Control of Complex Quantum Systems
KITP, Santa Barbara, January 24, 2013

Electrical Control of the Kondo Effect in a 2D Topological Insulator

Erik Eriksson (University of Gothenburg)
Anders Ström (University of Gothenburg)
Girish Sharma (École Polytechnique)
Henrik Johannesson (University of Gothenburg)

ÉCOLE POLYTECHNIQUE
GÖTEBORGS UNIVERSITET
VETENSKAPSRÅDET
supported by the
Swedish Research Council

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Nota Bene!
No closed-loop learning, feedback control, "quantum circuits", or anything of that kind...

Electrical "Control" of the Kondo Effect at the Edge of a Quantum Spin Hall System

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Outline

Quantum spin Hall system... some basics

At the edge: A new kind of electron liquid

Adding a magnetic impurity...

... and a Rashba spin-orbit interaction

Electrical control of the Kondo effect!

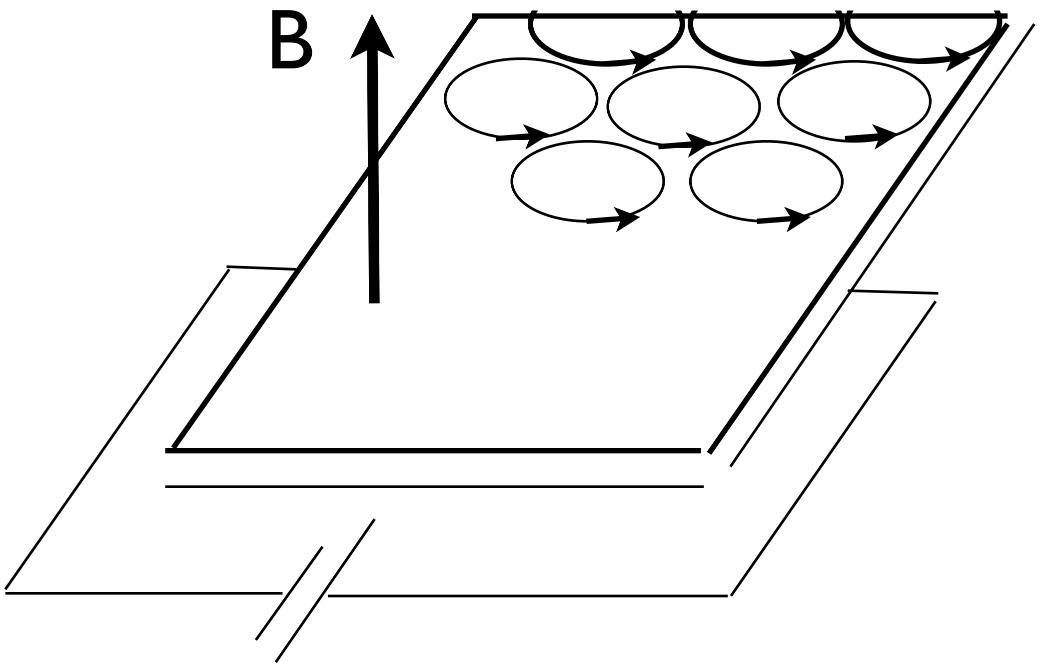
Why should people in quantum information/control/simulation bother?
Long-distance qubit entanglement using minimal control!

Quantum spin Hall system... some basics

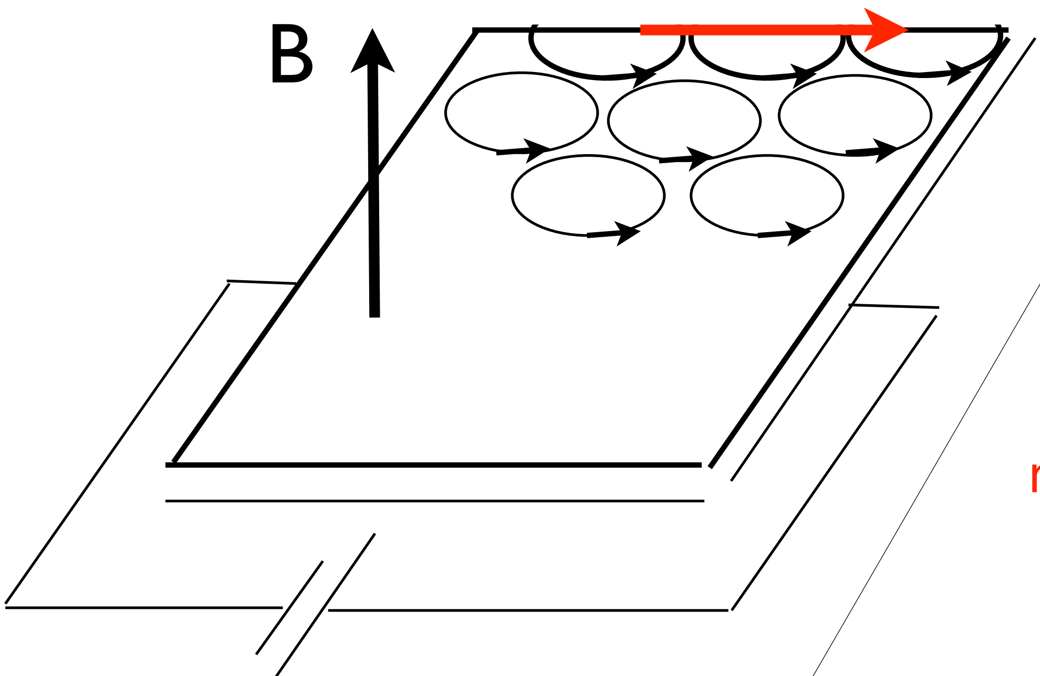
Quantum ~~spin~~ Hall system

Integer

Quantum Hall system



Quantum Hall system



"skipping currents"

quantization

chiral edge states

no channel for backscattering

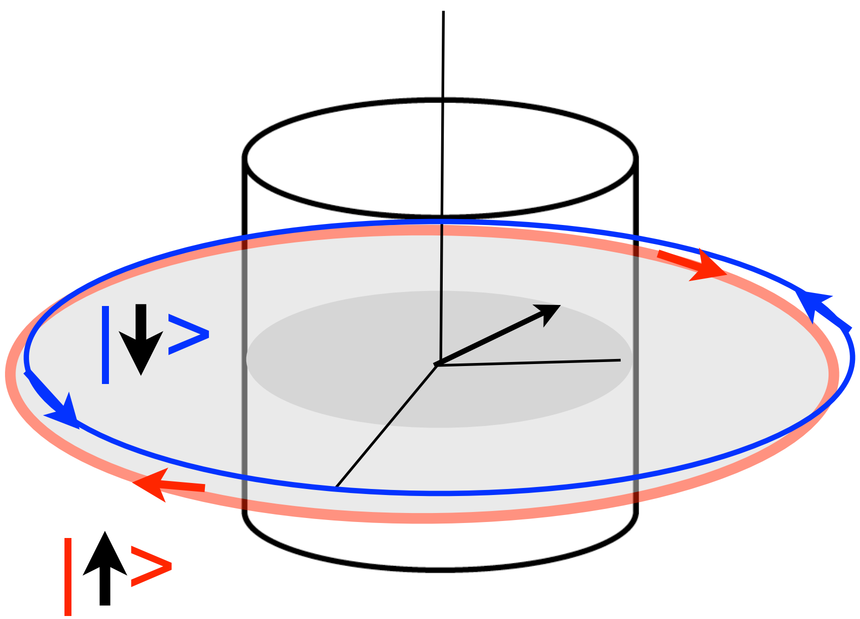
ballistic transport along the edge,
accounts for the quantization of
the Hall conductance

M. Büttiker, PRB **38**, 9375 (1988)

Can a many-body electronic state be stable against local perturbations without breaking time-reversal invariance?

Consider a Gedanken experiment...

