]}\rangle = |\uparrow\downarrow\rangle + e^{i\phi}|\downarrow\uparrow\rangle$$

QSH spin entangler



Efficiency?

Choose $|p_a| = |p_b| = |t_a| = |t_b|$ by fine tuning the local gates at the junctions

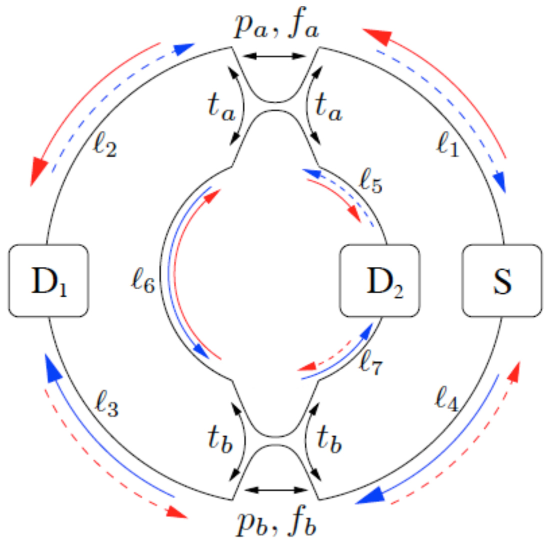


$$|f| = |f_a| = |f_b|$$

$$|\Psi_1^{[\phi]}\rangle = |\uparrow\downarrow\rangle + e^{i\phi}|\downarrow\uparrow\rangle$$

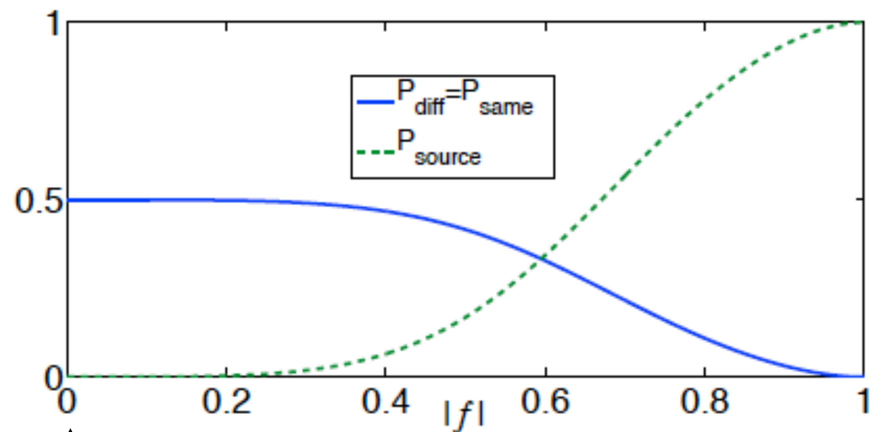
tunable phase
by a back gate

QSH spin entangler



Efficiency?

Choose $|p_a| = |p_b| = |t_a| = |t_b|$ by fine tuning the local gates at the junctions



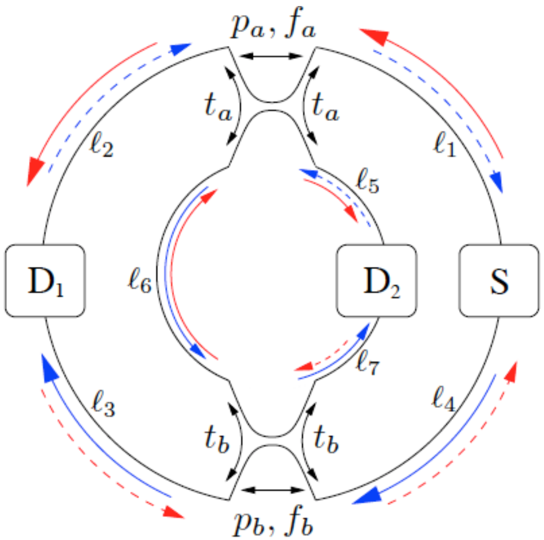
$|f| = |f_a| = |f_b|$

$$|\Psi_1^{[\phi]}\rangle = |\uparrow\downarrow\rangle + e^{i\phi}|\downarrow\uparrow\rangle$$

tunable phase
by a back gate

... more efficient entanglement production than in linear optics!

QSH spin entangler



Other cool stuff one can do...

