Workshop on Quantum Gases of Cold Atoms and Many-Body Correlations
Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan, China
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Synthesizing Majorana zero modes in a periodically gated quantum wire

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Universidade de Brasília

UNIVERSITY OF GOTHENBURG

CSRC

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Vetenskapsrådet

Outline

Introduction: Majorana zero modes

- What are they?
- Why are they interesting?
- How to get them?

Outline

Majorana zero modes

- What are they?
- Why are they interesting?
- How to get them?

A new proposal:

- Spin-orbit-coupled correlated electrons in one dimension
- Case study I: A periodically gated InAs quantum wire
- Case study II: Cold atoms?

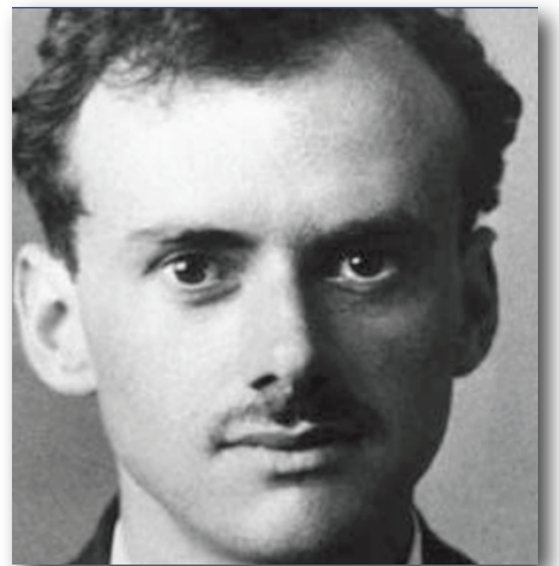
Introduction: Dirac, Weyl, and Majorana

Dirac fermions

what matter is made of...

$$\boxed{\left(i\gamma^\mu \partial_\mu - m\right)\psi = 0}$$

P. A. M. Dirac, Proc. Royal Soc. A (1928)



Paul Dirac

Introduction: Dirac, Weyl, and Majorana

Weyl fermions

$$(i\gamma^\mu \partial_\mu - \cancel{m})\,\psi = 0$$

$$\psi = \begin{pmatrix} u_+ \\ u_- \end{pmatrix}$$

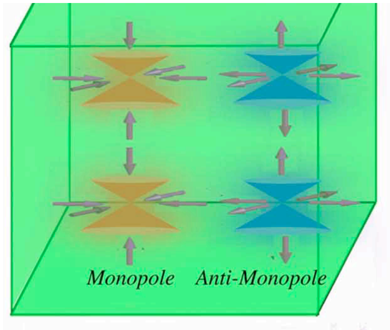
$$\boxed{i\sigma^\mu \partial_\mu u_- = 0}$$

H. Weyl, Z. Physik (1929)

Hermann Weyl

Weyl fermions

$$\boxed{i\sigma^{\mu}\partial_{\mu}u_{-} = 0}$$



Prediction: *emergent* particles in *Weyl semimetals*

S. Murakami, NJP (2007); X. Wan *et al.,* PRB (2011)

Introduction: Dirac, Weyl, and Majorana

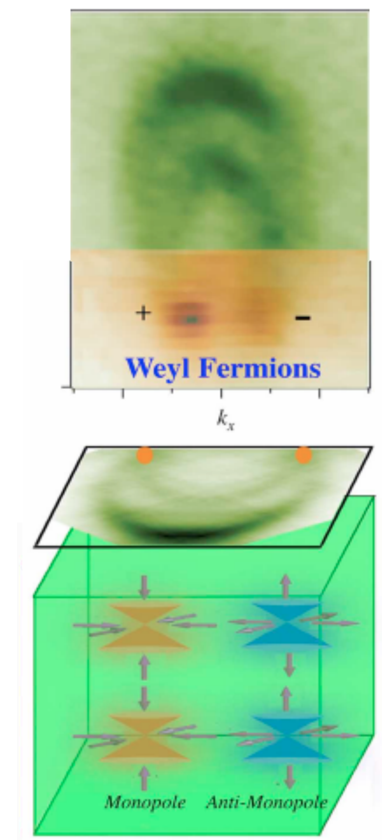
Weyl fermions

$$\boxed{i\sigma^{\mu}\partial_{\mu}u_{-} = 0}$$

Observed in ARPES experiments on TaAs

S.-Y. Xu *et al.*, Science (2015)

B. Q. Lv *et al.*, PRX (2015)



Prediction: *emergent* particles in *Weyl semimetals*

S. Murakami, NJP (2007); X. Wan *et al.*, PRB (2011)

Introduction: Dirac, Weyl, and Majorana

"A great deal more was hidden in the Dirac equation than the author had expected when he wrote it down in 1928. Dirac himself remarked in one of his talks that his equation was more intelligent than its author."

Victor Weisskopf on Dirac

Majorana fermions

$$(i\bar{\gamma}^\mu \partial_\mu - m)\,\bar{\psi} = 0$$

imaginary rep

real

E. Majorana, Nuovo Cim. (1937)

Ettore Majorana

Majorana fermions

$$\boxed{(i\bar{\gamma}^{\mu}\partial_{\mu} - m)\,\bar{\psi} = 0}$$

imaginary rep — real

$$\bar{\psi} = \bar{\psi}^{\star}$$

particle = antiparticle

Ettore Majorana

Are neutrinos Majoranas fermions?

Majorana fermions

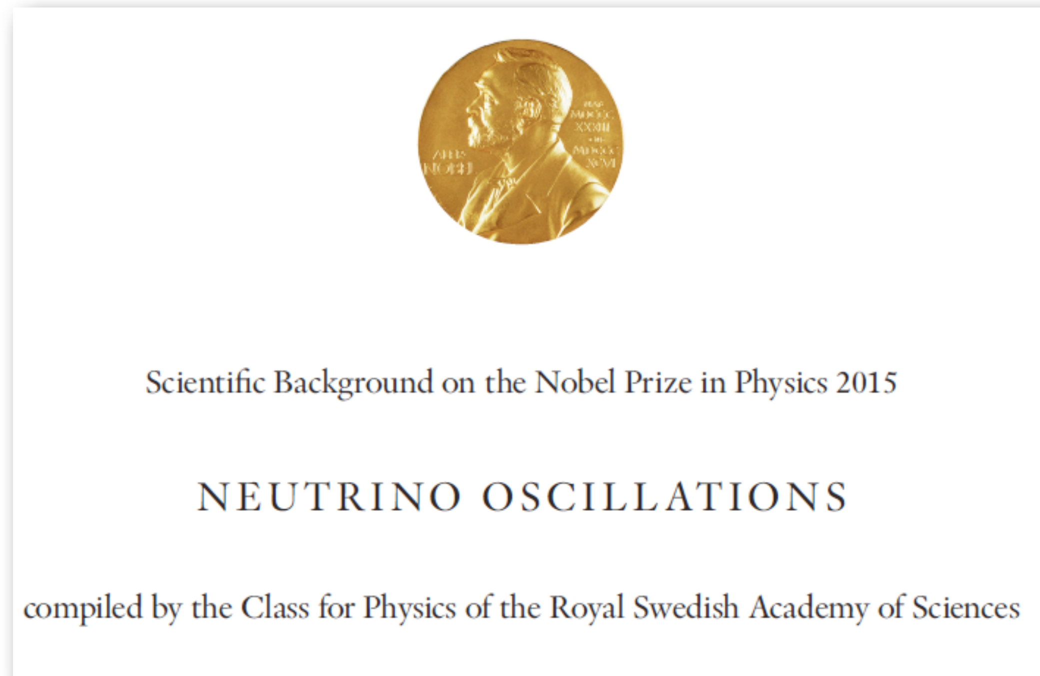
$$(i\bar{\gamma}^\mu \partial_\mu - m)\,\bar{\psi} = 0$$

imaginary rep

real

$$\bar{\psi} = \bar{\psi}^\star$$

particle = antiparticle


Scientific Background on the Nobel Prize in Physics 2015

NEUTRINO OSCILLATIONS

compiled by the Class for Physics of the Royal Swedish Academy of Sciences

The best way to investigate if neutrinos are indeed Majorana particles is believed to be neutrino-less double beta decay. These processes are forbidden in the Standard Model but could in principle occur for the handful of naturally occurring isotopes that normally decay through emission of two electrons (positrons) and two neutrinos. Many experiments search for neutrino-less double beta decay, so far without success.

Are neutrinos Majorana fermions?

Majorana fermions

$$\boxed{(i\bar{\gamma}^\mu \partial_\mu - m)\,\bar{\psi} = 0}$$

imaginary rep

real

$$\bar{\psi} = \bar{\psi}^\star$$

Where else to look...?

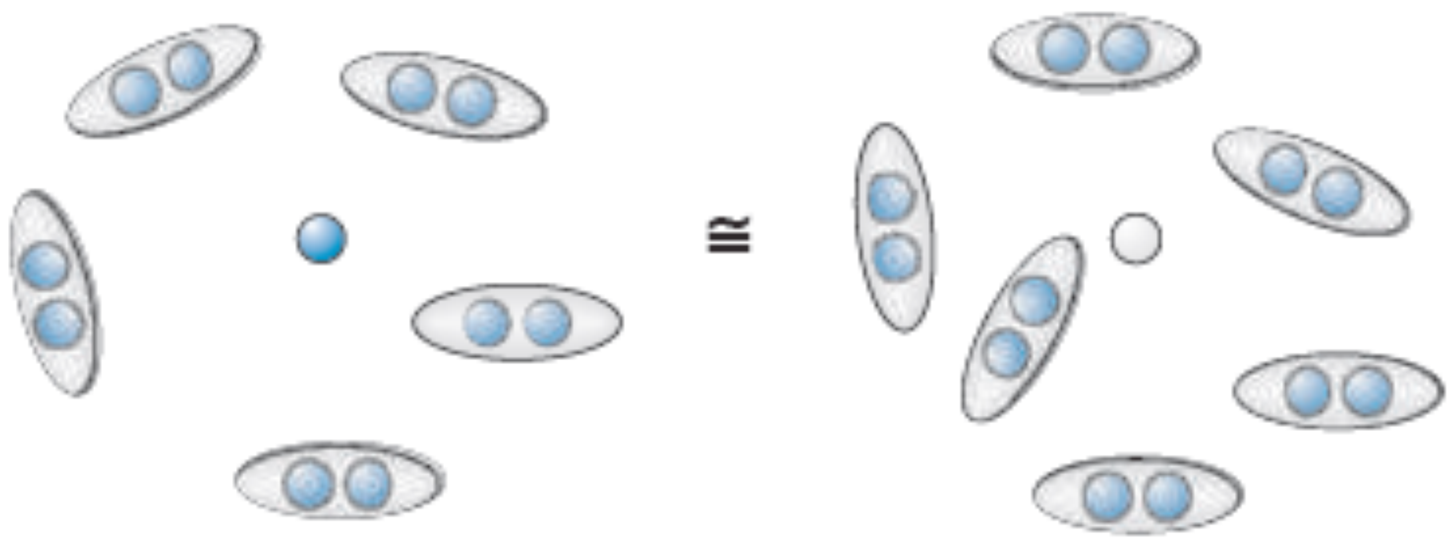
Introduction: Majorana zero-modes

spinless

1D p-wave

2D $p_x + ip_y$

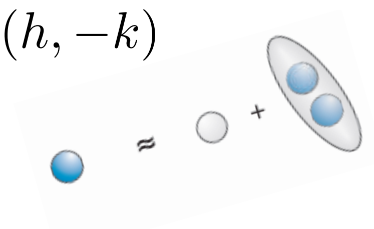
... in a superconductor!

