CPHT colloquium
École Polytechnique, 27 June 2014

Topological insulators, helical electrons, and Rashba interactions

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Stiftelsen för internationalisering av högre utbildning och forskning

Outline

Topological insulators... some basics

Outline

Topological insulators... *in 2D* some basics

Outline

Topological insulators... some basics (in 2D)

At the edge: a helical electron liquid

Outline

in 2D

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Some applications

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Some applications...

... and difficulties

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A synthetic helical liquid from Rashba interactions!

Outline

in 2D

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Outlook

Outline

Topological insulators *in 2D*... some basics

At the edge: a helical electron liquid

Some applications...

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A synthetic helical liquid from Rashba interactions!

Outlook

”An **electrical insulator** is a material whose internal electric charges do not flow freely... examples include glass, paper, Teflon, plastics, ceramic materials,... ”

from Wikipedia

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$$\int_M e(M) = \chi(M)$$

$$e(M) = \mathrm{Pf}(\mathcal{R}/2\pi) = \frac{(-1)^l}{(4\pi)^l l!} \sum_P \mathrm{sgn}(P) \mathcal{R}_{P(1)P(2)} \ldots \mathcal{R}_{P(2l-1)P(2l)}$$

”An **electrical insulator** is a material whose internal electric charges do not flow freely... examples include glass, paper, Teflon, plastics, ceramic materials,... ”

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?

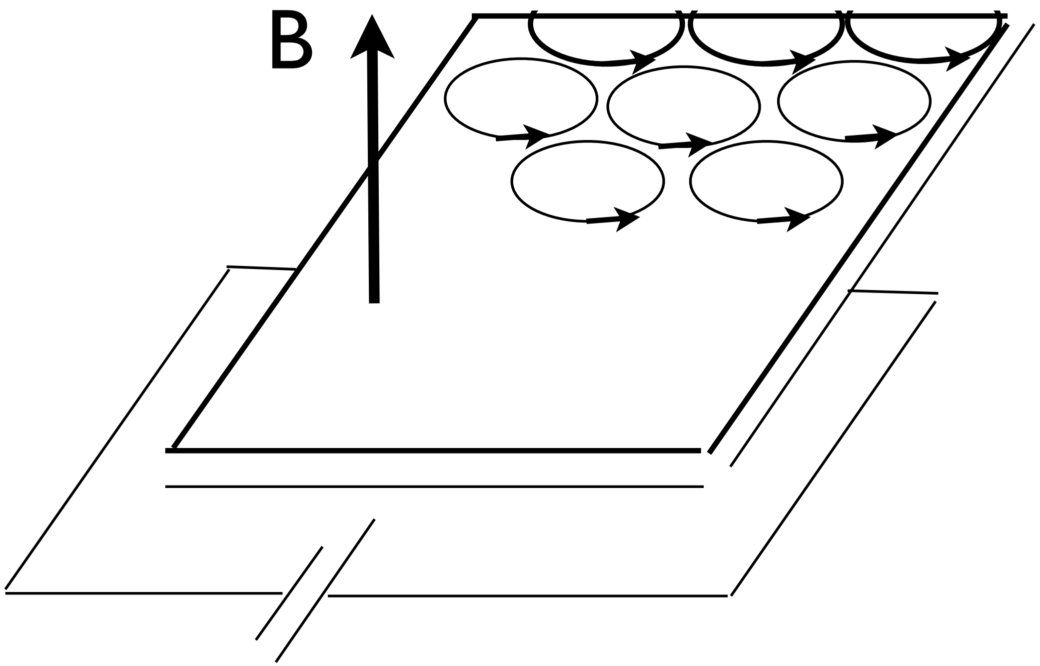
Ordinary insulators have nothing to do with topology, but *topological insulators* do!