B. A. Bernevig and S.-C. Zhang, PRL **96**, 106802 (2006)

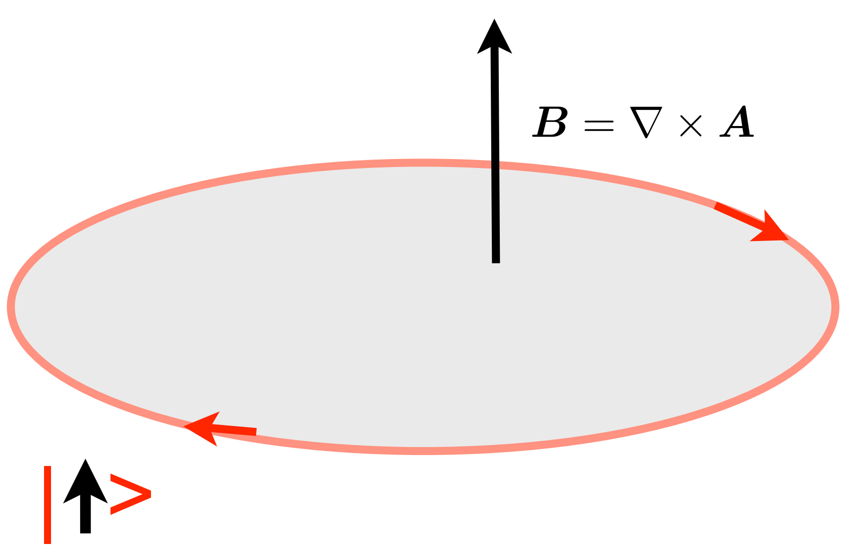


uniformly charged cylinder with electric field

$$\boldsymbol{E} = E(x, y, 0)$$

spin-orbit interaction

$$(\boldsymbol{E} \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} = E\sigma^z(k_y x - k_x y)$$



cf. with the IQHE in a symmetric gauge

$$\boldsymbol{A} = \frac{B}{2}(y, -x, 0)$$

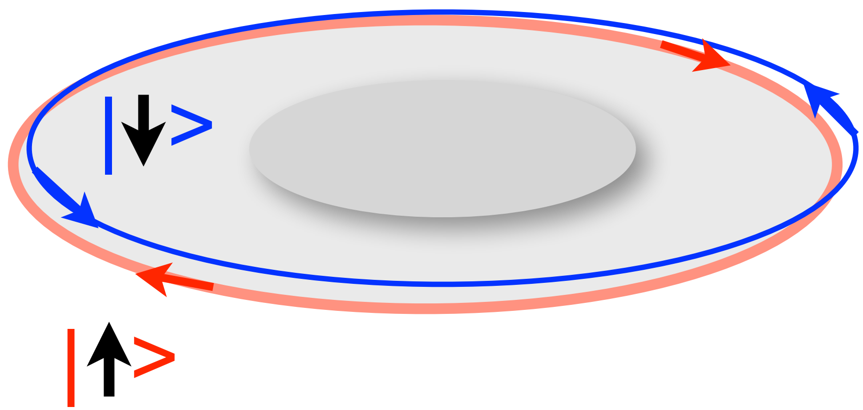
Lorentz force

$$\boldsymbol{A} \cdot \boldsymbol{k} \sim eB(k_y x - k_x y)$$

Quantum spin Hall (QSH) system

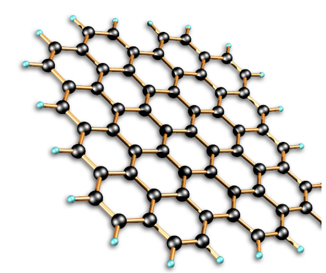
single Kramers pair

Two copies of an IQH system, bulk insulator with **helical edge states**



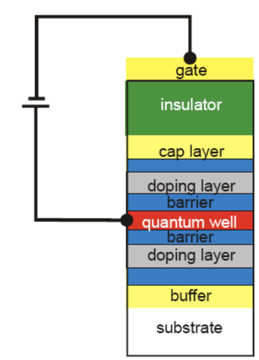
First proposed by Kane and Mele for graphene (2005)

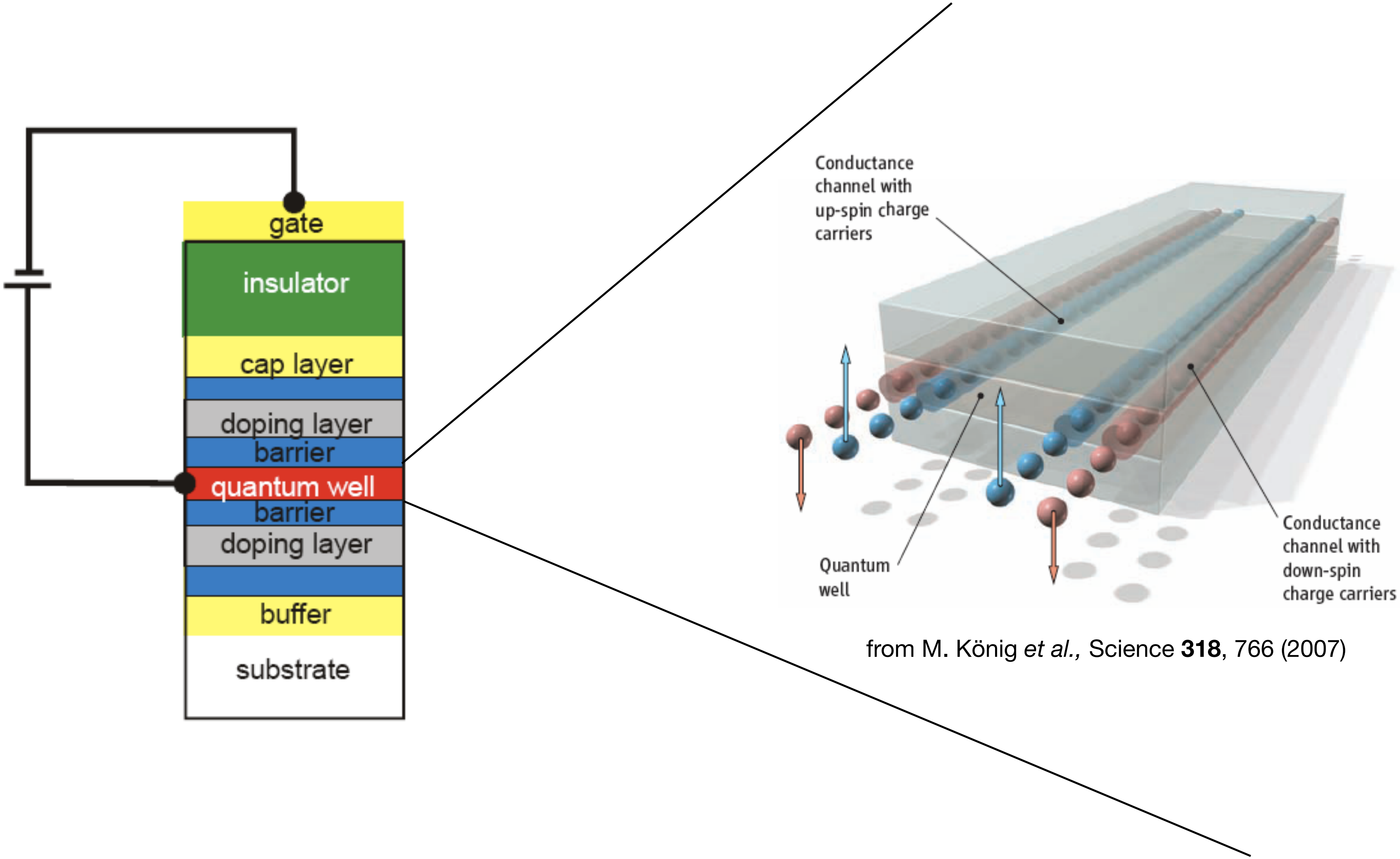
too weak spin-orbit interaction, doesn't quite work...



Bernevig *et al.* proposal for HgTe quantum wells (2006)

Experimental observation by König *et al.* (2007)





from M. König *et al.*, Science **318**, 766 (2007)

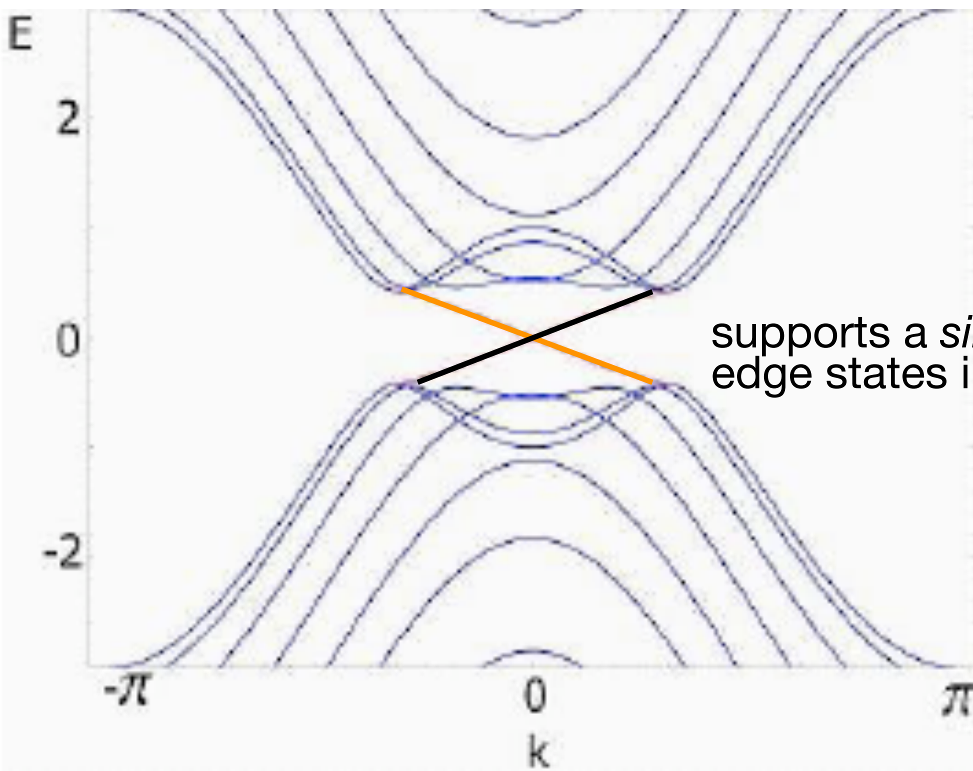
Bernevig *et al.* proposal for HgTe quantum wells (2006)
Experimental observation by König *et al.* (2007)

Are the helical edge states stable against local perturbations?