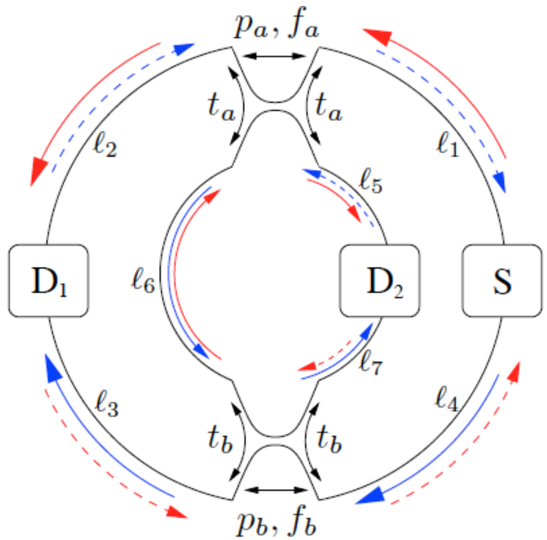
"Solid-state" Bell test:

$$B = E(\theta_1, \phi_1, \theta_2, \phi_2) - E(\theta_1', \phi_1', \theta_2, \phi_2) - E(\theta_1, \phi_1, \theta_2', \phi_2') - E(\theta_1', \phi_1', \theta_2', \phi_2')$$

$-2 \leq B \leq 2$ (CHSH inequality)

$$B_{max} = 2\sqrt{1 + C^2}$$

QSH spin entangler



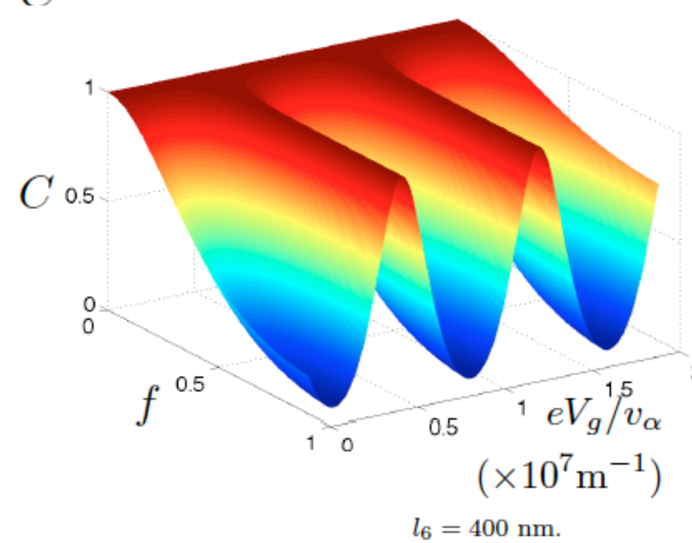
Other cool stuff one can do...

"Solid-state" Bell test:

$$B = E(\theta_1,\phi_1,\theta_2,\phi_2) - E(\theta_1',\phi_1',\theta_2,\phi_2) - E(\theta_1,\phi_1,\theta_2',\phi_2') - E(\theta_1',\phi_1',\theta_2',\phi_2')$$

$-2 \leq B \leq 2$ (CHSH inequality)

$$B_{max} = 2\sqrt{1+C^2}$$



But what about...

- materials and design?
- effects from interaction and disorder?
- ...

Questions...

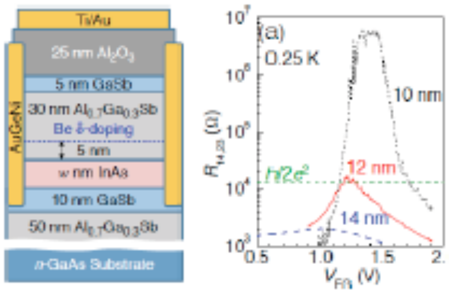
- materials and design?

Bad news: The best experimental realization of a 2D topological insulator – the HgTe/CdTe quantum well – is extremely tricky to handle!

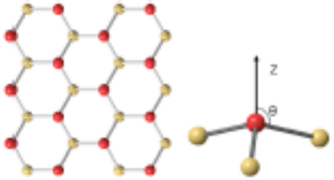
Other candidate 2D topological insulators:

"Stanene" (single atomic layer of tin)
Xu et al., PRL (2013)

InAs/GaSb quantum wells
Suzuki et al., PRB (2013)



Silicene
C.-C. Liu et al., PRL (2011)