2D topological insulators

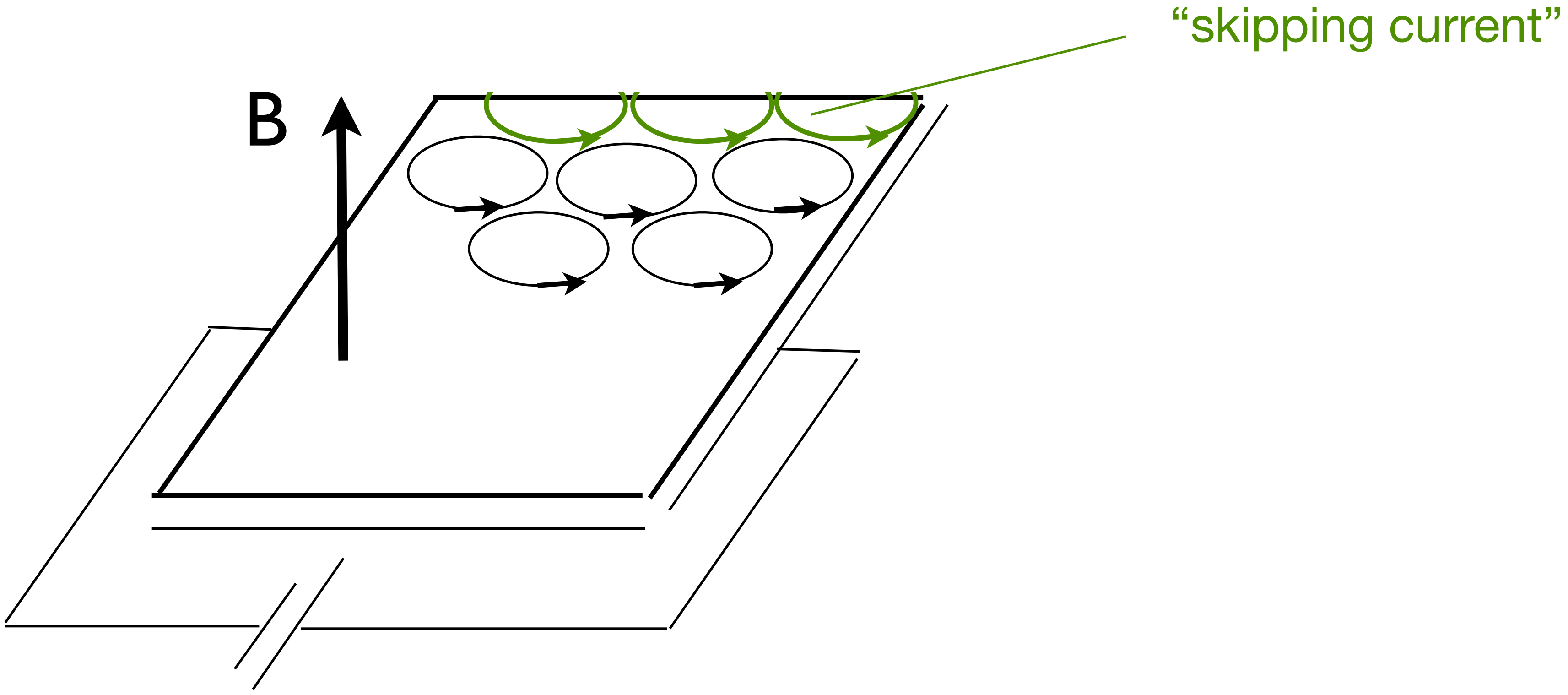
2D topological insulators...

taking off from the quantum Hall effect

quantum Hall effect



quantum Hall effect



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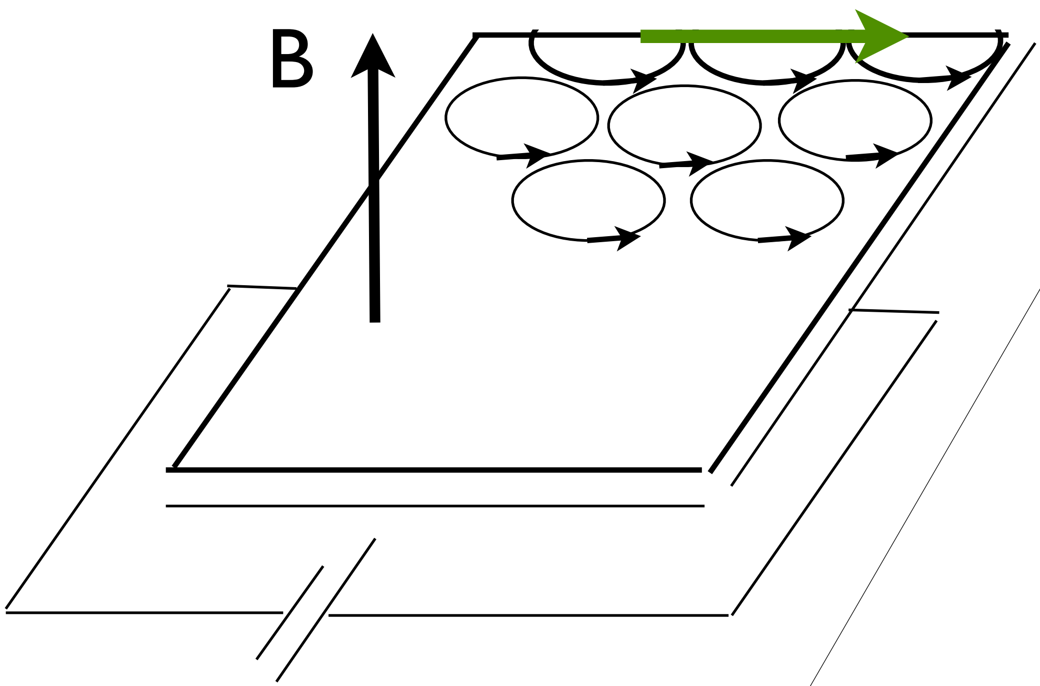