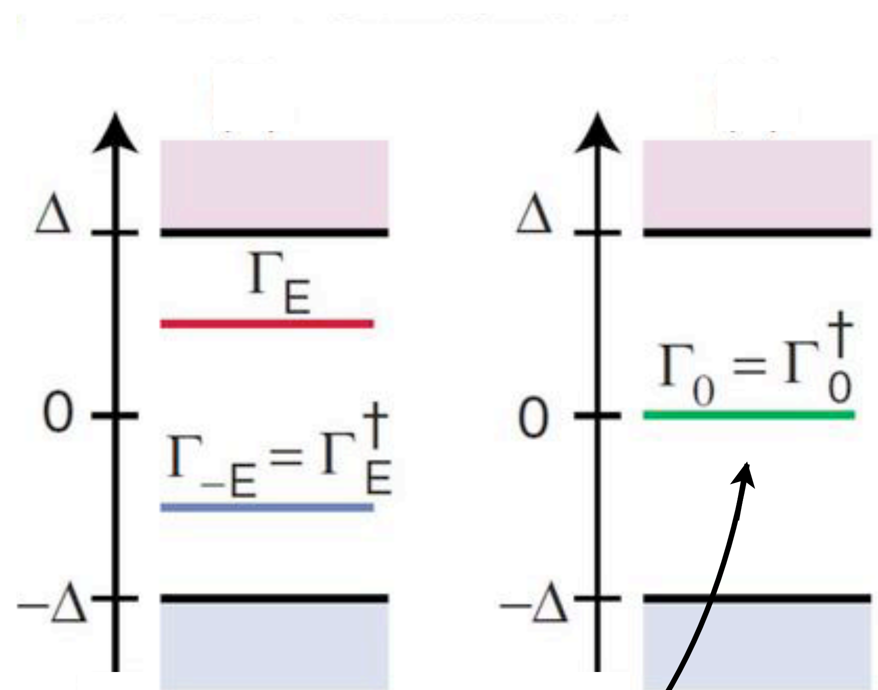


Introduction: Majorana zero-modes

Energy gap for quasiparticles

$$
\begin{aligned}
(e,k) \;&\rightarrow\; (e,k) + [(e,-k) + (h,-k)] \\
&=\; (2e,0) + (h,-k)
\end{aligned}
$$



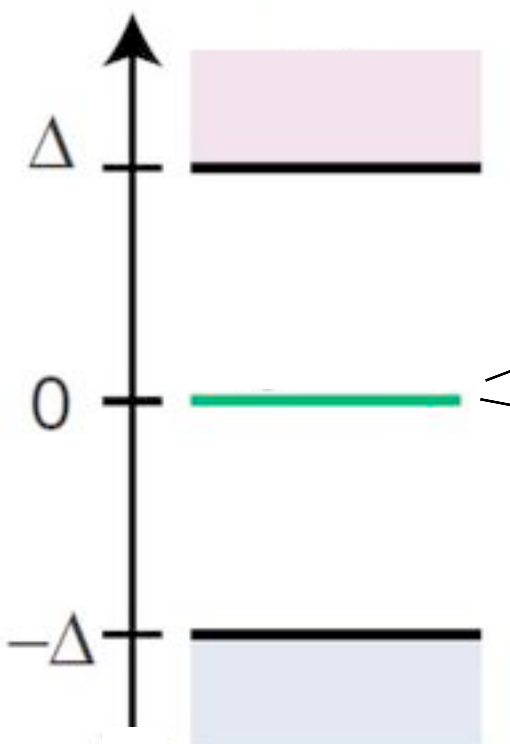


Intrinsic particle-hole symmetry!

Majorana zero(-energy) mode

$\gamma \equiv \Gamma_0$

Introduction: Majorana zero-modes



$$\gamma_A = c + c^\dagger$$

$$\gamma_B = i(c - c^\dagger)$$

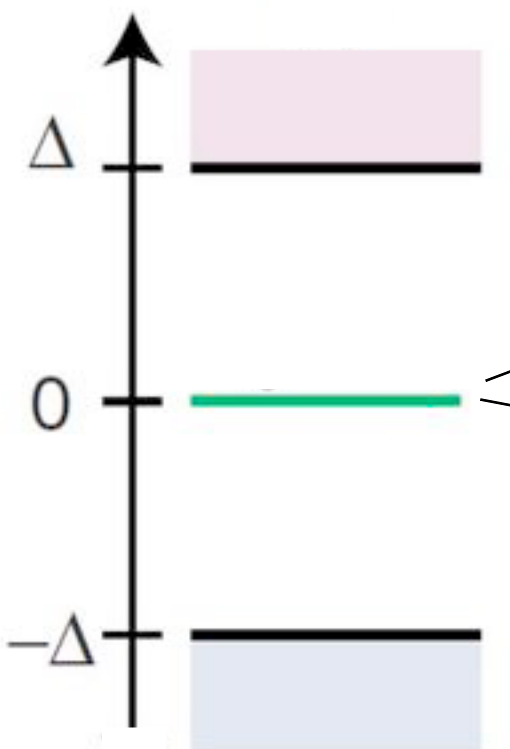
$$c = \frac{1}{2}(\gamma_A + i\gamma_B)$$

$$c^\dagger = \frac{1}{2}(\gamma_A - i\gamma_B)$$

$$\boxed{\begin{aligned} \gamma_A &= \gamma_A^\dagger \\ \gamma_B &= \gamma_B^\dagger \end{aligned}}$$

$$\boxed{\{\gamma_i, \gamma_j\} = 2\delta_{ij}}$$

Introduction: Majorana zero-modes



$$\gamma_A = c + c^\dagger$$

$$\gamma_B = i(c - c^\dagger)$$

$$c = \frac{1}{2}(\gamma_A + i\gamma_B)$$

$$c^\dagger = \frac{1}{2}(\gamma_A - i\gamma_B)$$

The Majorana modes
may separate spatially!
Nonlocal single-particle states!

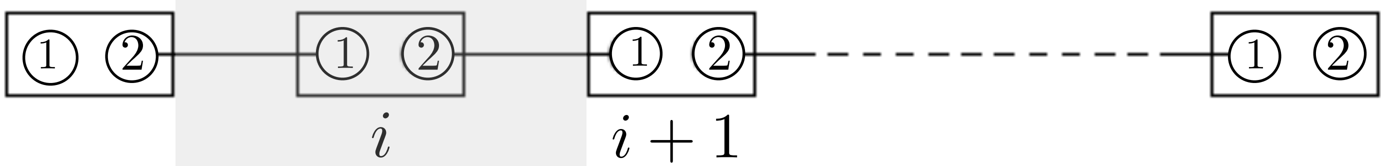
What's the
big deal?

Introduction: Majorana zero-modes

Kitaev chain: toy model for a 1D p-wave superconductor

A. Y. Kitaev, Physics-Uspekhi (2001)

$$H = -\mu \sum_i c_i^\dagger c_i - \frac{1}{2} \sum_i (t c_i^\dagger c_{i+1} + \Delta e^{i\phi} c_i c_{i+1} + \mathrm{H.c.})$$



$$c_i = (\gamma_{i,1} + i\gamma_{i,2})/2$$

$$c_i^\dagger = (\gamma_{i,1} - i\gamma_{i,2})/2$$

Introduction: Majorana zero-modes

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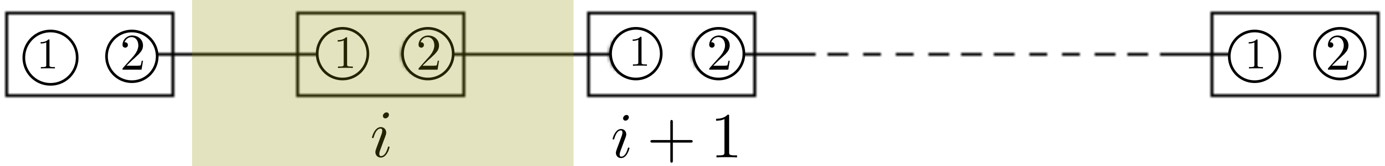

$$\gamma_{i,1} = c_i^\dagger + c_i,$$
$$\gamma_{i,2} = i\left(c_i^\dagger - c_i\right)$$

$$H = -it \sum_{i=1}^{N-1} \gamma_{i,2}\gamma_{i+1,1} \qquad \mu = 0,\ t = \Delta.$$

Introduction: Majorana zero-modes

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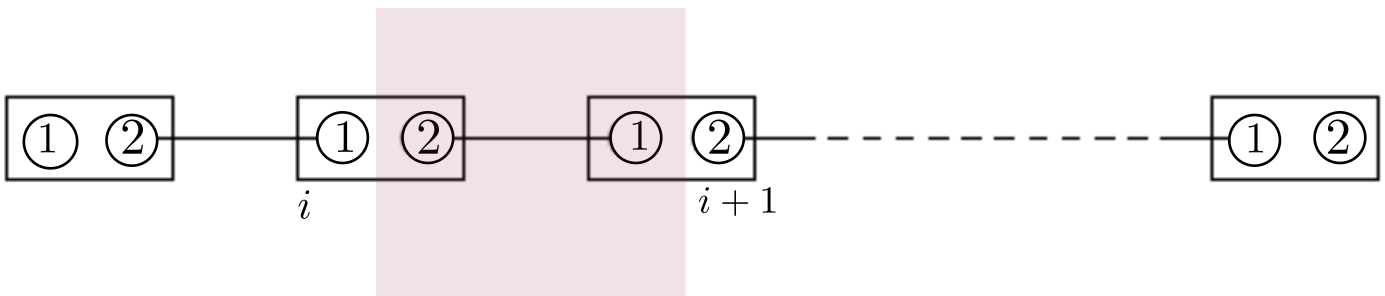


$$\tilde{c}_i = (\gamma_{i+1,1} + i\gamma_{i,2})/2$$

$$\tilde{c}_i^\dagger = (\gamma_{i+1,1} - i\gamma_{i,2})/2$$

$$H = -it \sum_{i=1}^{N-1} \gamma_{i,2}\gamma_{i+1,1} \qquad \mu = 0,\ t = \Delta,$$

$$H = 2t \sum_{i=1}^{N-1} \tilde{c}_i^\dagger \tilde{c}_i \qquad \mu = 0,\ t = \Delta,$$

Introduction: Majorana zero-modes

Kitaev chain: toy model for a 1D p-wave superconductor

A. Y. Kitaev, Physics-Uspekhi (2001)

$$H = 2t \sum_{i=1}^{N-1} \tilde{c}_i^\dagger \tilde{c}_i \quad \mu = 0, t = \Delta,$$

$\tilde{c}_M = (\gamma_{N,2} + i\gamma_{1,1})/2.$ is absent from the Hamiltonian!