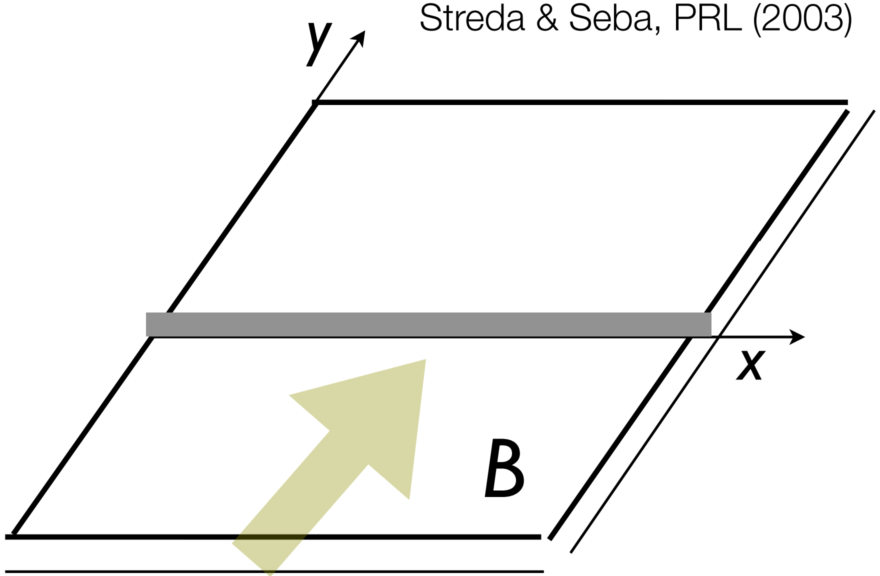


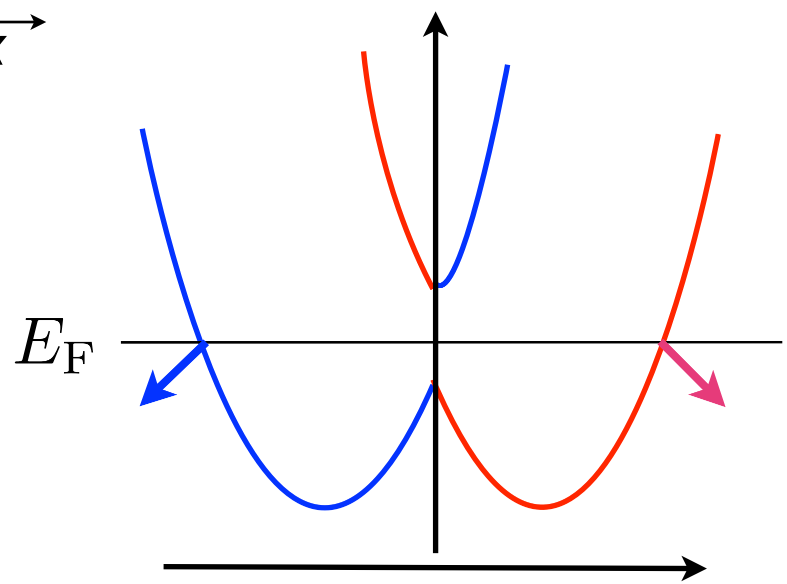
- materials and design?

Alternative realizations of 1D helical electrons in high demand!

Start with a quantum wire, spin split by a Rashba interaction, and add a magnetic field

Streda & Seba, PRL (2003)

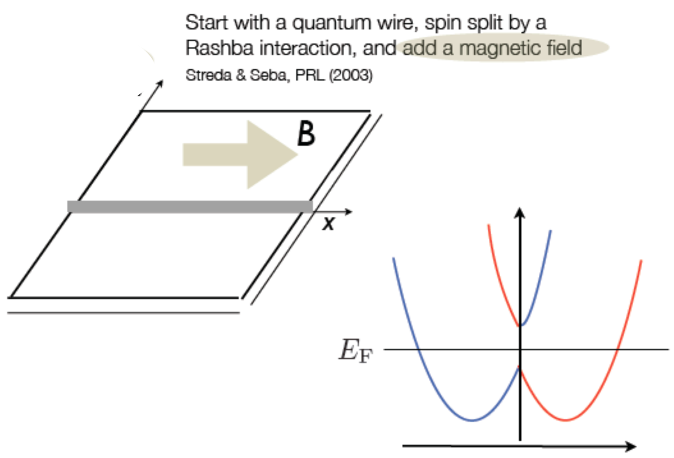




Questions...

- materials and design?

Alternative realizations of 1D helical electrons in high demand!