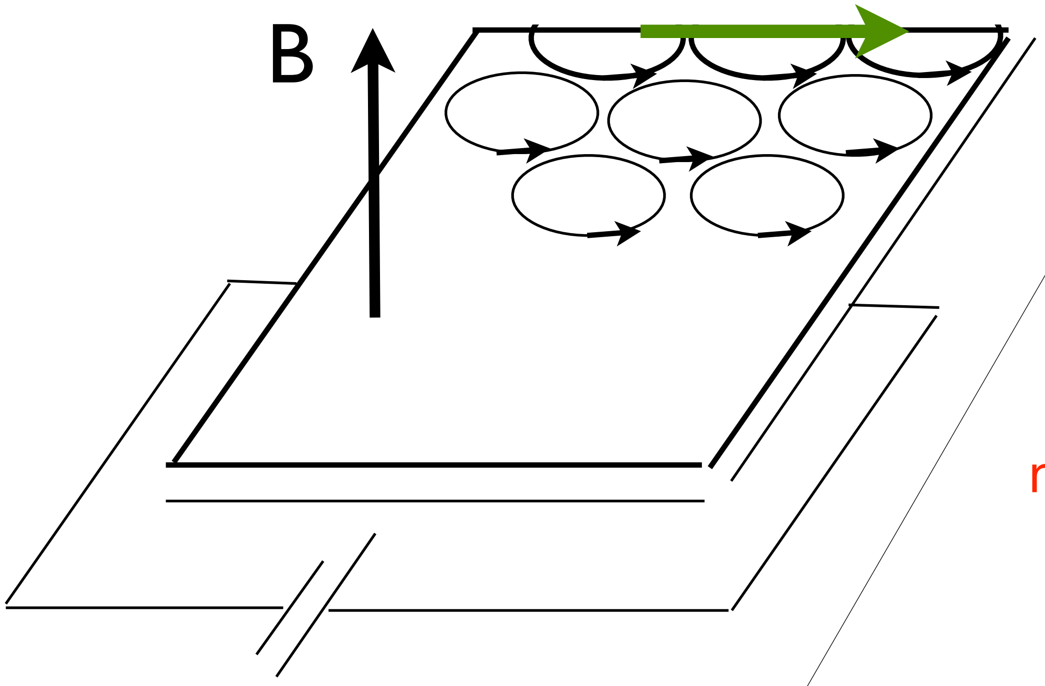
"skipping current"

quantization

chiral edge states

Halperin, PRB (1982)

quantum Hall effect



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no channel for backscattering

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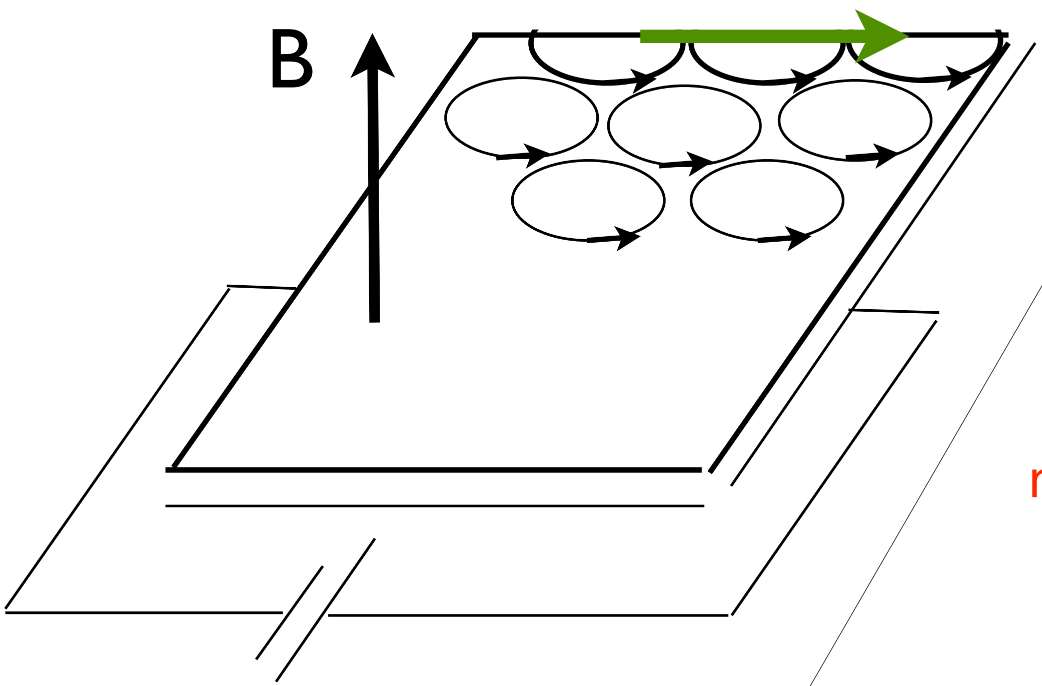