Pair of Majorana zero-energy modes bound to the edges.

Two-fold degenerate ground state.

Introduction: Majorana zero-modes

Kitaev chain: toy model for a 1D p-wave superconductor

A. Y. Kitaev, Physics-Uspekhi (2001)

Topological phase

$|\mu| < 2t.$

Pair of Majorana zero-energy modes bound to the edges.
Two-fold degenerate ground state.

Introduction: Majorana zero-modes

Kitaev chain: toy model for a 1D p-wave superconductor

Topological phase

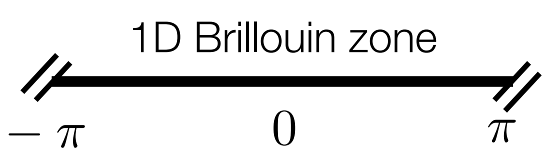
$|\mu| < 2t$

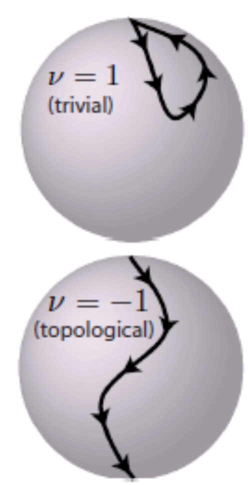
Z_2 topological index $\nu = \pm 1$
"D symmetry class"
Schnyder *et al.*, PRB (2008)
Kitaev, AIP Conf. Proc. (2009)

$$H + \text{translational invariant perturbation} = \sum_{k \in BZ} C_k^\dagger \mathcal{H}_k C_k$$

$$C_k^\dagger = (c_k^\dagger \; c_{-k})$$
$$\mathcal{H}_{\boldsymbol{k}} = \boldsymbol{h}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}$$
$$\hat{\boldsymbol{h}}(k) = \boldsymbol{h}(k) / |\boldsymbol{h}(k)|$$

$\hat{\boldsymbol{h}}(k)$:

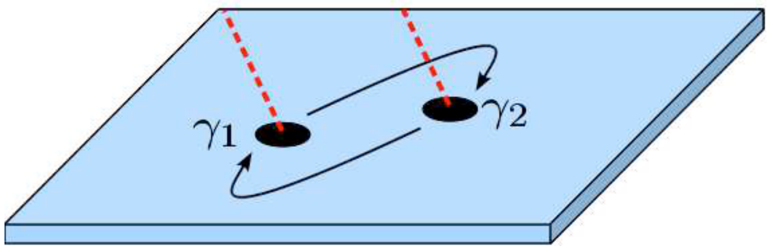




Majorana zero-modes for a chain with open boundaries

Introduction: Majorana zero-modes

2D $p_x + ip_y$ superconductors also host Majorana zero-modes, bound to vortices
Read & Green, PRB (2000)



$\gamma_1 \rightarrow -\gamma_2$ (one crossing of the branch cut)

$\gamma_2 \rightarrow +\gamma_1$

$$\gamma_i \rightarrow B_{12}\gamma_i B_{12}^\dagger, \;\; i = 1, 2$$

Braid operator $B_{12} = \frac{1}{\sqrt{2}}(1 + \gamma_1\gamma_2)$

Non-abelian statistics

also for Majorana zero-modes in 1D p-wave superconductors
Alicea et al., Nat. Phys. (2011)...

...as well as for ν=5/2 FQHE and cold atom proposals for Majoranas
Moore & Read, NPB (1991)
Sato et al., PRL (2009)

$$[B_{i-1,i}, B_{i,i+1}] = \gamma_{i-1}\gamma_{i+1}$$

Exchange of *several* Majorana modes do not commute!

Introduction: Majorana zero modes

Braiding which involves Majoranas from different fermions produce superpositions of *different* number states.

Look at $|11\rangle = c_a^\dagger c_b^\dagger \, |00\rangle$

$$\begin{aligned} c_a &= (\gamma_1 + i\gamma_2)/2 \\ c_b &= (\gamma_3 + i\gamma_4)/2 \end{aligned}$$

Topological quantum computing?

$$B_{23}\,|00\rangle = (|00\rangle + i\,|11\rangle)/\sqrt{2}$$

Introduce *qubit states* $\begin{aligned} |\bar{0}\rangle &= |00\rangle \\ |\bar{1}\rangle &= |11\rangle \end{aligned}$

Then, $B_{23} = \exp(-i\pi\sigma_x/4)$ on the Bloch sphere.

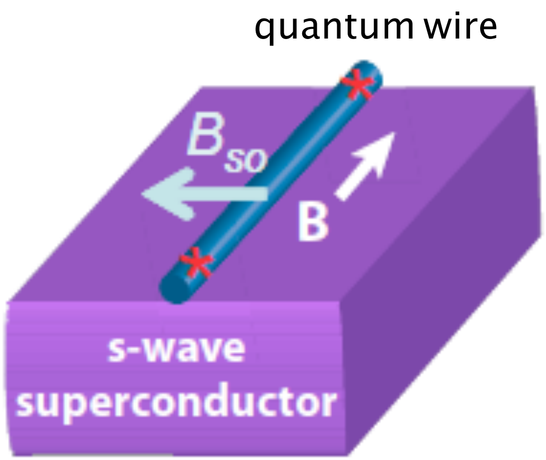
Majorana braidings = single-qubit rotations.

The number states are delocalized and hence robust against local perturbations.

Introduction: Majorana zero modes

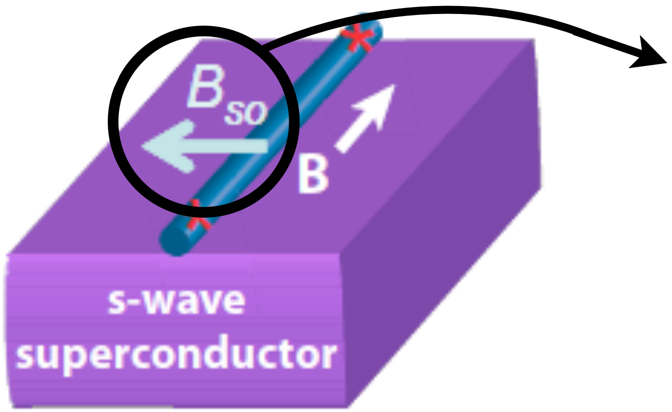
Experimental search for Majoranas in *1D p-wave superconductors*

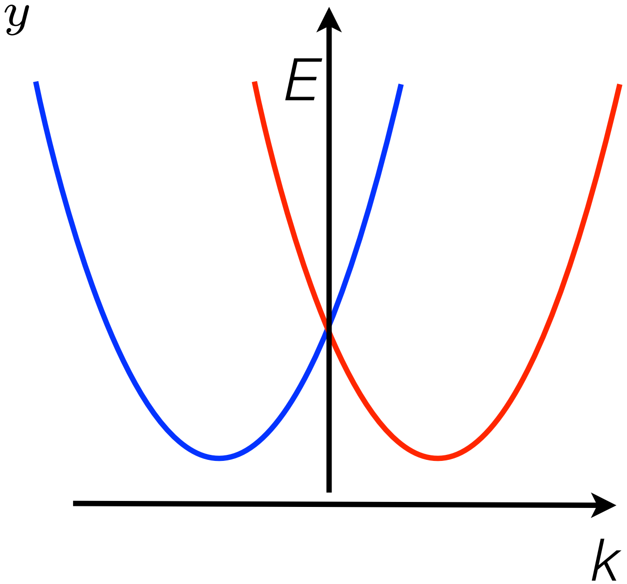
simplest, most easily accessible...