Are the helical edge states stable against local perturbations?

Yes! As long as the perturbations are time-reversal invariant! Look at the band structure of an HgTe quantum well...

strong spin-orbit interactions in atomic p-orbitals create an *inverted* band gap (p-band on top of s-band)



B. A. Bernevig *et al.*, PRL **95**, 066601 (2005)

supports a *single* Kramers pair of helical edge states inside the inverted gap

Kramers degeneracy at k=0 protects the stability of the edge states

ballistic transport

$$G = \frac{2e^2}{h}$$

4-terminal measurement, equilibration in contacts

At the edge: A new kind of electron liquid

of Kramers pair in a **nontrivial** (trivial) helical liquid

$$N_K = 1 \bmod 2 \qquad = 0 \bmod 2$$

$\nu = 0, 1$ is a "Z_2" **topological invariant** and can be calculated from the band structure of the bulk ("bulk-edge correspondence")

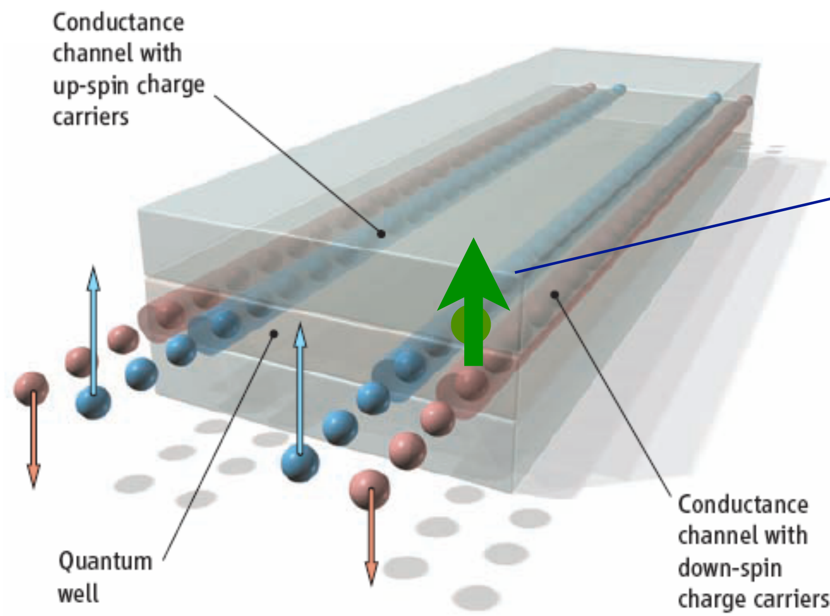
L. Fu and C. L. Kane, PRB **76**, 045302 (2007)

2D topological insulator

(a.k.a. quantum spin Hall system)

What if time-reversal symmetry is broken...?

... for example, by the presence of a magnetic impurity?



from M. König *et al.*, Science **318**, 766 (2007)

case study:

Mn^{2+}

large and positive single-ion anisotropy $(S^z)^2$

$$S = 5/2 \underset{\text{low } T}{\longrightarrow} S_{\text{eff}} = 1/2$$

anisotropic spin exchange with the edge electrons

R. Zitko *et al.*, PRB **78**, 224404 (2008)

$$H_{\text{K}} = \Psi^\dagger(0)\left[J_\perp(\sigma^+ S^-_{\text{eff}} + \sigma^- S^+_{\text{eff}}) + J_z \sigma^z S^z_{\text{eff}}\right]\Psi(0)$$

$$\Psi^T = (\psi_\uparrow, \psi_\downarrow)$$

Adding a magnetic impurity...

The Kondo interaction is time-reversal invariant!
Could it still cause a *spontaneous* breaking of time reversal invariance and collapse the QSH state?

Adding a magnetic impurity...

Recall the Kondo effect

One-loop RG equations:

P. W. Anderson, J. Phys. C **3**, 2436 (1970)

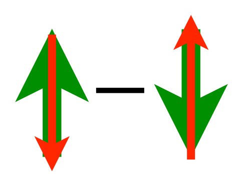
$$\frac{\partial J_\perp}{\partial D} = -\nu J_\perp J_z + \dots$$
$$\frac{\partial J_z}{\partial D} = -\nu J_\perp^2 + \dots$$

strong-coupling physics for $T << T_K$

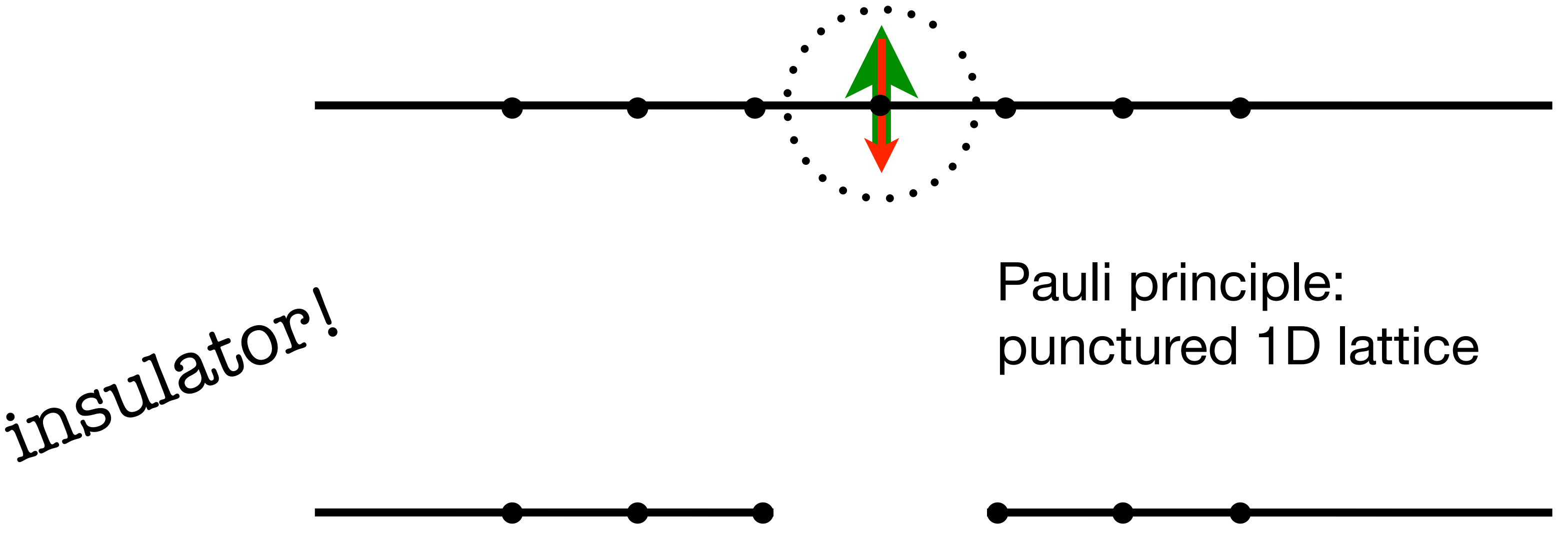
$$T_K = D_0 \exp(-\text{const.}/J_0)$$

$$J_0 \equiv \max(J_\perp, J_z)_{D=D_0}$$

formation of impurity-electron singlet **("Kondo screening")**



Adding a magnetic impurity...