"skipping current"

↓ *quantization*

chiral edge states

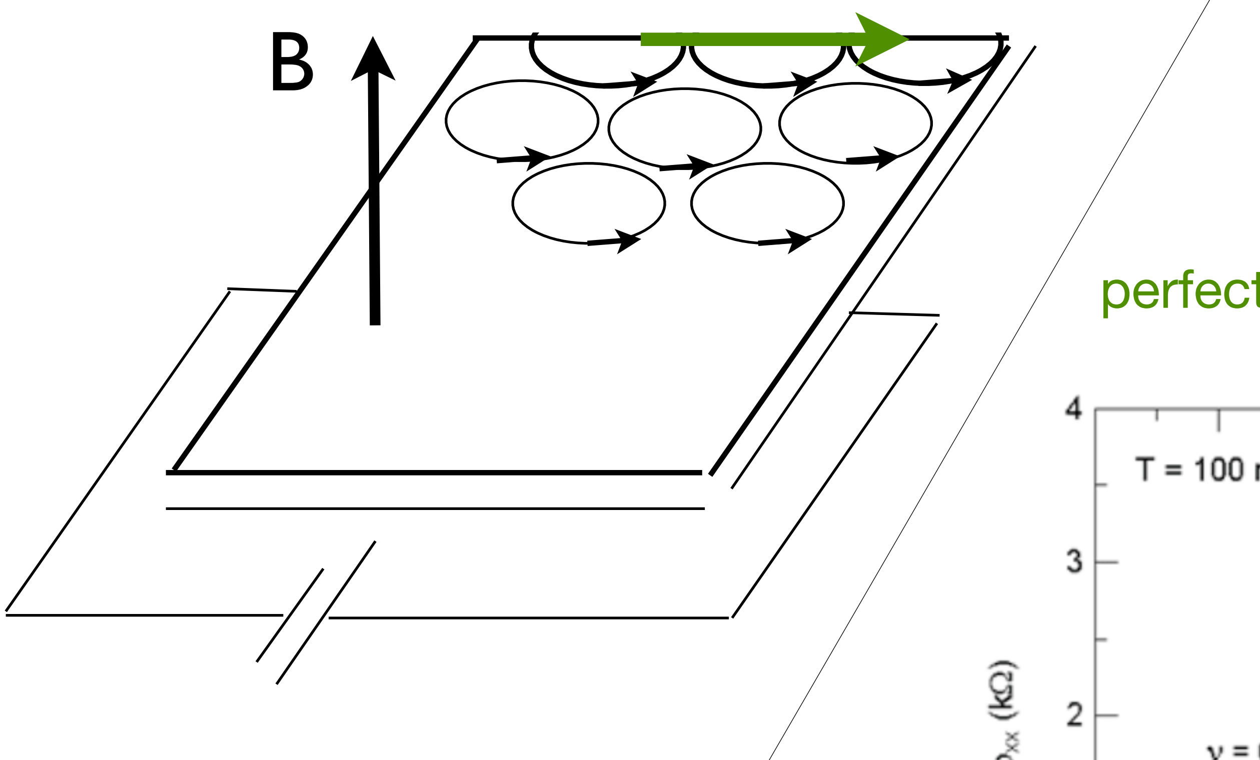
↓

no channel for backscattering

↓

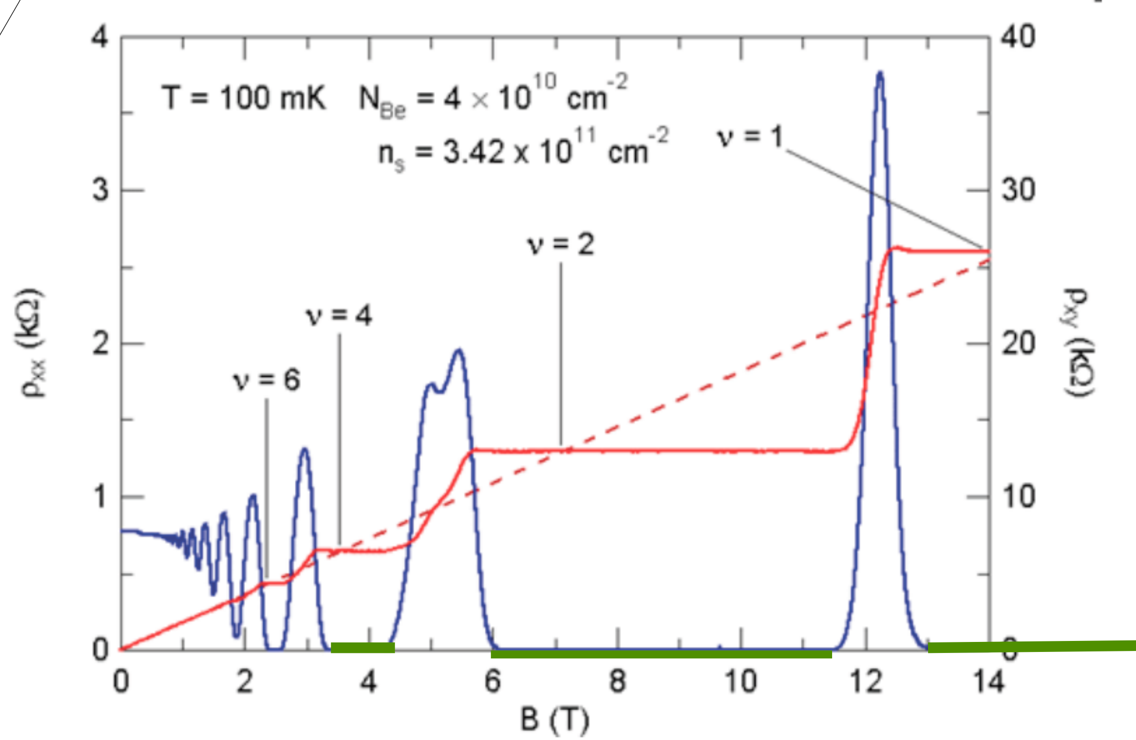
perfect conductance along the edge

quantum Hall effect



perfect conductance along the edge

von Klitzing *et al.*, PRL (1980)



a bulk insulator with perfectly
conducting edge states

Is this kind of physics possible without a magnetic field?

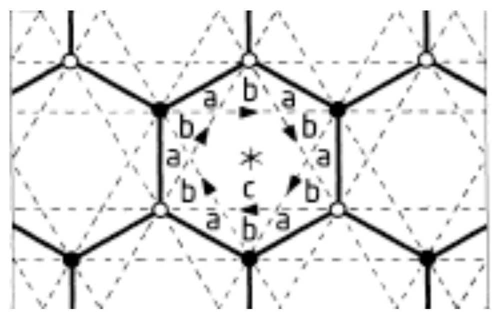
a bulk insulator with perfectly conducting edge states

Is this kind of physics possible without a magnetic field?

Duncan Haldane