Introduction: Majorana zero modes

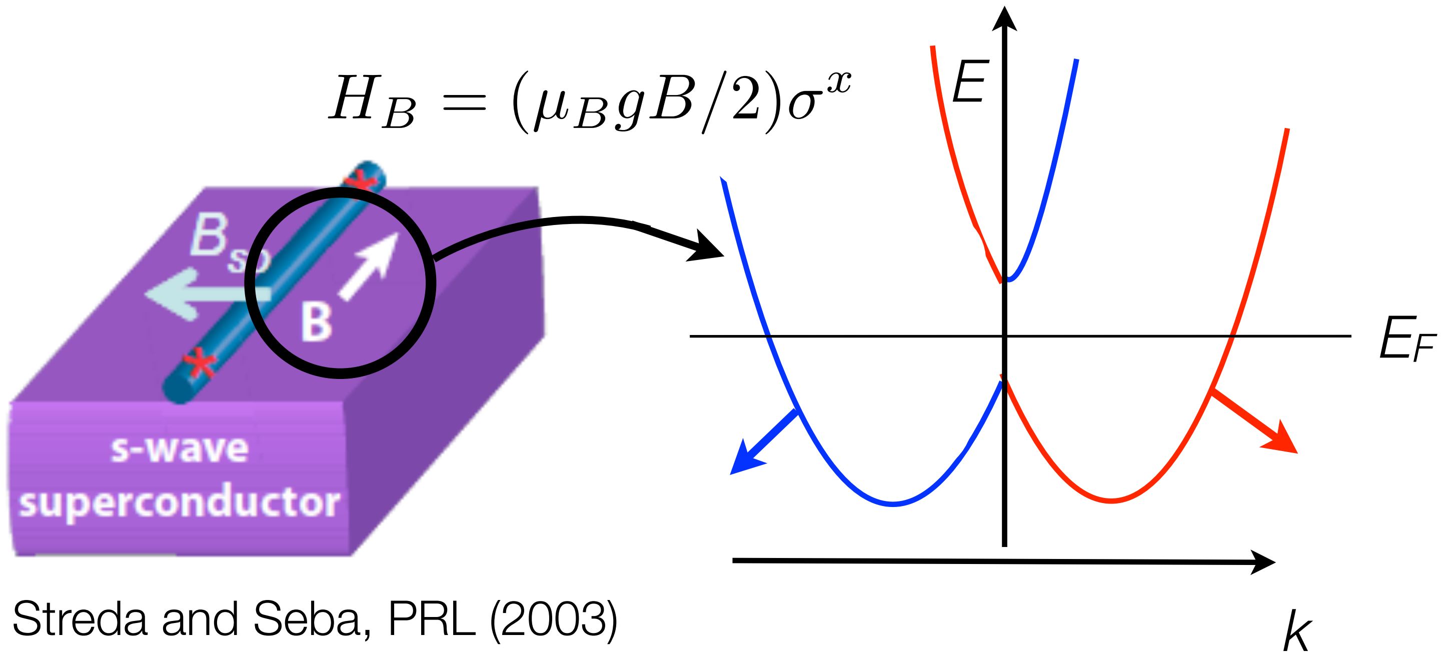
$$H_{B_{SO}} = \alpha k_x \sigma^y$$





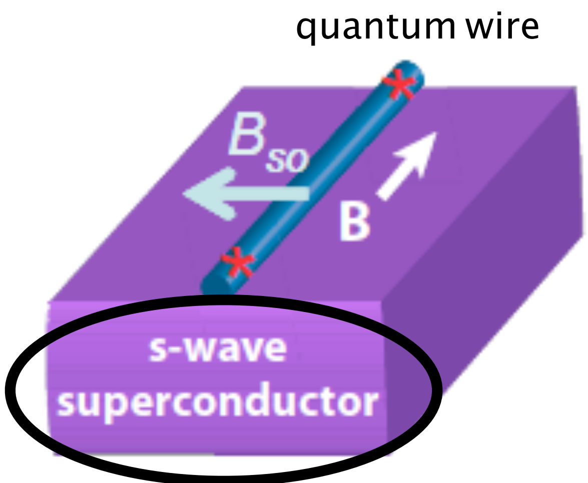
Introduction: Majorana zero modes

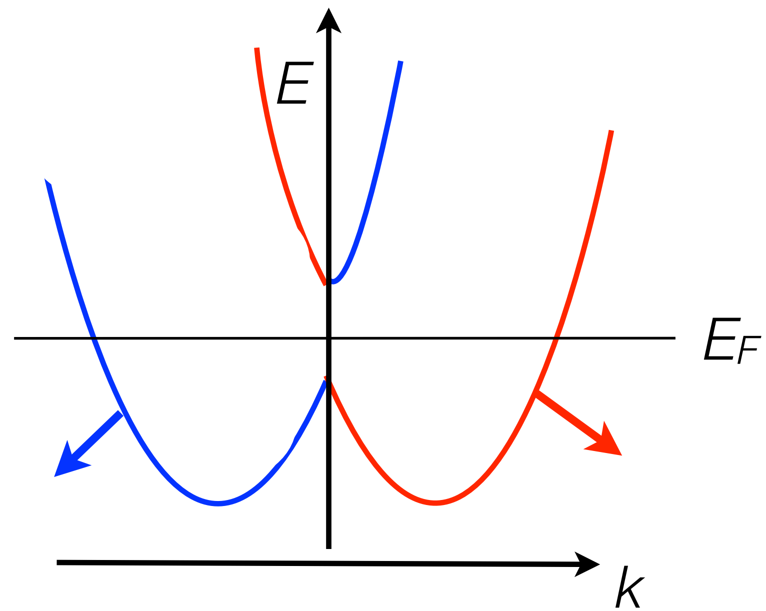
$$H_B = (\mu_B g B/2)\sigma^x$$



Streda and Seba, PRL (2003)

Introduction: Majorana zero modes



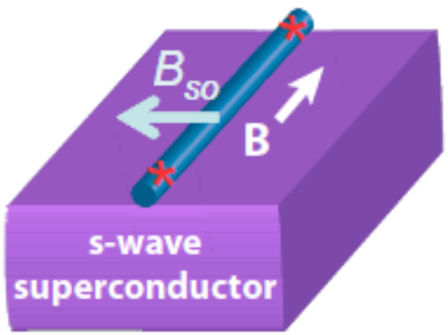


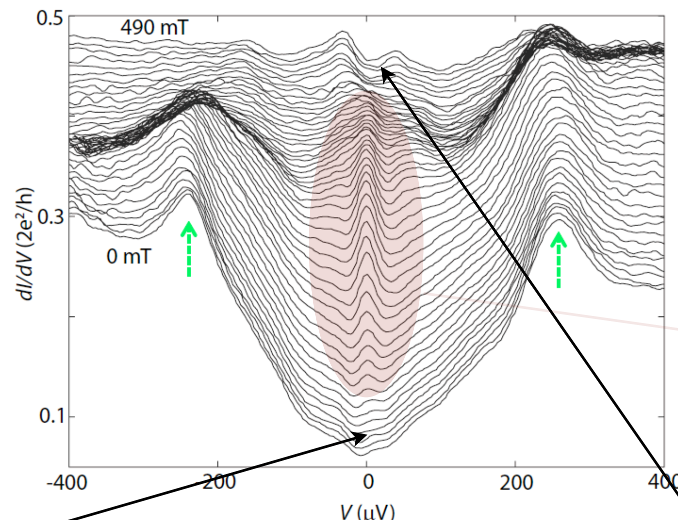
”spinless” 1D *p*-wave superconductor
by s-wave proximity effect

J. D. Sau *et al.*, PRL (2010)
Y. Oreg *et al.*, PRL (2010)
J. Alicea, PRB (2010)

Introduction: Majorana zero modes

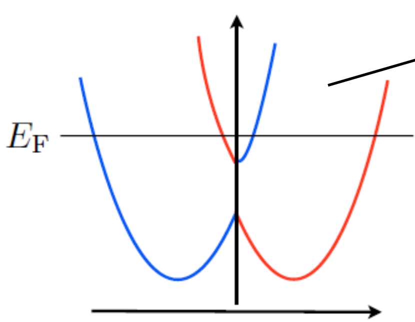
Experimental search for Majoranas in *1D p-wave superconductors*



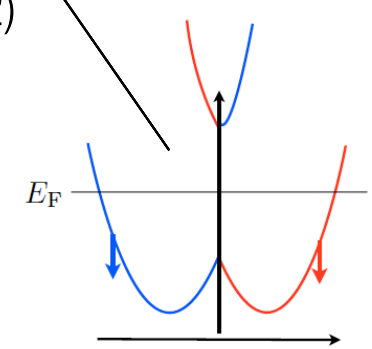


conductance peaks
indicate a midgap state...
Majorana zero mode?

V. Mourik *et al.*, Science (2012)
reproduced in other experiments



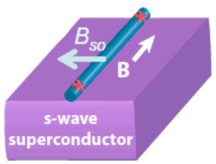
spinful electrons
no Majoranas

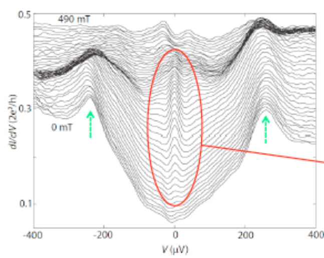


no proximity pairing
no Majoranas

Introduction: Majorana zero modes

Experimental search for Majoranas in *1D p-wave superconductors*





conductance peaks
indicate a midgap state...
Majorana zero mode?

Mourik *et al.*, Science (2012)
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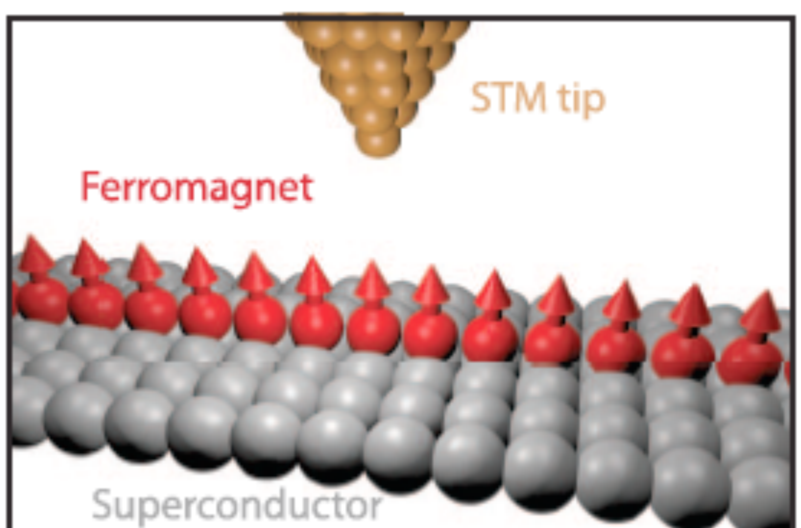
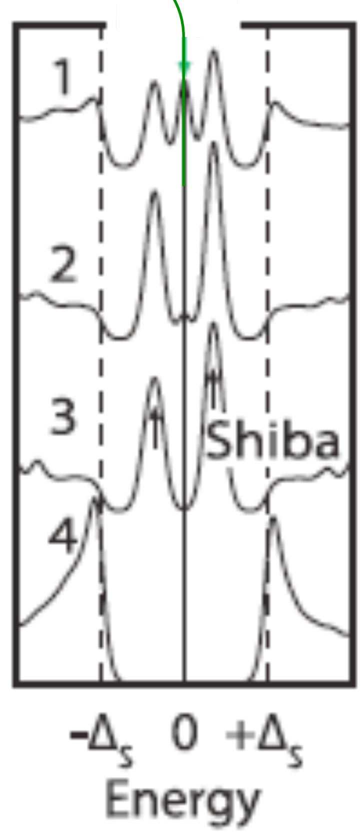
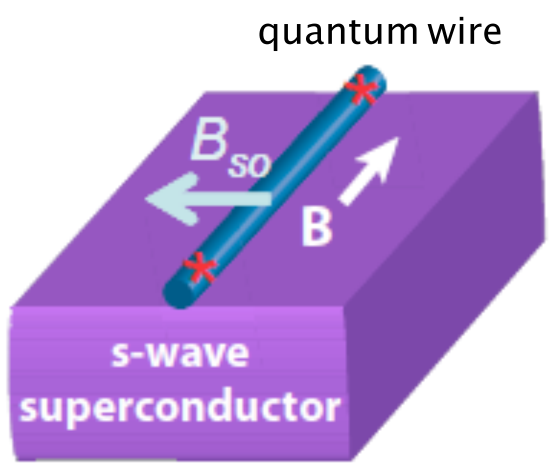




S. Nadj-Perge *et al.*, Science (2014)

Introduction: Majorana zero modes

Experimental search for Majoranas in *1D p-wave superconductors*



The **magnetic field** allows for tuning through the topological phase...

... but also creates some problems!