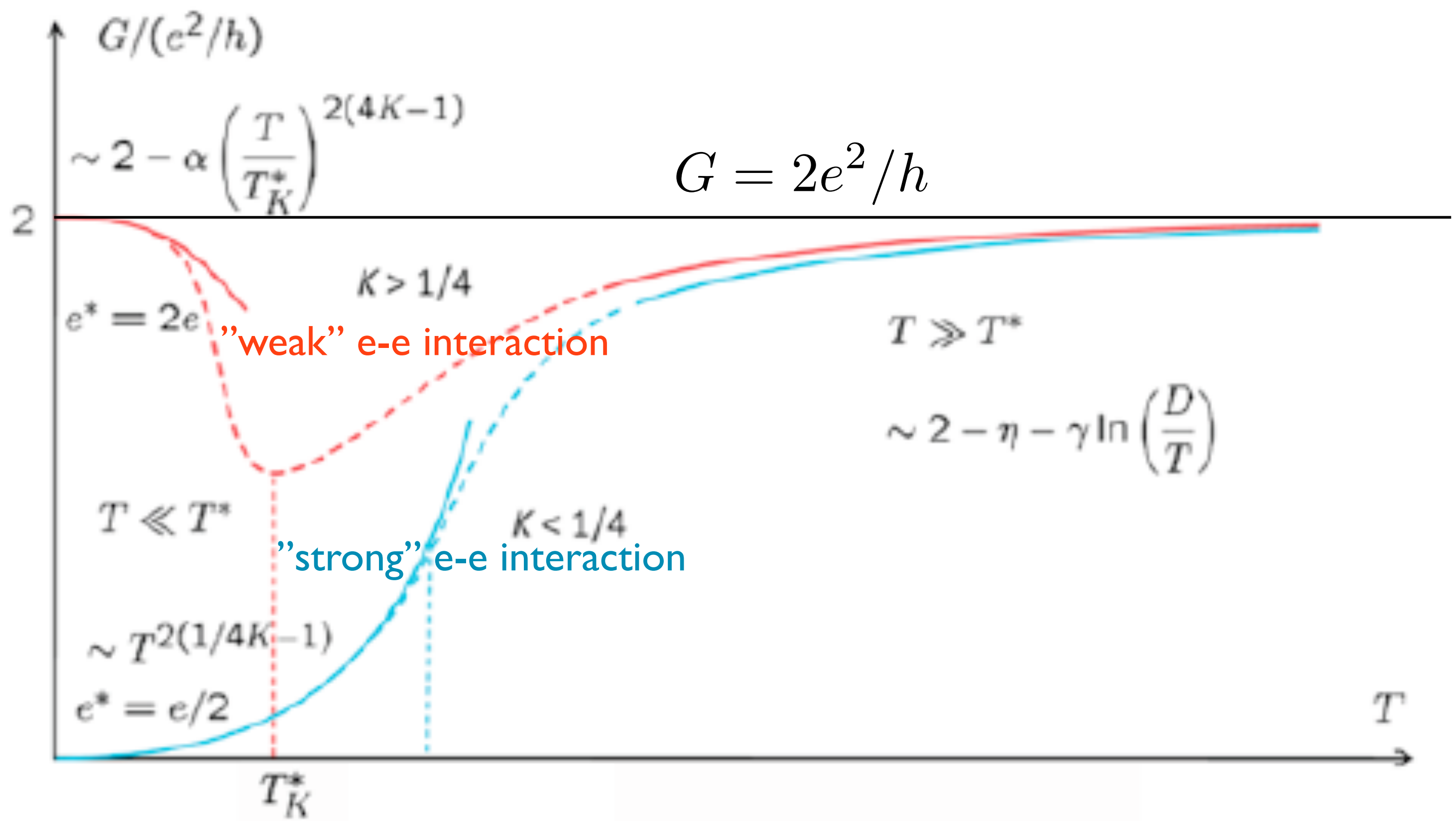


Adding a magnetic impurity...

Does this really happen for the helical liquid?
To find out, add e-e interactions.... important in 1D!

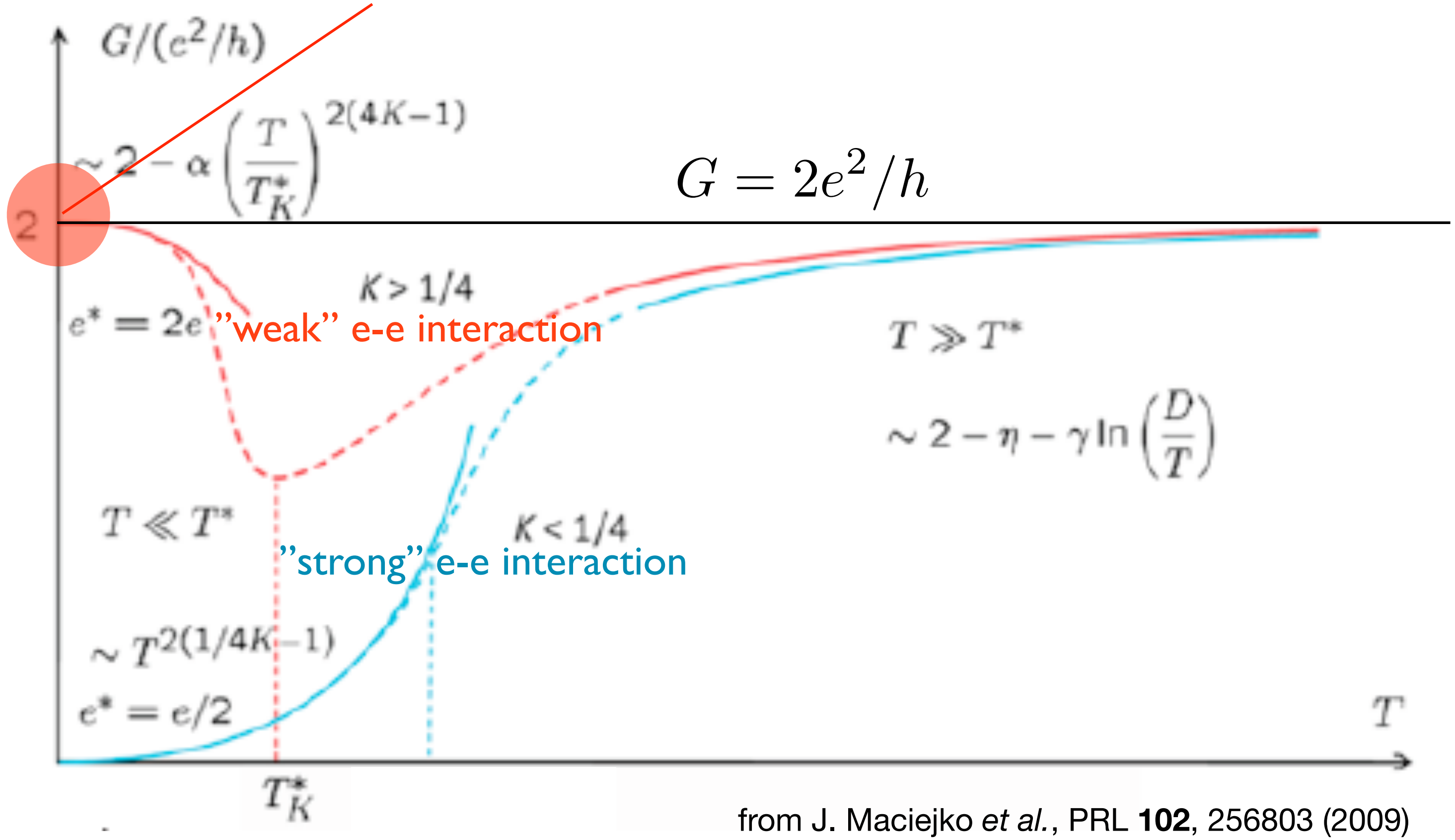
bosonization, RG, and linear response

J. Maciejko *et al.*, PRL **102**, 256803 (2009)



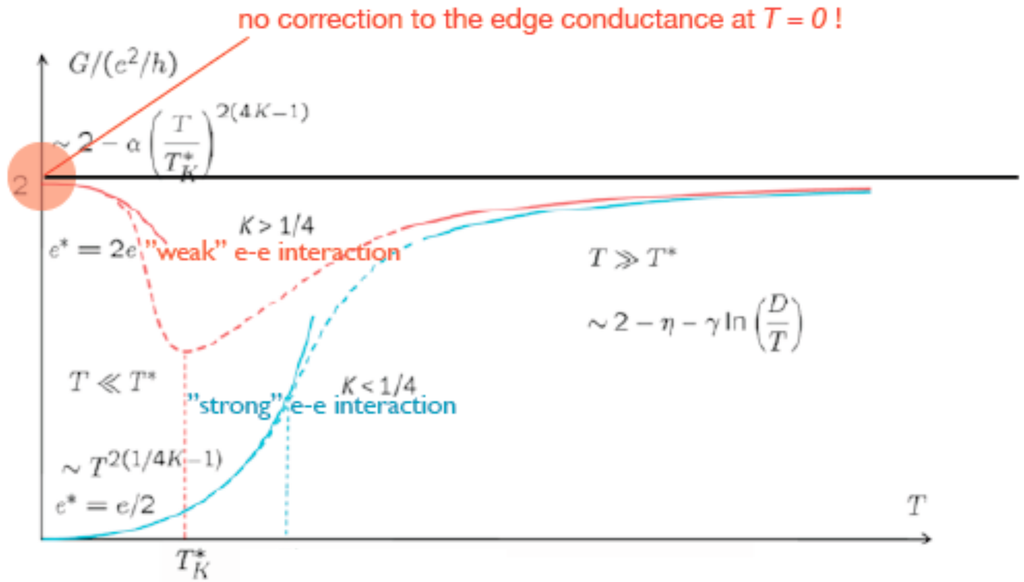
Adding a magnetic impurity...