Well..., at least one doesn't need a *net* magnetic field... *PRL, 1988*

Haldane's toy model: tight-binding electrons on a honeycomb lattice with a staggered magnetic field



"Chern insulator"

a bulk insulator with perfectly conducting edge states

Is this kind of physics possible without a magnetic field?

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Well..., at least one doesn't need a *net* magnetic field... *PRL, 1988*

Charlie Kane

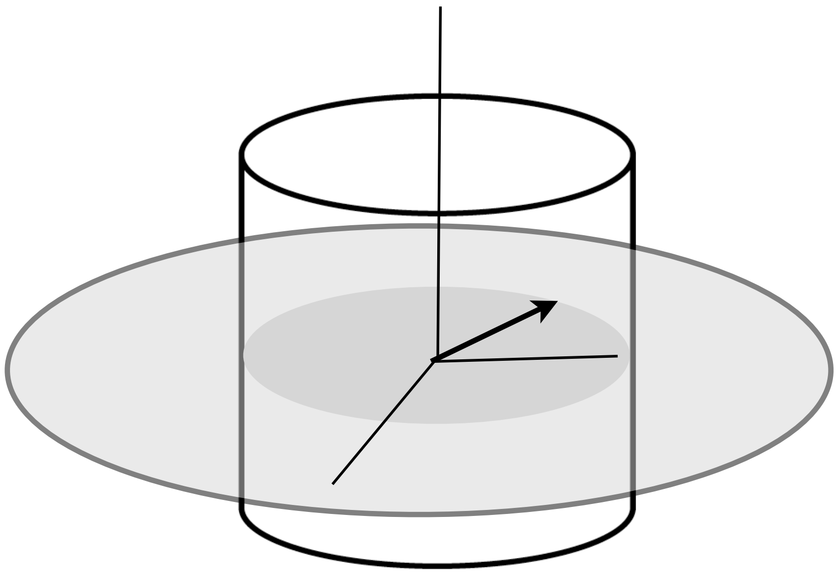
Gene Mele

In fact, one can do away with the magnetic field altogether! *PRLs, 2005*

To see how this is possible, consider a Gedanken experiment...