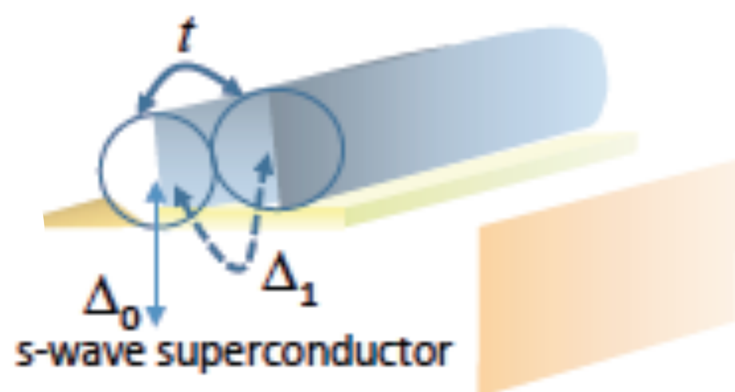
- weakened proximity effect
- enhanced susceptibility to disorder
- impractical for applications
- ....

Can one do without the magnetic field?

Introduction: Majorana zero modes

Many "magnet-free" schemes exploiting time-reversal symmetry, allowing for *paired* Majoranas at the ends of the wire...
("DIII symmetry class", A. Schnyder *et al.*, PRB (2008); A. Kitaev, AIP Conf. Proc. (2009))

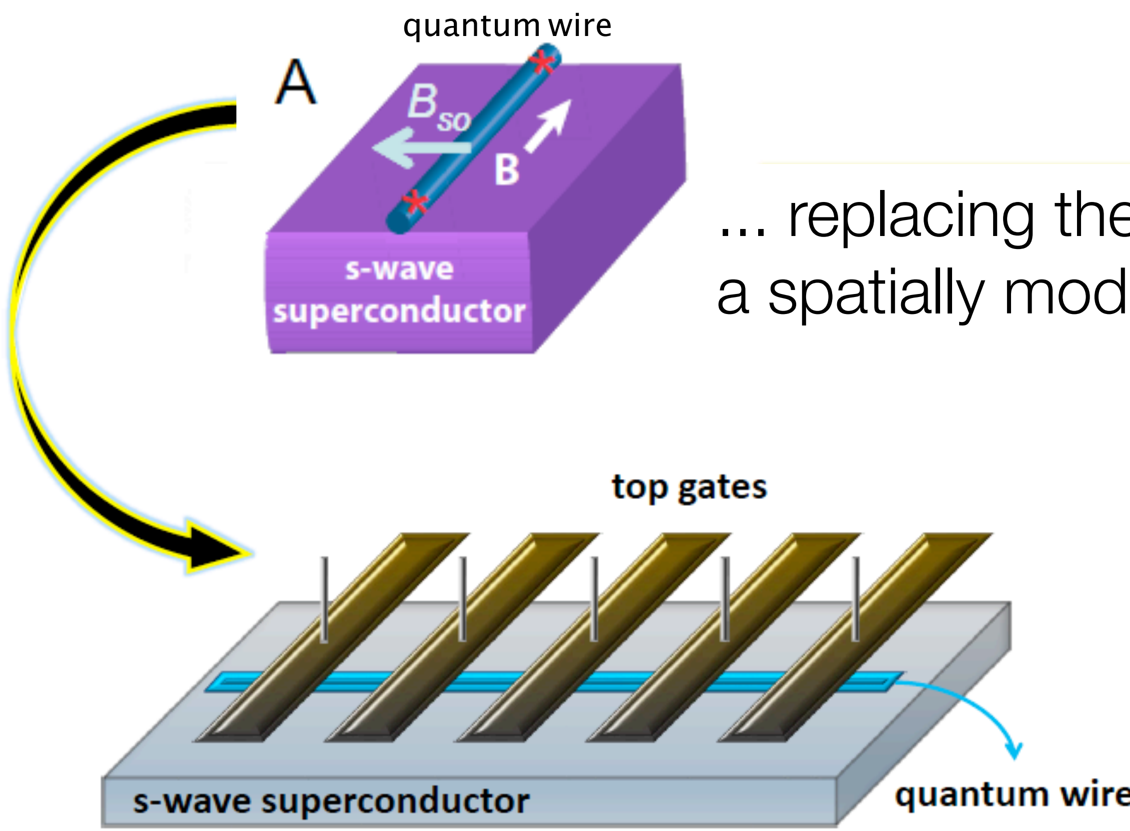
Deng *et al.*, PRL (2102)
Wong & Law, PRB (2012)
Zhang *et al.*, PRL (2013)
Nakosai *et al.*, PRL (2013)
Keselman *et al.*, PRL (2013)
Sticlet *et al.*, PRB (2013)
Chung *et al.*, PRB (2013)
Liu *et al.*, PRX (2014)
Dumitrescu *et al.*, PRB (2014)
Gaidamauskas *et al.* PRL (2014)
Haim *et al.*, PRB (2014)
Klinovaja & Loss, PRB (2014)
Kotetes, PRB (2015)
... and more?



two channels with time-reversal symmetry

A new proposal for a topological superconductor...

M. Malard, G. I. Japaridze, and H. J., *to appear*

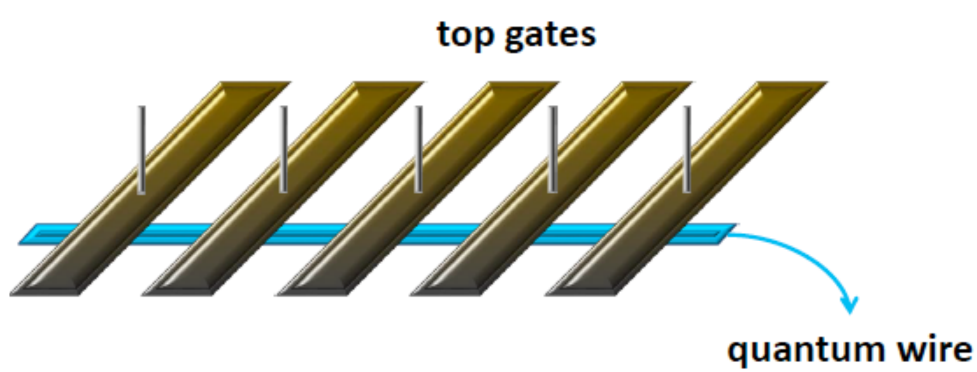


... replacing the magnetic field by a spatially modulated electric field

”D symmetry class” with *unpaired* Majoranas at the end of the wire (robust against time-reversal breaking)

First step: generate a 1D helical electron liquid

G. I. Japaridze, H. J., and M. Malard, PRB (2014)

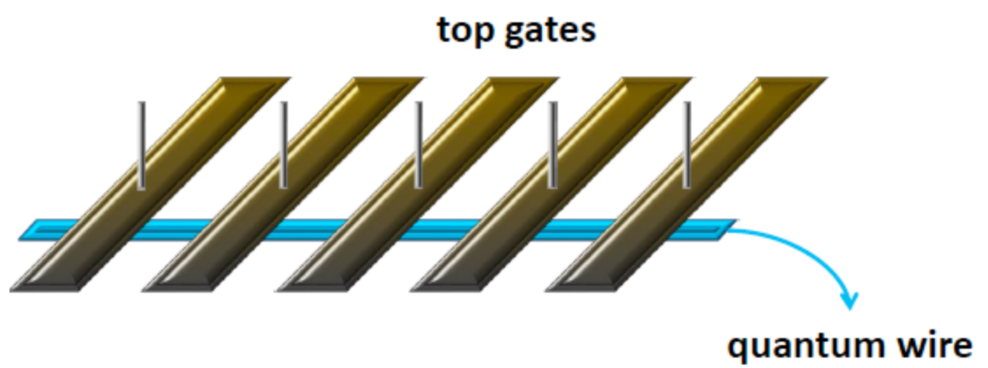


A new proposal for a topological superconductor...

First step: generate a 1D helical electron liquid, using:

a material with strong intrinsic Dresselhaus spin-orbit interaction

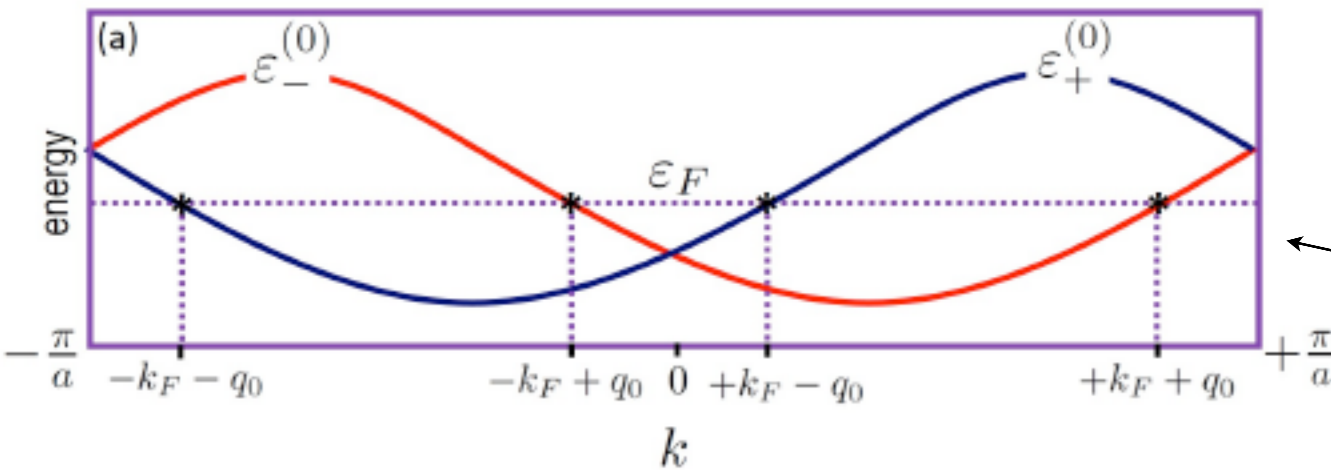
a modulated Rashba spin-orbit interaction from a ”keyboard” of charged top gates

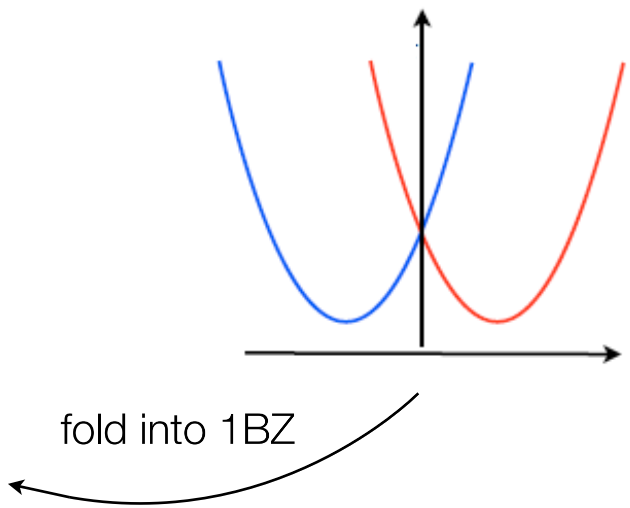


weakly screened electron-electron interactions

A new proposal for a topological superconductor...