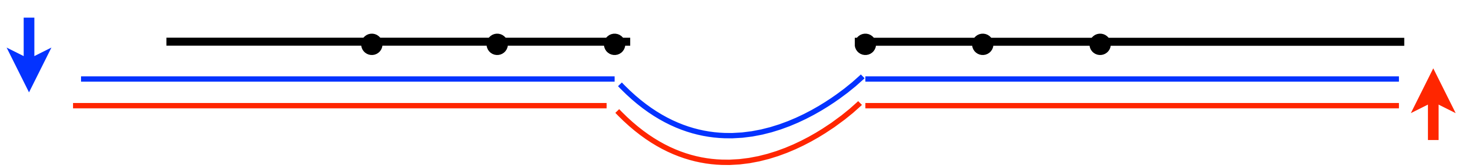
no correction to the edge conductance at *T = 0* !



from J. Maciejko *et al.*, PRL **102**, 256803 (2009)

Adding a magnetic impurity...





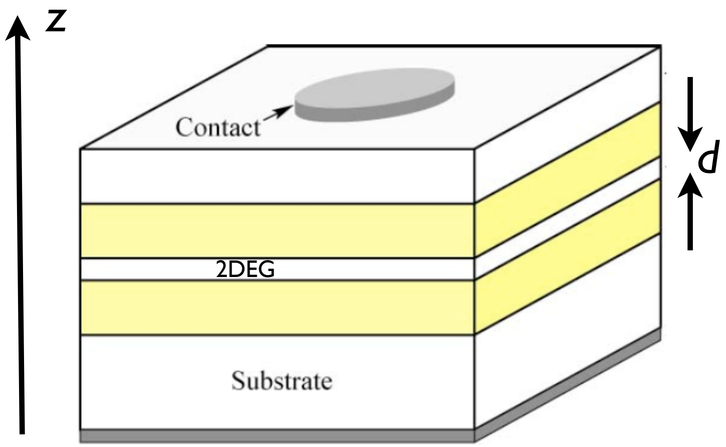
Due to its topological nature, the QSH state follows the new shape of the edge.

Weak coupling QSH states are robust against local breaking of time-reversal symmetry!

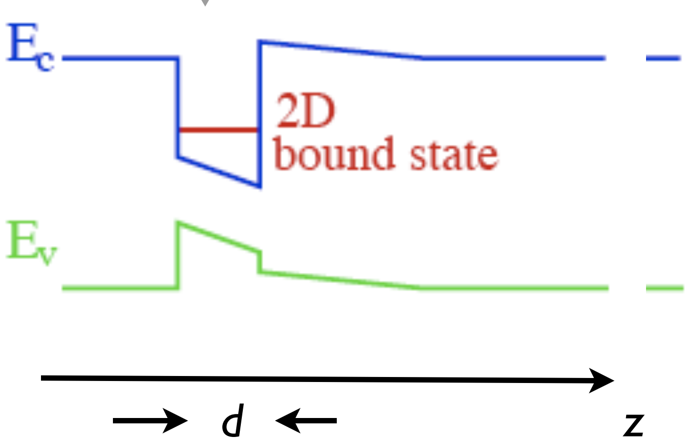
J. Maciejko *et al.*, PRL **102**, 256803 (2009)

But… one important thing is missing from the analysis

Rashba spin-orbit interaction!



semiconductor heterostructure

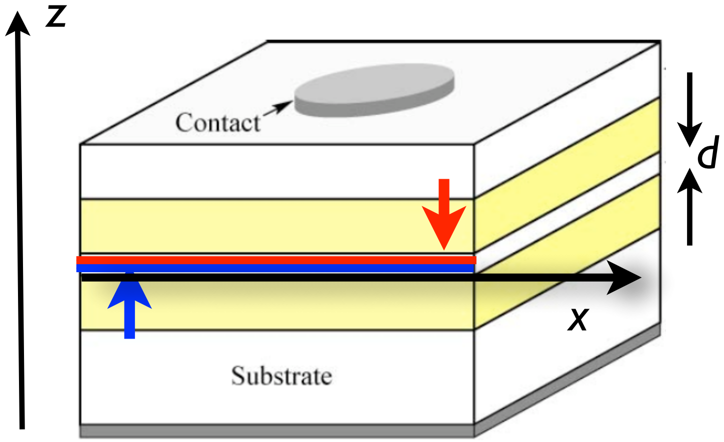


Spatial asymmetry of band edges mimics an ***E***-field in the *z*-direction

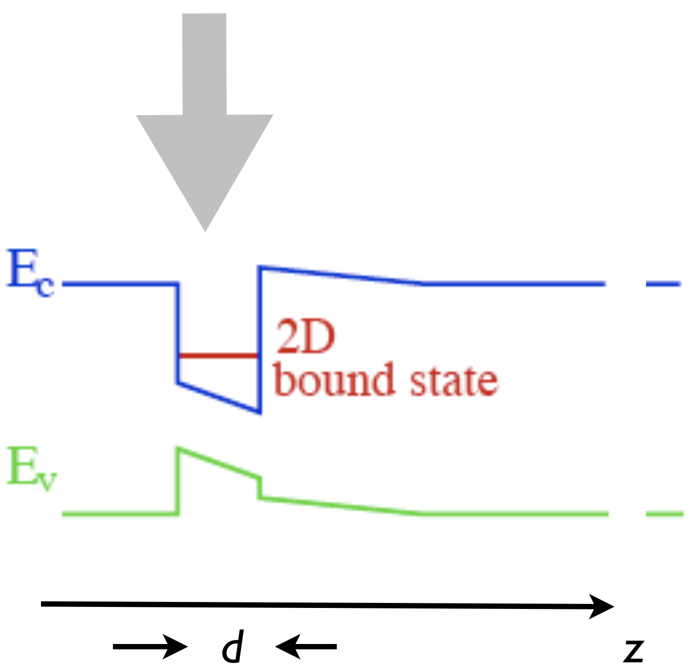
$$H_R = \alpha(k_x\sigma^y - k_y\sigma^x)$$

Yu. A. Bychkov and E. I. Rashba, J. Phys. C **17**, 6039 (1984)

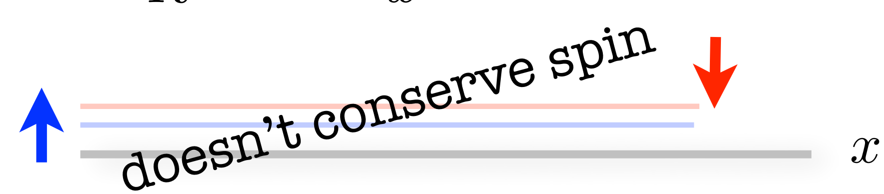
Rashba spin-orbit interaction!



semiconductor heterostructure



$$H_R = \alpha\, k_x \sigma^y$$



Adding the Rashba interaction...