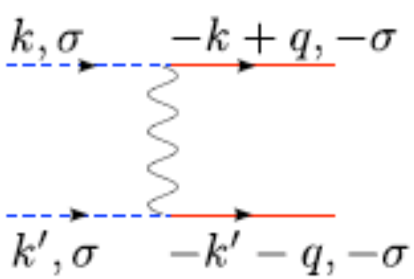
A novel proposal:
Mariana Malard's talk
today 4:40 pm

Questions...

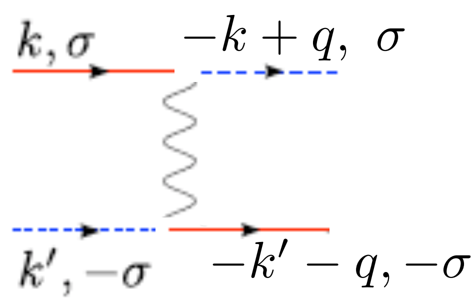
- e-e interaction effects?

- e-e interaction effects?

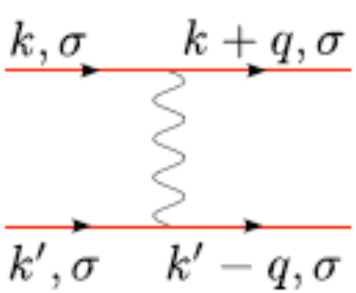
Scattering channels for 1D helical electrons



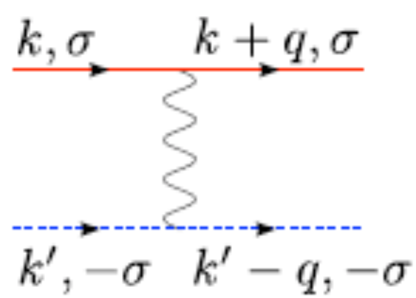
umklapp



backscattering



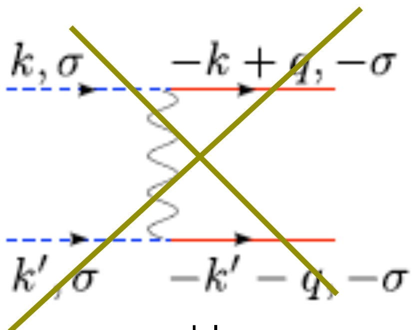
forward



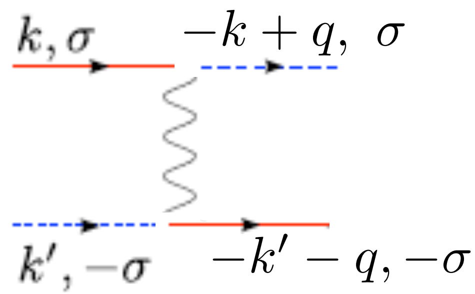
dispersive

- e-e interaction effects?

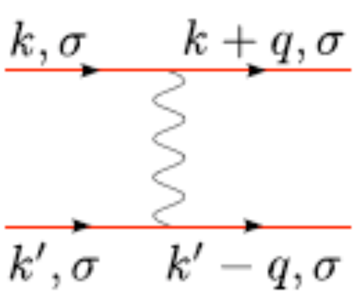
Scattering channels for 1D helical electrons



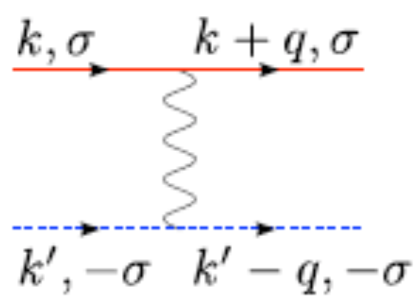
no umklapp
away from half-filled band



backscattering



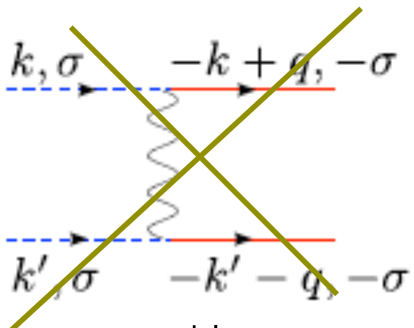
forward



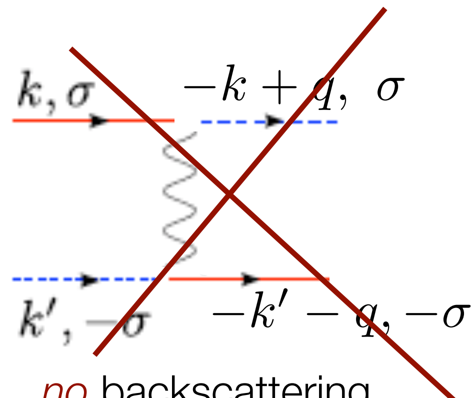
dispersive

- e-e interaction effects?