Start with a spin-orbit coupled quantum wire.
Lowest spin-split bands:





A new proposal for a topological superconductor...

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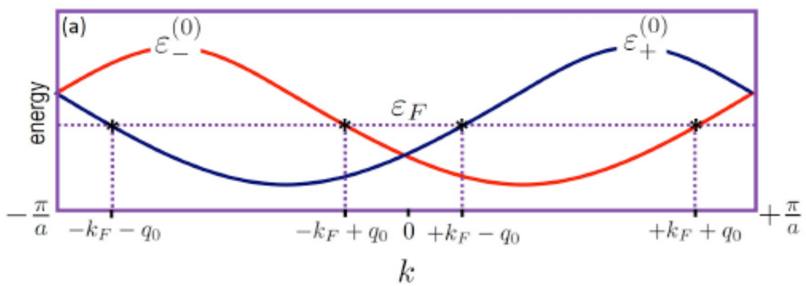
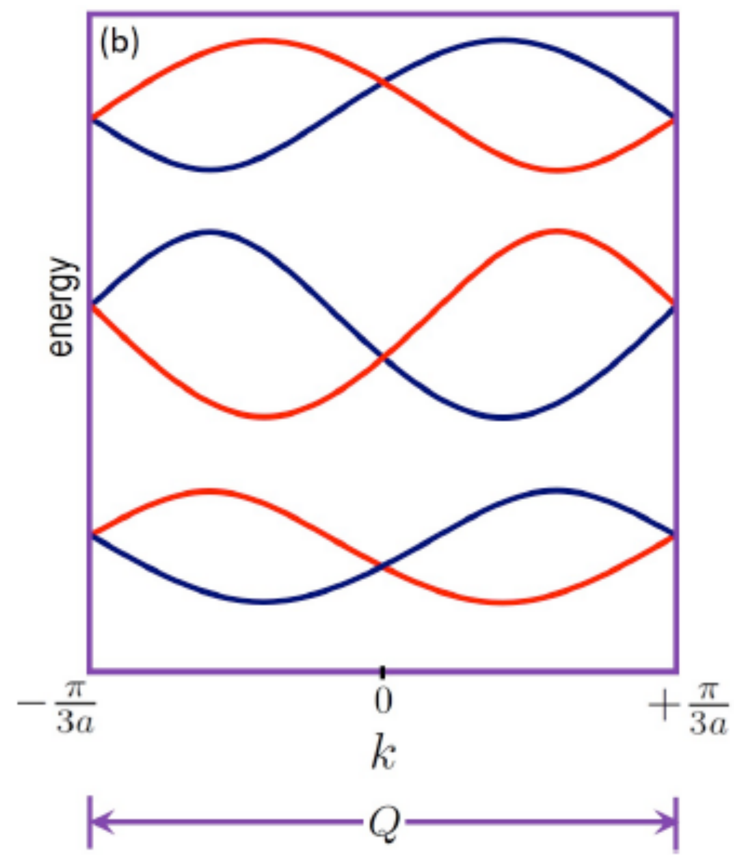
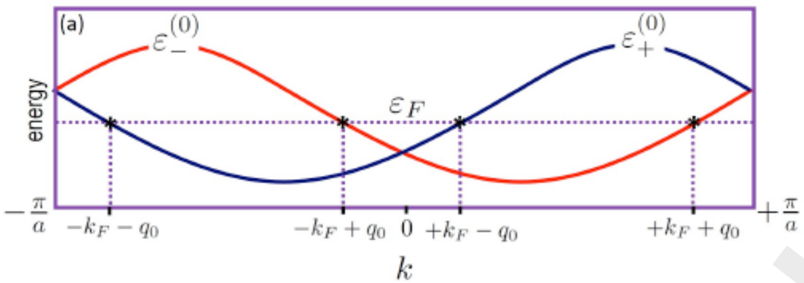
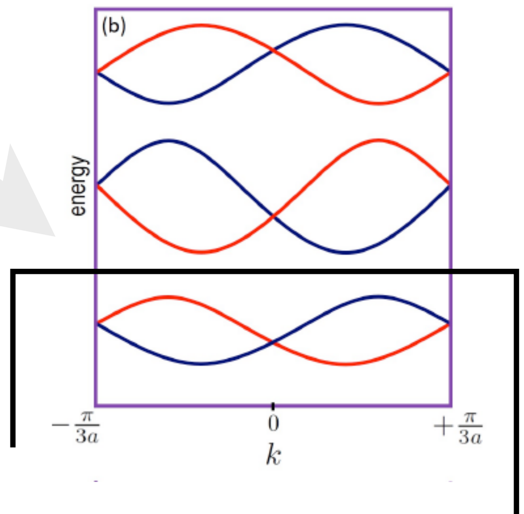


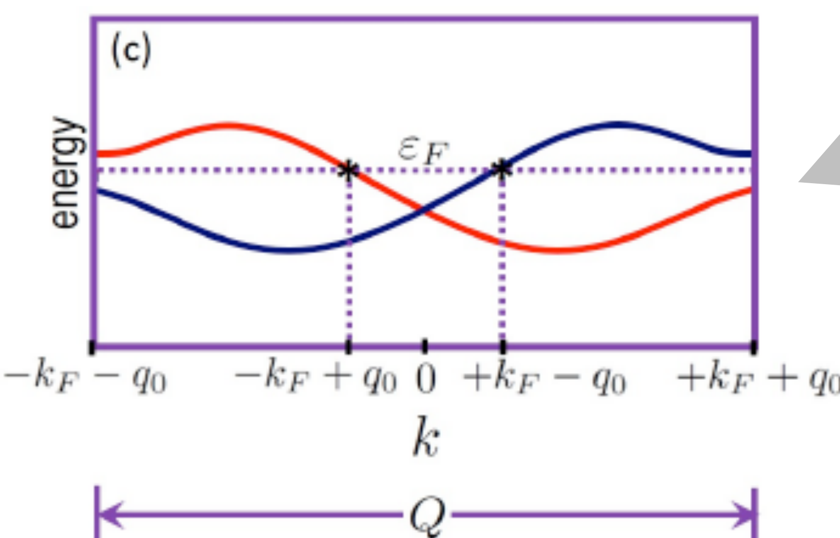
"Turn on" the keyboard of top gates:
Modulated chemical potential & Rashba spin-orbit interaction of wave number.
$Q = (2\pi/a)(p/r)$ (here $p = 1$, $r = 3$)



A new proposal for a topological superconductor...







Tune the Fermi level so that the outer Fermi points coincide with the BZ boundaries.

Add e-e interaction.

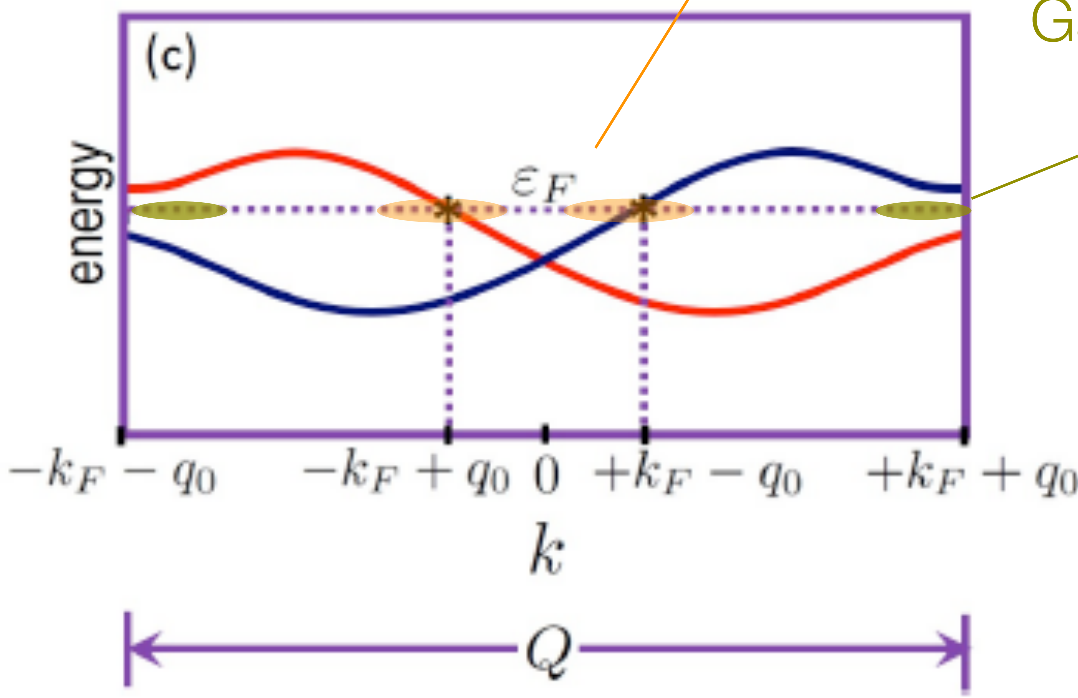
Spontaneous breaking of time-reversal symmetry!

Gap opening!

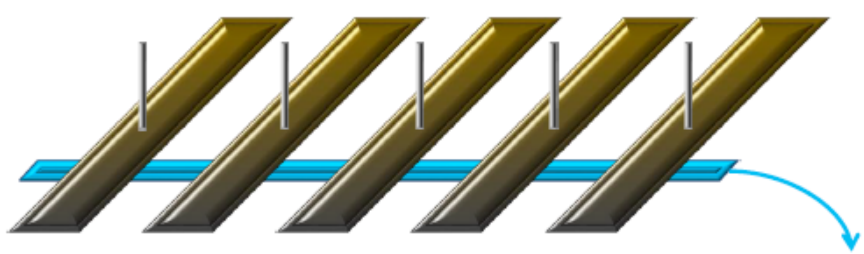
A new proposal for a topological superconductor...

Low-energy "inner branch": *Helical electron liquid*

Gapped "outer branch"



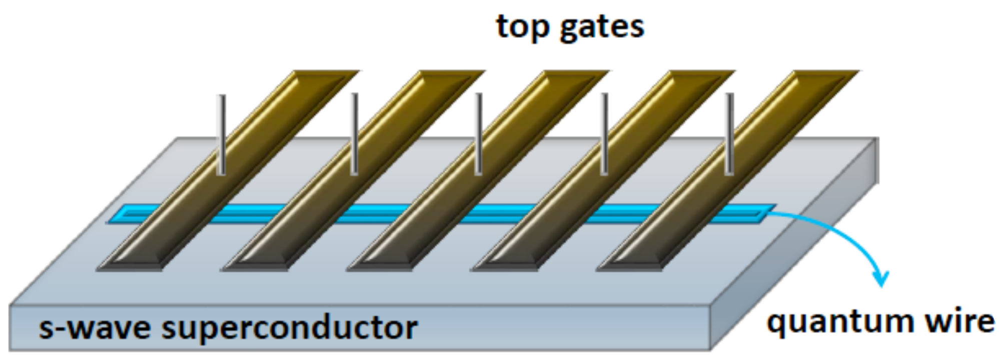
A new proposal for a topological superconductor...



with a helical electron
liquid at low energies

A new proposal for a topological superconductor...