... breaks the locking of spin to momentum. However, there is still a single Kramers pair on the QSH edge, and this is all that matters!

kinetic term Rashba

$$H = v_F \int \mathrm{d}x \; \Psi^\dagger(x) \left[-\mathrm{i}\sigma^z \partial_x\right] \Psi(x) + \alpha \int \mathrm{d}x \; \Psi^\dagger(x) \left[-\mathrm{i}\sigma^y \partial_x\right] \Psi(x)$$

$$\Psi^T = (\psi_\uparrow, \psi_\downarrow)$$

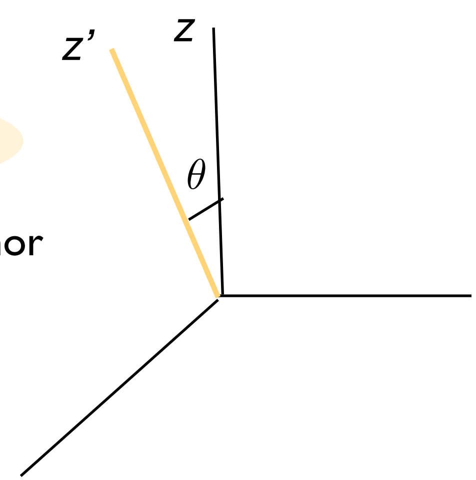
$$\Psi' = \mathrm{e}^{-\mathrm{i}\sigma^x \theta/2} \Psi$$

$$\cos\theta = v_F / v_\alpha \qquad v_\alpha = \sqrt{v_F^2 + \alpha^2}$$

$$H' = v_\alpha \int \mathrm{d}x \; \Psi'^\dagger(x) \left[-\mathrm{i}\sigma^{z'} \partial_x\right] \Psi'(x)$$

$$\Psi'^T = (\psi_{\uparrow'}, \psi_{\downarrow'})$$

The "Rashba-rotated" spinor still defines a Kramers pair



Adding the Rashba interaction...

e-e interaction is invariant under $\Psi \rightarrow \Psi'$

Kondo interaction $H_{\mathrm{K}} \;=\; \Psi^{\dagger}(0)\left[J_{\perp}(\sigma^{+}S^{-}_{\mathrm{eff}} + \sigma^{-}S^{+}_{\mathrm{eff}}) + J_{z}\sigma^{z}S^{z}_{\mathrm{eff}}\right]\Psi(0)$

$$\Psi' = \mathrm{e}^{-\mathrm{i}\sigma^{x}\theta/2}\Psi$$
$$\boldsymbol{S}' = \mathrm{e}^{-\mathrm{i}S^{x}\theta/2}\boldsymbol{S}\mathrm{e}^{\mathrm{i}S^{x}\theta/2}$$

$$H'_{K} = \Psi'^{\dagger}(0)[J_{x}\sigma^{x}S^{x} + J'_{y}\sigma^{y'}S^{y'} + J'_{z}\sigma^{z'}S^{z'} + J_{\mathrm{NC}}(\sigma^{y'}S^{z'} + \sigma^{z'}S^{y'})]\Psi'(0)$$

XYZ Kondo

Non-Collinear term

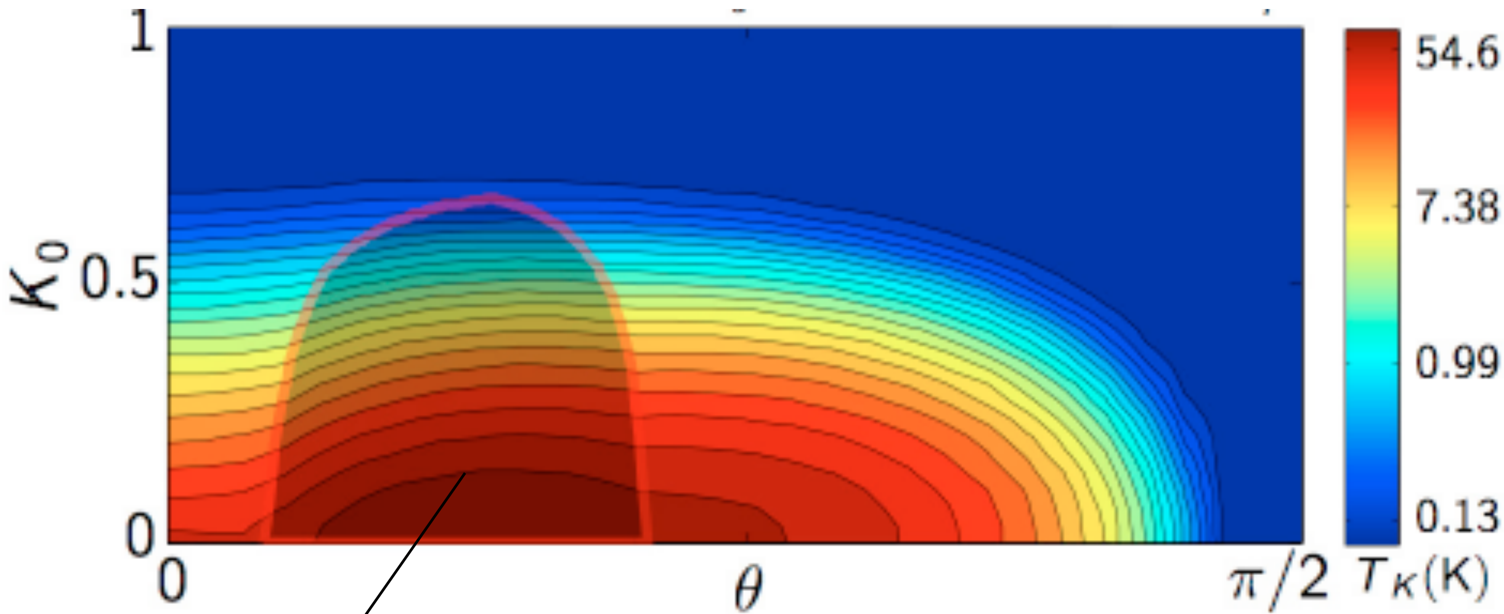
depend on the Rashba coupling α
controllable by a gate voltage

...bosonization and RG

E. Eriksson, A. Ström, G. Sharma, H.J., PRB **86**, 161103(R) (2012)

Electrical control of the Kondo temperature

via the "Rashba angle" θ ~ gate voltage



easy-axis Kondo

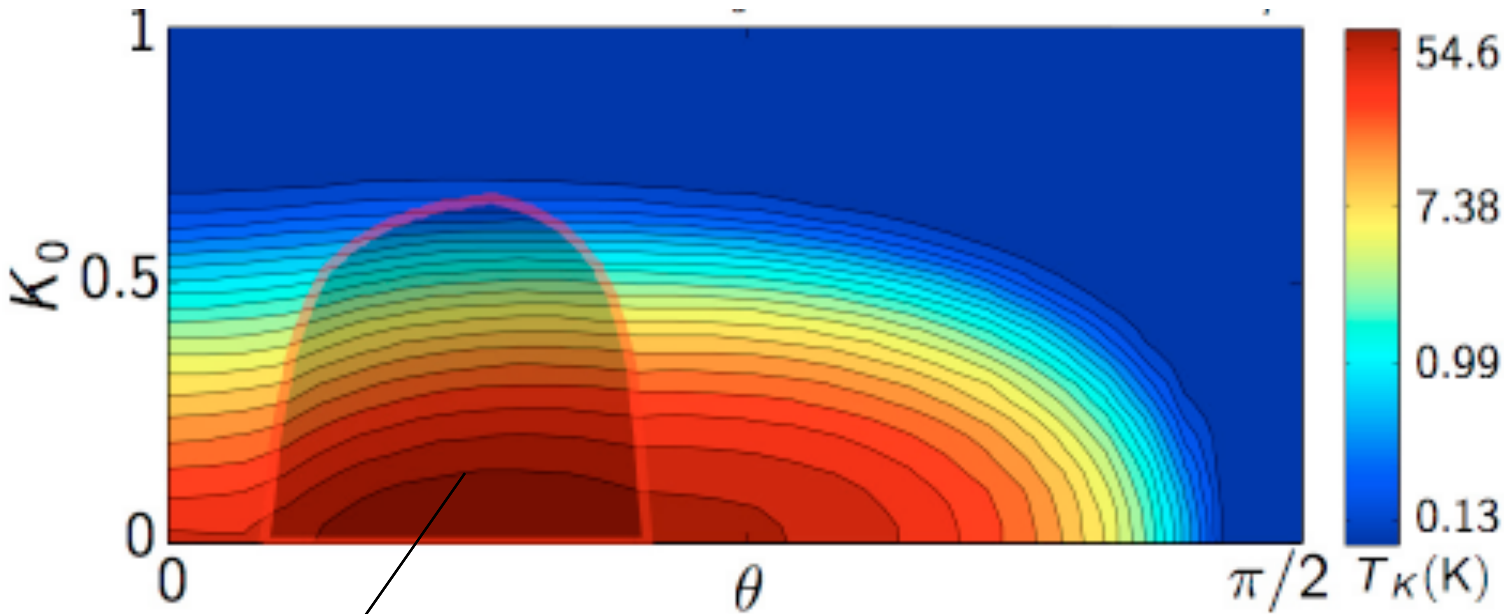
$J_x = J_y = 5 \text{ meV}, \ J_z = 50 \text{ meV}$

Region where $J_{\rm NC}$ dominates the RG flow

→ obstruction of Kondo screening**!**

Electrical control of the Kondo temperature

via the "Rashba angle" θ ~ gate voltage



easy-axis Kondo

$J_x = J_y = 5\ \mathrm{meV},\ J_z = 50\ \mathrm{meV}$

Region where J_{NC} dominates the RG flow

→ obstruction of Kondo screening!