Bernevig & Zhang, PRL (2006)

To see how this is possible,
consider a Gedanken experiment...



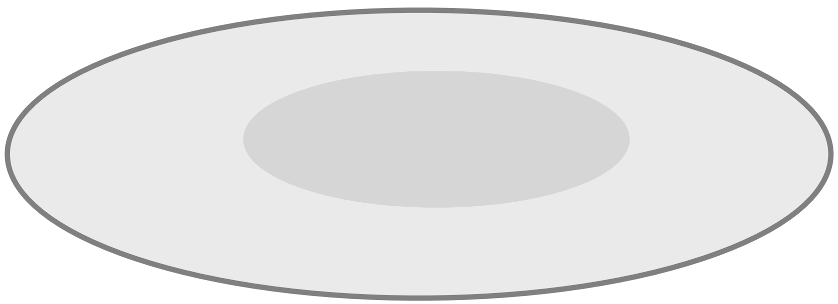
uniformly charged cylinder with electric field
$\boldsymbol{E} = E(x, y, 0)$

spin-orbit interaction

time-reversal invariant

$$(\boldsymbol{E} \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} = E\sigma^z(k_y x - k_x y)$$

To see how this is possible, consider a Gedanken experiment...



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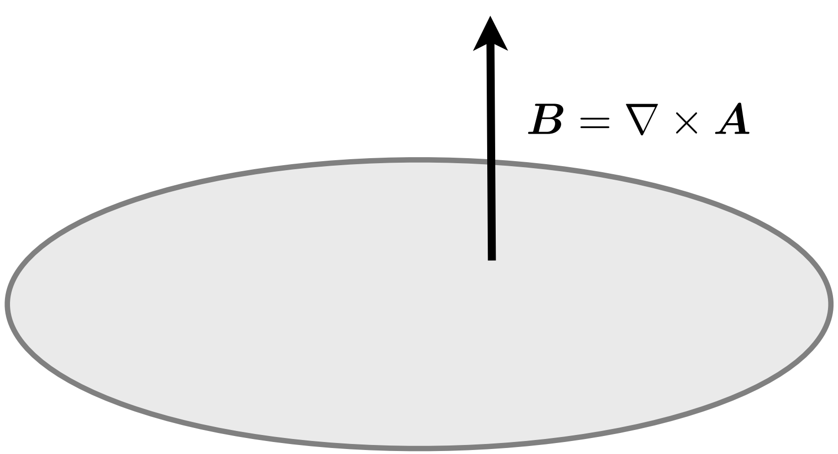

compare with an integer quantum Hall system

Lorentz force

$$\boldsymbol{A} \cdot \boldsymbol{k} \sim eB(k_y x - k_x y)$$

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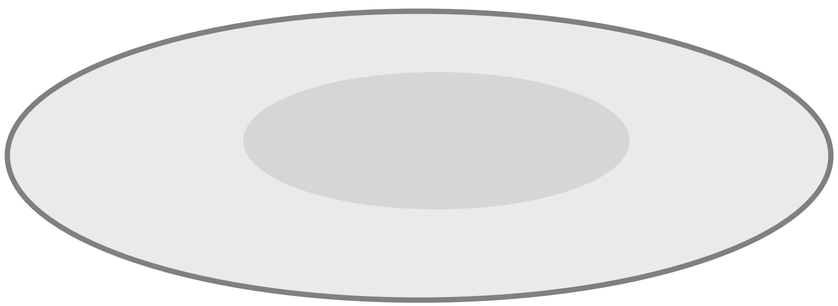