1D p-wave superconductor hosting Majorana zero modes?



Competition between superconducting proximity effect and opening of an insulating gap in the ”outer branch”!

Does it work?

A new proposal for a topological superconductor...

To find out, consider the **lattice model**

kinetic energy + chemical potential + uniform Rashba and Dresselhaus spin-obit interactions:

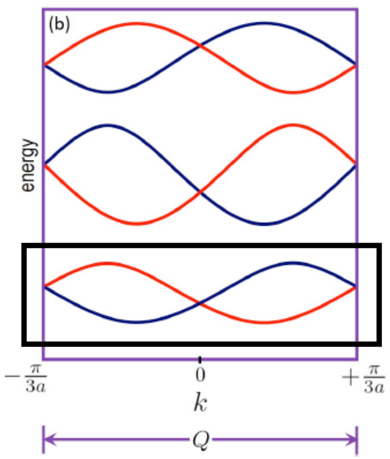
$$H_0 = \sum_{n,\alpha} [-t c^\dagger_{n,\alpha} c_{n+1,\alpha} + (\mu/2) c^\dagger_{n,\alpha} c_{n,\alpha}] - i \sum_{n,\alpha,\alpha'} c^\dagger_{n,\alpha} [\gamma_R \sigma^y_{\alpha\alpha'} + \gamma_D \sigma^x_{\alpha\alpha'}] c_{n+1,\alpha'} + H.c.$$

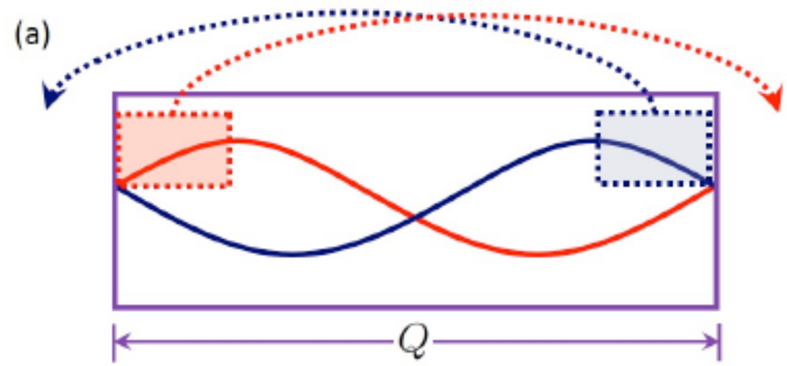
modulated Rashba spin-orbit interaction: $H_{mod} = -i \sum_{n,\alpha,\alpha'} c^\dagger_{n,\alpha}\, \gamma'_R \cos(Qna) \sigma^y_{\alpha\alpha'}\, c_{n+1,\alpha'} + H.c.$

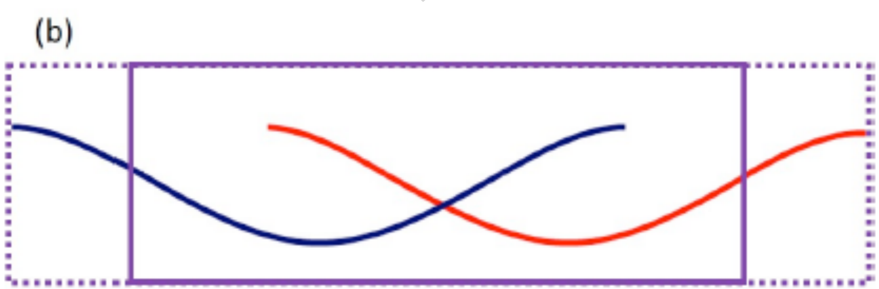
s-wave superconducting pairing potential: $H_{sc} = \sum_n [\,\Delta\, c_{n,\uparrow} c_{n,\downarrow} + H.c.\,]$

e-e interactions: $H_{e-e} = \sum_{n,n',\alpha,\alpha'} V(n-n') c^\dagger_{n,\alpha} c^\dagger_{n',\alpha'} c_{n',\alpha'} c_{n,\alpha}$

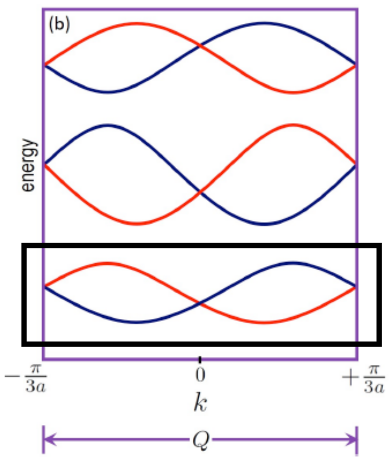
A new proposal for a topological superconductor...

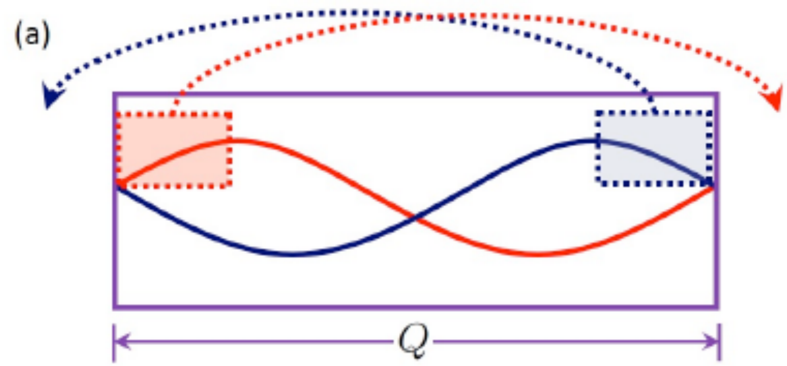






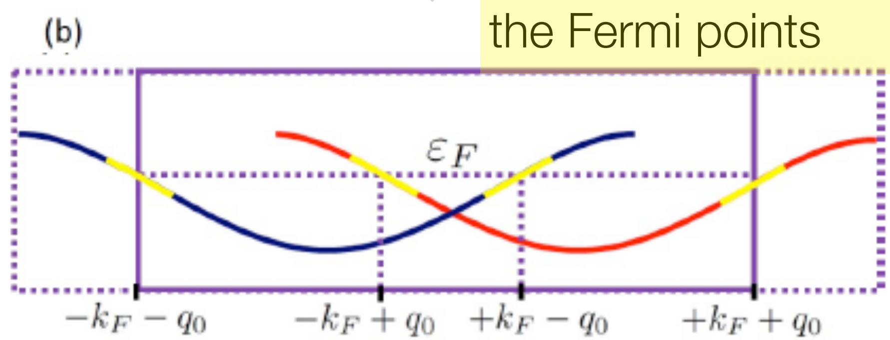
A new proposal for a topological superconductor...





tune the Fermi level

linearization around the Fermi points



A new proposal for a topological superconductor...

Low-energy limit and bosonization:

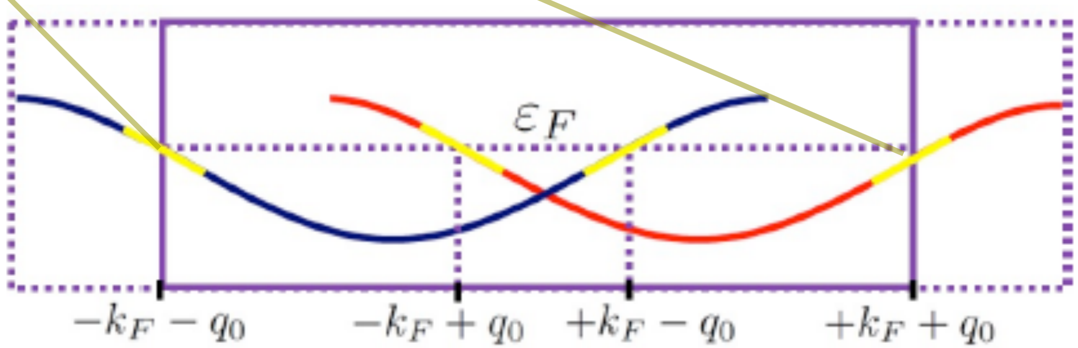
proximity pairing

effective SO coupling

$$\mathcal{H}_1 = u[(\partial_x\theta_1)^2 + (\partial_x\phi_1)^2] - \frac{\Delta}{\pi a}\sin\left(\sqrt{\frac{4\pi}{K}}\theta_1\right) + \frac{\lambda}{\sqrt{\pi K}a}\cos(\sqrt{4\pi K}\phi_1)\partial_x\theta_1$$

"Luttinger liquid" interaction parameter

"nonstandard"!



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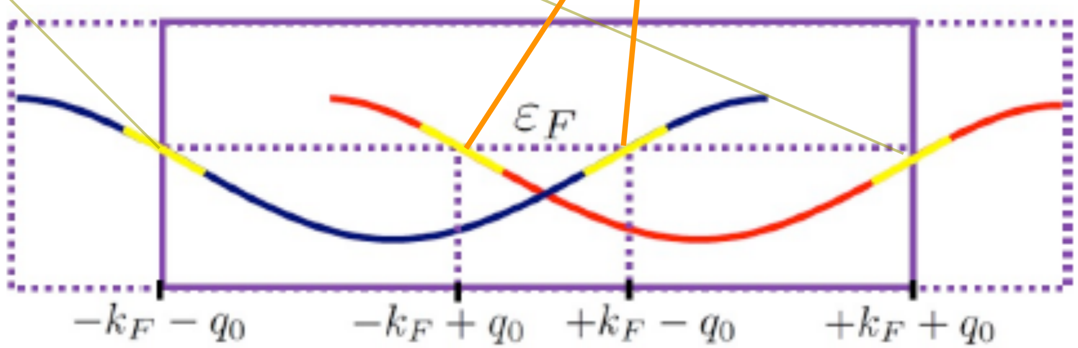
$$\mathcal{H}_2 = u[(\partial_x\theta_2)^2 + (\partial_x\phi_2)^2] - \frac{\Delta}{\pi a}\sin\left(\sqrt{\frac{4\pi}{K}}\theta_2\right)$$



A new proposal for a topological superconductor...

Low-energy limit and bosonization:

outer branch

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$$\mathcal{H}_2 = u[(\partial_x\theta_2)^2 + (\partial_x\phi_2)^2] - \frac{\Delta}{\pi a}\sin\left(\sqrt{\frac{4\pi}{K}}\theta_2\right)$$

inner branch

branch mixing

$$\mathcal{H}_{12} = \frac{g_2 K}{\pi}\partial_x\phi_1\partial_x\phi_2$$

A new proposal for a topological superconductor...