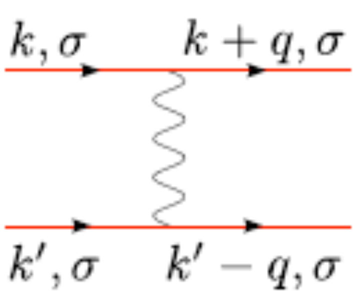
Scattering channels for 1D helical electrons



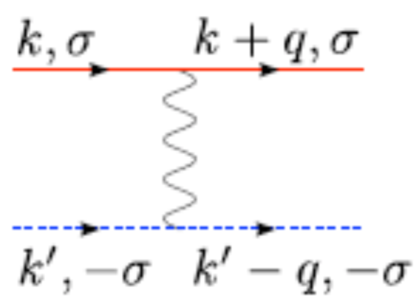
no umklapp
away from half-filled band



no backscattering
due to time-reversal invariance



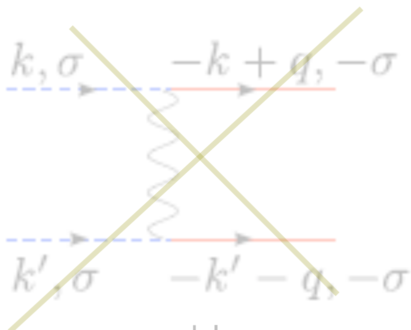
forward



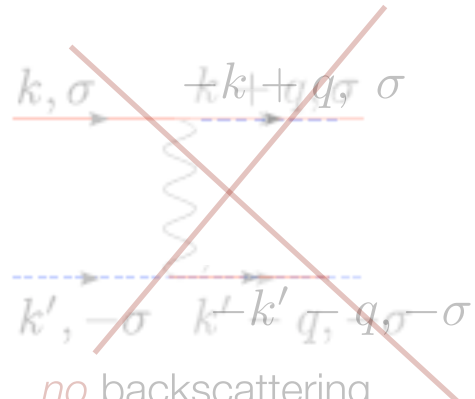
dispersive

- e-e interaction effects?

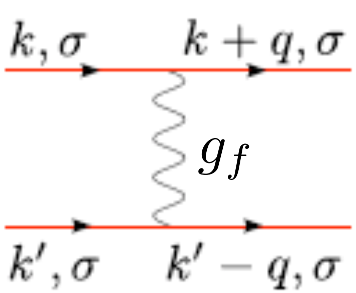
Scattering channels for 1D helical electrons



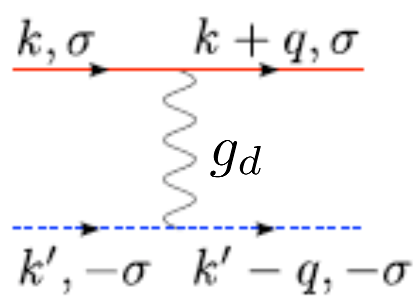
no umklapp
away from half-filled band



no backscattering
due to time-reversal invariance



forward



dispersive

$$K = \sqrt{\frac{\pi v_F + g_f - g_d}{\pi v_F + g_f + g_d}}$$

"Luttinger liquid parameter"

Questions...

- e-e interaction effects?

Estimates of *K*

for the Würzburg experiments on HgTe/CdTe quantum wells

$$0.55 \leq K \leq 0.98$$

Hou *et al.*, PRL (2009)   Lezmy *et al.*, PRB (2012)

Questions...

- e-e interaction effects?

Estimates of *K*
for the Würzburg experiments on HgTe/CdTe quantum wells

$$0.55 \leq K \leq 0.98$$

Hou *et al.*, PRL (2009)   Lezmy *et al.*, PRB (2012)

!

Questions...

- e-e interaction effects?

Estimates of *K*

for the Würzburg experiments on HgTe/CdTe quantum wells

$$0.55 \leq K \leq 0.98$$

Hou *et al.*, PRL (2009)   Lezmy *et al.*, PRB (2012)

?   !

More work needed to assess the viability of *electron quantum optics* for QSH edge states!

Thanks to my collaborators

Anders Ström, Braunschweig

Patrik Recher, Braunschweig

Phys. Rev. B **91**, 245406 (2015)

...and thank you!