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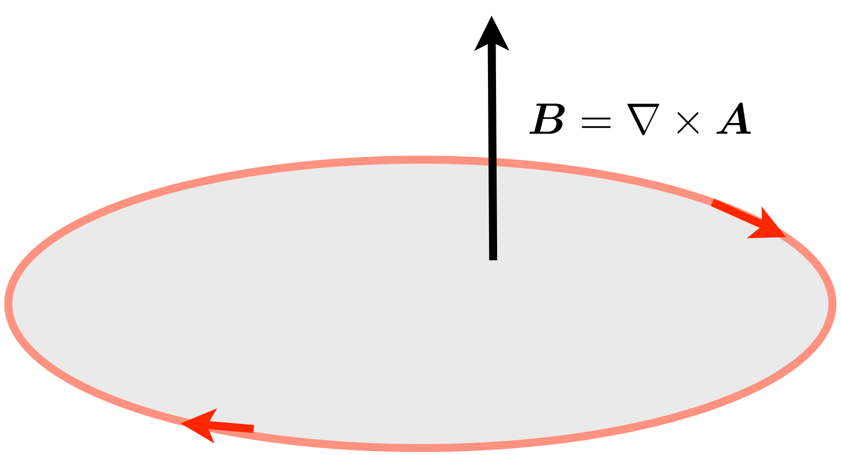

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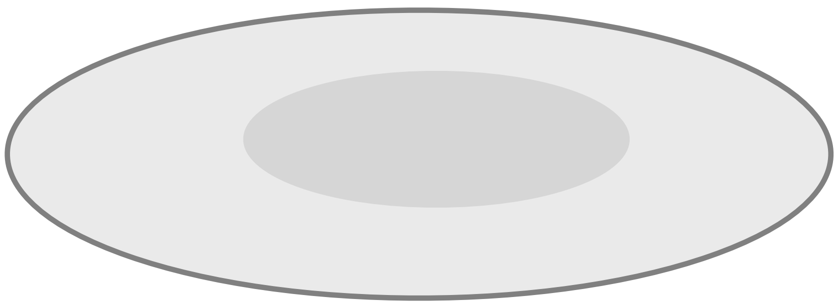

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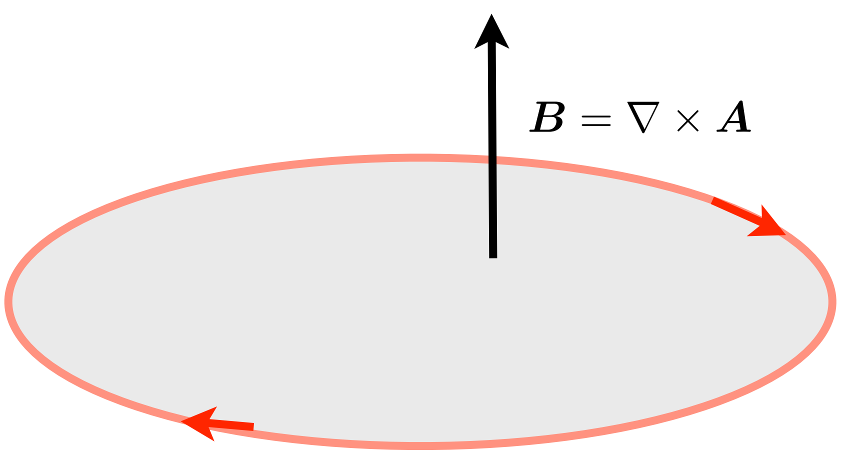

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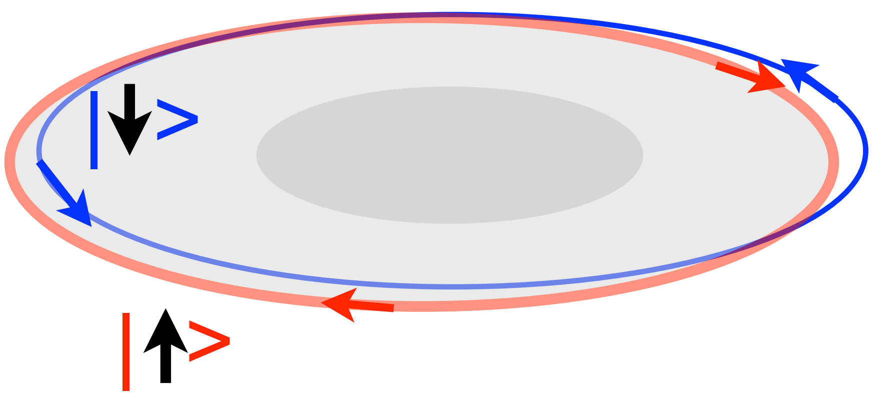