Low-energy limit and bosonization:

outer branch

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inner branch

The two branches **decouple** when the ϕ_1-field gets pinned by opening of an insulating gap in the outer branch.

branch mixing

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A new proposal for a topological superconductor...

Low-energy limit and bosonization:

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p-wave topological phase appears in the inner branch by **opening of an insulating gap in the outer branch.**

$$\boxed{\mathcal{H}_2 = u[(\partial_x\theta_2)^2 + (\partial_x\phi_2)^2] - \frac{\Delta}{\pi a}\sin\left(\sqrt{\frac{4\pi}{K}}\theta_2\right)}$$

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A new proposal for a topological superconductor...

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integrate out $\partial_x\theta_1$

Extended sine-Gordon model,
"self-dual" Gaussian fixed point at K=1/2 !

Can use perturbative RG

to determine the parameter intervals supporting a topological phase

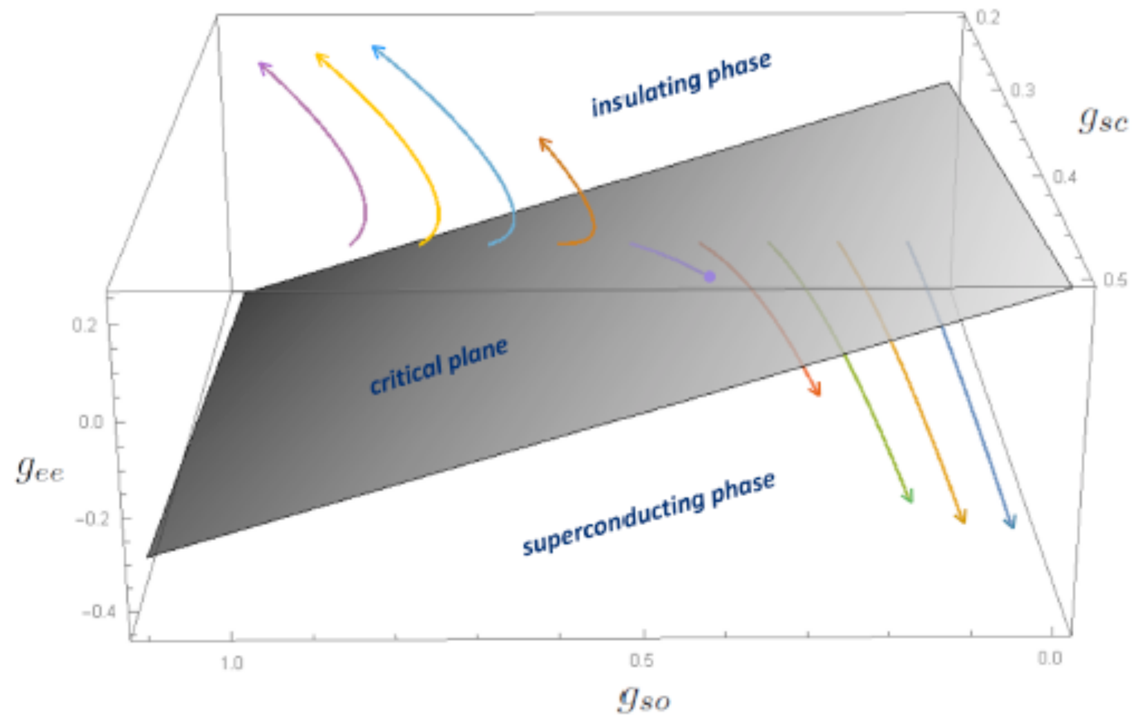
A new proposal for a topological superconductor...

1-loop RG equations

$$\frac{dg_{ee}}{dl} = g_{so}^2 - g_{sc}^2$$

$$\frac{dg_{so}}{dl} = 4g_{so}g_{ee}\ ,$$

$$\frac{dg_{sc}}{dl} = -4g_{sc}g_{ee}$$



$$g_{ee} = \frac{1}{2} - K$$

$g_{so} \sim$ modulated Rashba $\times$ Dresselhaus amplitude

$g_{sc} \sim$ proximity pairing amplitude

Case studies

Case study I: A periodically gated InAs quantum wire proximity coupled to a Nb superconductor

$$T \approx 0.1 \text{ K}$$

$\Delta \approx 0.3 \text{ meV}$ V. Mourik *et al.*, Science (2012)

$\ell_{\text{loc}} \approx 10\mu\text{m}$ D. Liu & S. Das Sarma, PRB (1995)

$a \approx 5 \text{ Å}$ P. Bhattacharaya, Semicond. Data Rev. (1992)

$$v \approx 10^5 \text{ m/s}$$

$[\alpha_{\text{Rashba}}/\beta_{\text{Dresselhaus}}]_{\text{uniform}} \approx 2$ S. Giglberger *et al.*, PRB (2007)

$$\text{K} \approx 0.7$$

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↓ *RG numerics*

$$\boxed{\begin{array}{l} M_{\text{insulating}} \geq M_{\text{supercond}} \approx 10 \ \mu\text{eV} \\ \text{requires } [\alpha_{\text{Rashba}}]_{\text{modulated}} \geq 1.5 \times 10^{-10} \text{ eVm} \end{array}}$$

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requires $[\alpha_{\mathrm{Rashba}}]_{\mathrm{modulated}} \geq 1.5 \times 10^{-10}$ eVm

Implementation with cold atoms?

Many proposals for creating $p_x + i\,p_y$ superfluids in cold fermionic atom optical traps...

PRL **101**, 160401 (2008) PHYSICAL REVIEW LETTERS week ending 17 OCTOBER 2008

$p_x + ip_y$ Superfluid from s-Wave Interactions of Fermionic Cold Atoms

Chuanwei Zhang,[1,2] Sumanta Tewari,[1,3] Roman M. Lutchyn,[1,4] and S. Das Sarma[1]
[1]*Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
[2]*Department of Physics and Astronomy, Washington State University, Pullman, Washington 99164, USA*
[3]*Department of Physics and Astronomy, Clemson University, Clemson, South Carolina 29634, USA*
[4]*Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
(Received 30 May 2008; revised manuscript received 5 September 2008; published 17 October 2008)

Two-dimensional ($p_x + ip_y$) superfluids or superconductors offer a playground for studying intriguing physics such as quantum teleportation, non-Abelian statistics, and topological quantum computation. Creating such a superfluid in cold fermionic atom optical traps using p-wave Feshbach resonance is turning out to be challenging. Here we propose a method to create a $p_x + ip_y$ superfluid directly from an s-wave interaction making use of a topological Berry phase, which can be artificially generated. We discuss ways to detect the spontaneous Hall mass current, which acts as a diagnostic for the chiral p-wave superfluid.

DOI: 10.1103/PhysRevLett.101.160401 PACS numbers: 03.75.Ss, 03.65.Vf, 03.67.Lx, 73.43.Fj

PHYSICAL REVIEW A **84**, 013603 (2011)

Topological $p_x + ip_y$ superfluid phase of fermionic polar molecules

J. Levinsen,[1,2] N. R. Cooper,[1,2] and G. V. Shlyapnikov[2,3,4]
[1]*T.C.M. Group, University of Cambridge, Cavendish Laboratory, J.J. Thomson Ave., Cambridge CB3 0HE, UK*
[2]*Laboratoire de Physique Théorique et Modèles Statistiques, CNRS and Université Paris Sud, UMR8626, 91405 Orsay, France*
[3]*Van der Waals-Zeeman Institute, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands*
[4]*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030, USA*
(Received 20 March 2011; published 7 July 2011)

We discuss the topological $p_x + ip_y$ superfluid phase in a two-dimensional (2D) gas of single-component fermionic polar molecules dressed by a circularly polarized microwave field. This phase emerges because the molecules may interact with each other via a potential $V_0(r)$ that has an attractive dipole-dipole $1/r^3$ tail, which provides p-wave superfluid pairing at fairly high temperatures. We calculate the amplitude of elastic p-wave scattering in the potential $V_0(r)$ taking into account both the anomalous scattering due to the dipole-dipole tail and the short-range contribution. This amplitude is then used for the analytical and numerical solution of the renormalized BCS gap equation which includes the second-order Gor'kov-Melik-Barkhudarov corrections and the correction related to the effective mass of the quasiparticles. We find that the critical temperature T_c can be varied within a few orders of magnitude by modifying the short-range part of the potential $V_0(r)$. The decay of the system via collisional relaxation of molecules to dressed states with lower energies is rather slow due to the necessity of a large momentum transfer. The presence of a constant transverse electric field reduces the inelastic rate, and the lifetime of the system can be of the order of seconds even at 2D densities $\sim 10^9$ cm^{-2}. This leads to T_c of up to a few tens of nanokelvins and makes it realistic to obtain the topological $p_x + ip_y$ phase in experiments with ultracold polar molecules.

Implementation with cold atoms?

What about a 1D *p*-wave superfluid from modulated spin-orbit and atom interactions?
The present scheme requires:

- *Repulsively interacting fermionic atoms trapped in a 1D optical lattice.* ✓

  Experiments on ^{40}K: Hubbard-like interactions in 3D from Feshbach resonance;
  Jördens *et al.,* Nature (2008); Schneider *et al.,* Science (2008). 1D; Moritz *et al.,* PRL (2005)

- *Uniform coupling to Dresselhaus-type spin-orbit fields.* ✓

  Equal mixture of Rashba and Dresselhaus couplings from two-photon Raman transitions.
  Experiments on ^{40}K and ^{6}Li cold atoms; Wang *et al.,* PRL (2012); Cheuk *et al., PRL* (2012)

- *A spatially modulated Rashba-type spin-orbit interaction.* **?**

  Theoretical proposal: Detuning from two-photon Raman resonance using a spatially inhomogeneous magnetic field; Su *et al.,* New. J. Phys. (2015)

- *Effective s-wave proximity pairing.* **?**

  Theoretical proposal: coupling to a surrounding BEC of Feshbach molecules via a pulsed RF field; Jiang *et al.,* PRL (2011).

Summary

”Proof-of-concept”: 1D p-wave superconducting phase hosting Majoranas possible in an ordinary quantum wire using an all-electric device.

/

periodic gating (modulated Rashba), intrinsic Dresselhaus, and weakly screened *e*-*e* interaction

Realization requires large values of the gate-controlled Rashba coupling. (Unattainable in present-day hybrid semiconductor-superconductor devices?)

1D *p*-wave superfluid from modulated Rashba, Dresselhaus and Feshbach?

For more, see

G. I. Japaridze, H. J., and M. Malard, Phys. Rev. B **89**, 201403(R) (2014);
M. Malard, G. I. Japaridze, and H. J., *to appear*