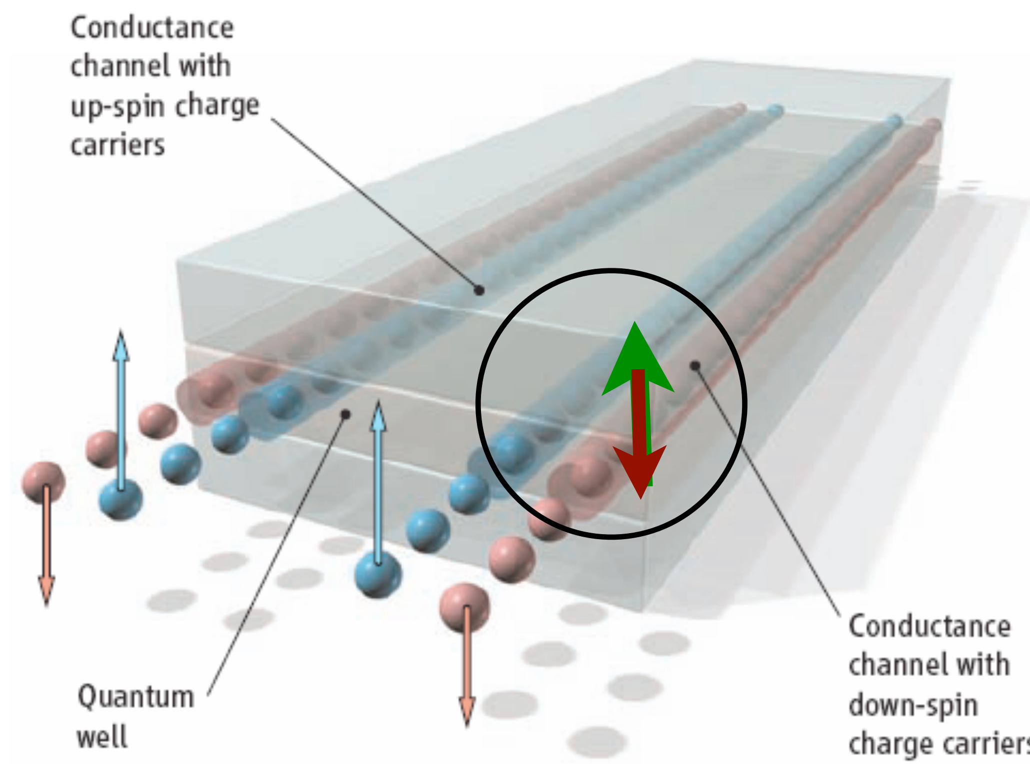
challenges the 'folklore' that the Kondo effect is blind to time-reversal invariant perturbations!

drawn from
Y. Meir and N.S. Wingreen, PRB **50**, 4947 (1994)

Conclusion

E. Eriksson *et al.*, PRB **86**, 161103(R) (2012)

The Kondo effect in a 2D topological insulator can be ”controlled” via a tunable gate voltage
(given the ”right” setup: weak e-e screening, easy-axis Kondo exchange,...)



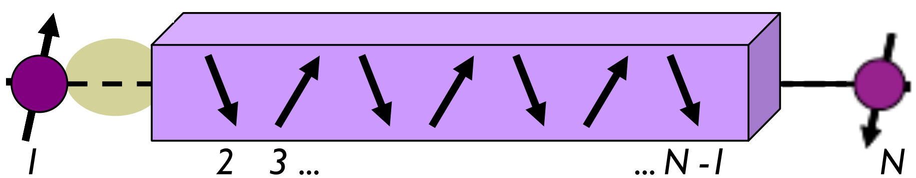
Epilogue

Is this kind of thing of any use? Possible applications in quantum information/simulation/control/...? Why bother about the Kondo effect?

An intriguing scenario:
Generating long-distance qubit entanglement via the Kondo effect

P. Sodano, A. Bayat, S. Bose, Phys. Rev. B **81**, 100412(R) (2010)

'Kondo spin chain'



$$H = J'(J_1 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + J_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3) + J_1 \sum_{i=2}^{N-1} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+1} + J_2 \sum_{i=2}^{N-2} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+2}$$

same low-energy physics as the spin sector of the Kondo model

S. Eggert and I. Affleck, PRB **46**, 10866 (1992)

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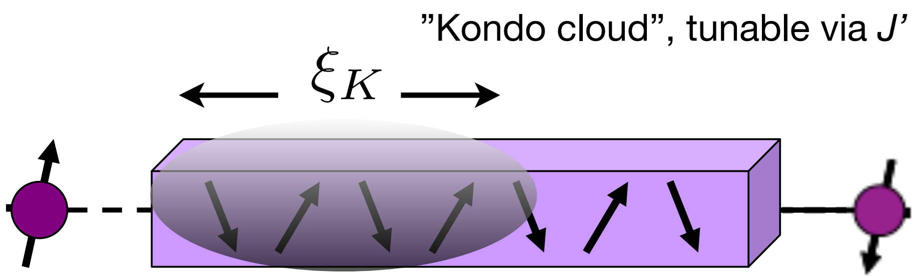
Generating long-distance qubit entanglement via the Kondo effect

P. Sodano, A. Bayat, S. Bose, Phys. Rev. B **81**, 100412(R) (2010)



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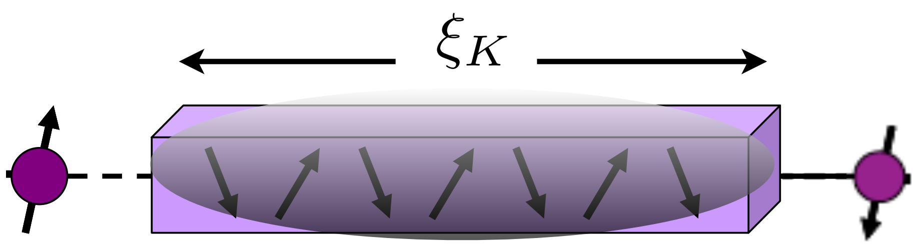


$$H = J'(J_1 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + J_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3) + J_1 \sum_{i=2}^{N-1} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+1} + J_2 \sum_{i=2}^{N-2} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+2}$$

$$| \, \Psi_o \rangle$$

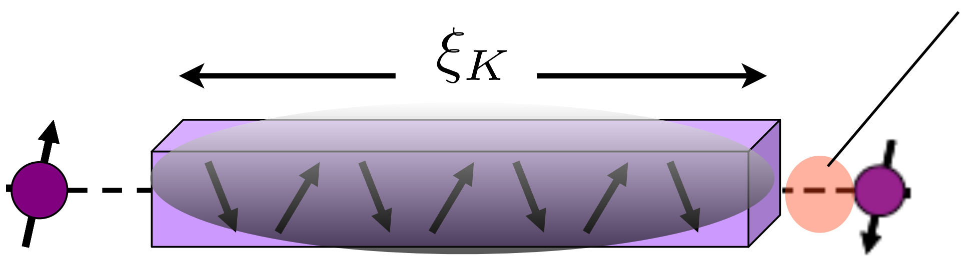
ground state with ”optimal” Kondo cloud

An intriguing scenario:

Generating long-distance qubit entanglement via the Kondo effect

P. Sodano, A. Bayat, S. Bose, Phys. Rev. B **81**, 100412(R) (2010)

local quantum quench at t = t_0



$$H = J'(J_1 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + J_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3) + J_1 \sum_{i=2}^{N-1} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+1} + J_2 \sum_{i=2}^{N-2} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+2}$$

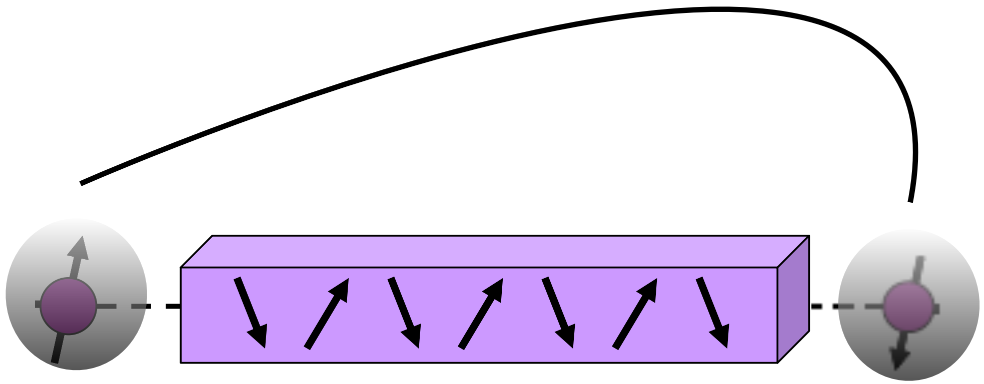
$$\to H_q = J'(J_1 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + J_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3) + J'(J_1 \boldsymbol{\sigma}_{N-1} \cdot \boldsymbol{\sigma}_N + J_2 \boldsymbol{\sigma}_{N-2} \cdot \boldsymbol{\sigma}_N) + J_1 \sum_{i=2}^{N-2} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+1} + J_2 \sum_{i=2}^{N-3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+2}$$

$$|\,\Psi_o\,\rangle \to |\,\Psi_q(t)\,\rangle \equiv e^{-iH_q t}\,|\,\Psi_o\,\rangle \qquad t > t_0$$

An intriguing scenario:

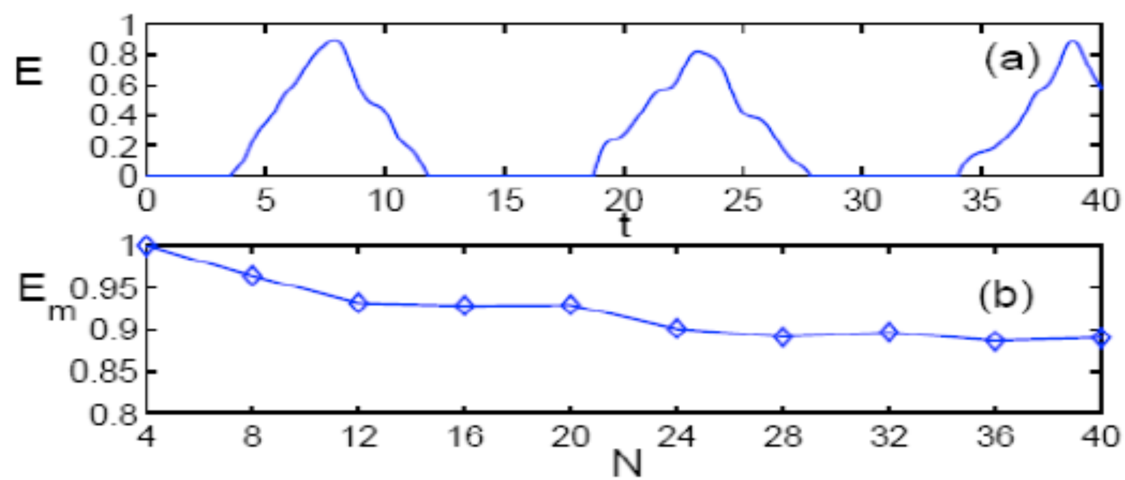
Generating long-distance qubit entanglement via the Kondo effect

P. Sodano, A. Bayat, S. Bose, Phys. Rev. B **81**, 100412(R) (2010)



Entanglement dynamics in $\langle \Psi_q(t) |$:

fast oscillatory and long-lived entanglement between the end spins!



Appendix

Low-temperature transport, $T \ll T_K$ (away from the "dome" on slide 54)

"weak" e-e interaction $K > 1/4$

$T = 0$

$$G = \frac{2e^2}{h}$$

Low-temperature transport, $T \ll T_K$ (away from the ”dome” on slide 54)

”weak” e-e interaction $K > 1/4$

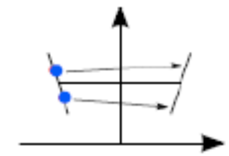
$$G = \frac{2e^2}{h} - \delta G$$

Low-temperature transport, $T \ll T_K$ (away from the ”dome” on slide 54)

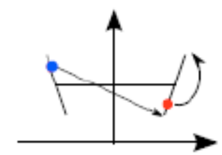
”weak” e-e interaction

$$G = \frac{2e^2}{h} - \delta G$$

$1/4 < K < 2/3$: $\delta G \sim (T/T_K)^{8K-2}$



$K > 2/3$: $\delta G \sim (T/T_K)^{2K+2}$

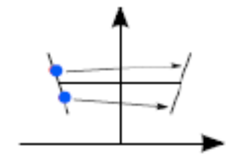


Low-temperature transport, $T \ll T_K$ (away from the ”dome” on slide 54)

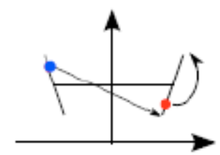
”weak” e-e interaction

$$G = \frac{2e^2}{h} - \delta G$$

$1/4 < K < 2/3$: $\sim (T/T_K)^{8K-2}$



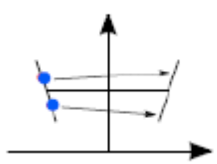
$K > 2/3$: $\sim (T/T_K)^{2K+2}$



”strong” e-e interaction $K < 1/4$

$T = 0$

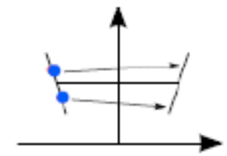
$G = 0$

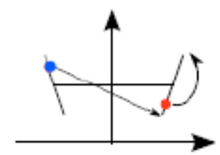


Low-temperature transport, $T \ll T_K$ (away from the "dome" on slide 54)

"weak" e-e interaction

$$G = \frac{2e^2}{h} - \delta G \begin{cases} \sim (T/T_K)^{8K-2} & 1/4 < K < 2/3 \\ \sim (T/T_K)^{2K+2} & K > 2/3 \end{cases}$$





"strong" e-e interaction $K < 1/4$

$$\mathrm{G} \sim (T/T_K)^{2(1/4K-1)}$$ from instanton processes

J. Maciejko *et al.*, PRL **102**, 256803 (2009)

”High-temperature” transport, $T \gg T_K$

$$I = G_0 V - \delta I$$

$$J_{\perp,z} \ll \omega \ll T$$

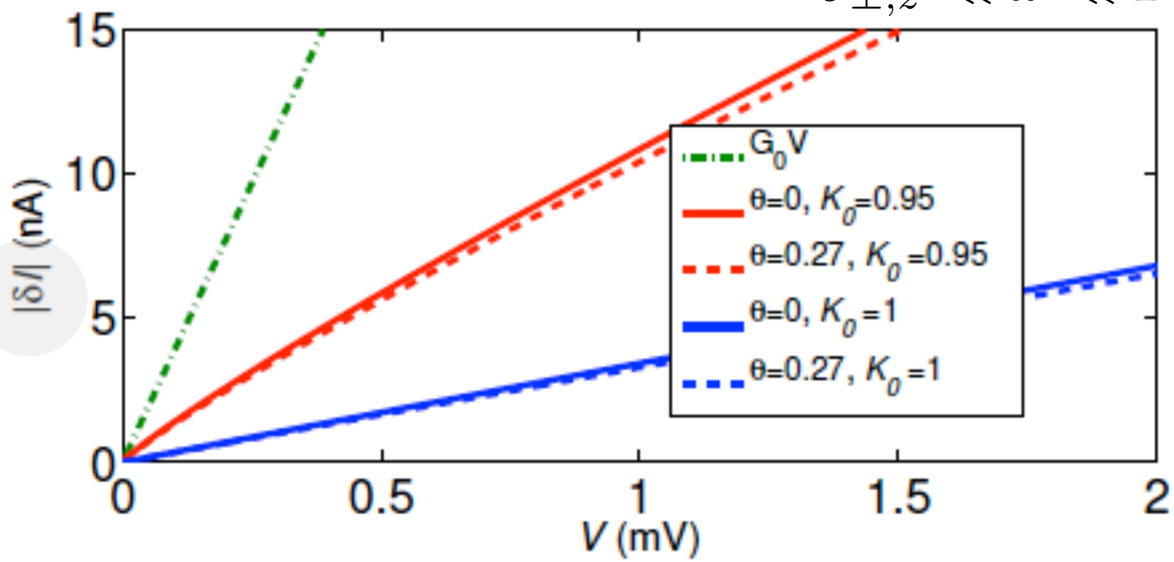


FIG. 2: The RG-improved current correction (12) at $T = 30$ mK as a function of applied voltage, for different values of K_0 and θ. The dashed lines represent $\theta \approx 0.27$, corresponding to $\hbar\alpha = 10^{-10}$ eVm. Other parameters are defined in the text. The QSH edge current $G_0 V$ is plotted as a reference.

E. Eriksson, A. Ström, G. Sharma, H.J., PRB **86**, 161103(R) (2012); *erratum*, to be published