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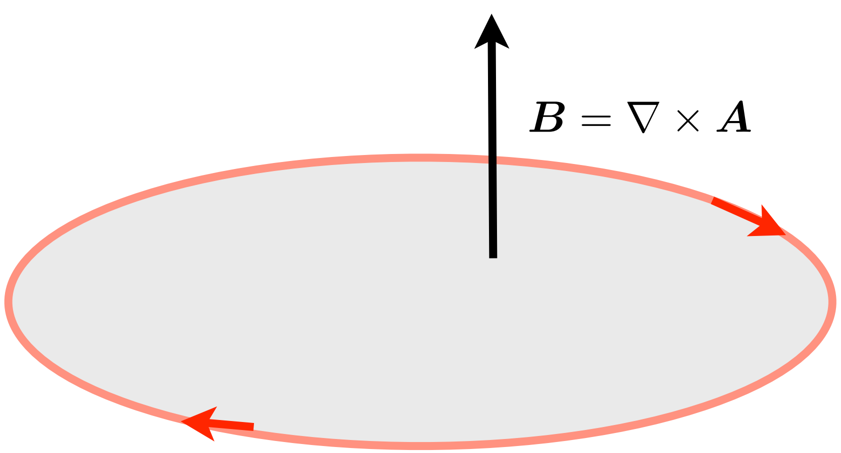

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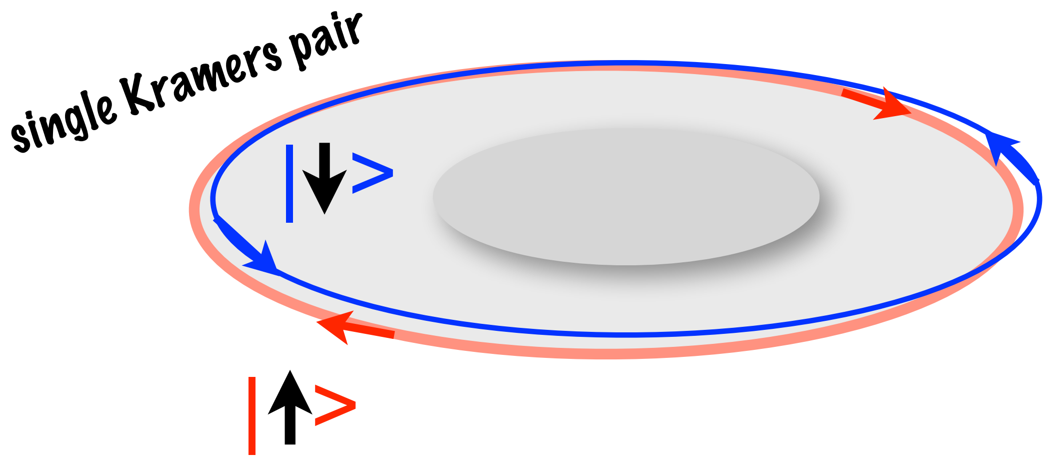
Quantum spin Hall (QSH) insulator

Two copies of a quantum Hall system, bulk insulator with **helical edge states**



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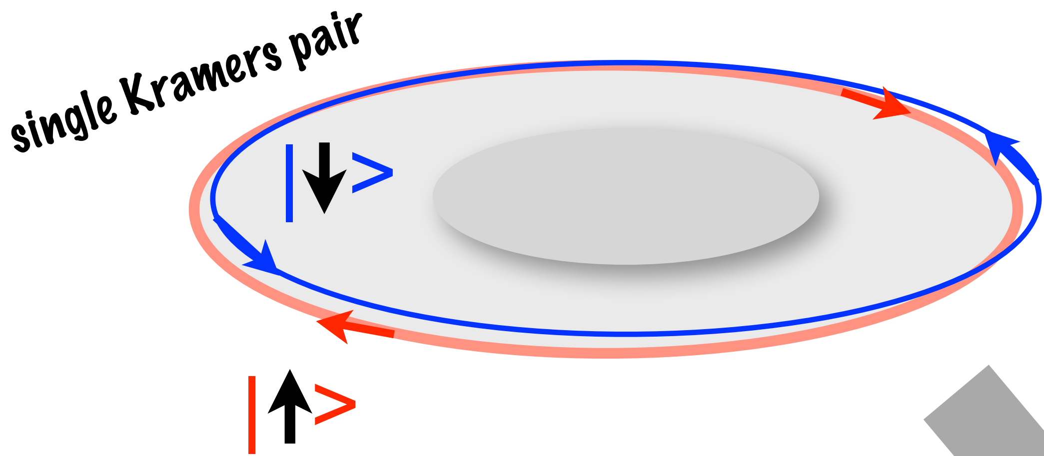
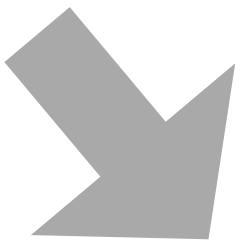


perturb with a time-reversal invariant spin-nonconserving interaction



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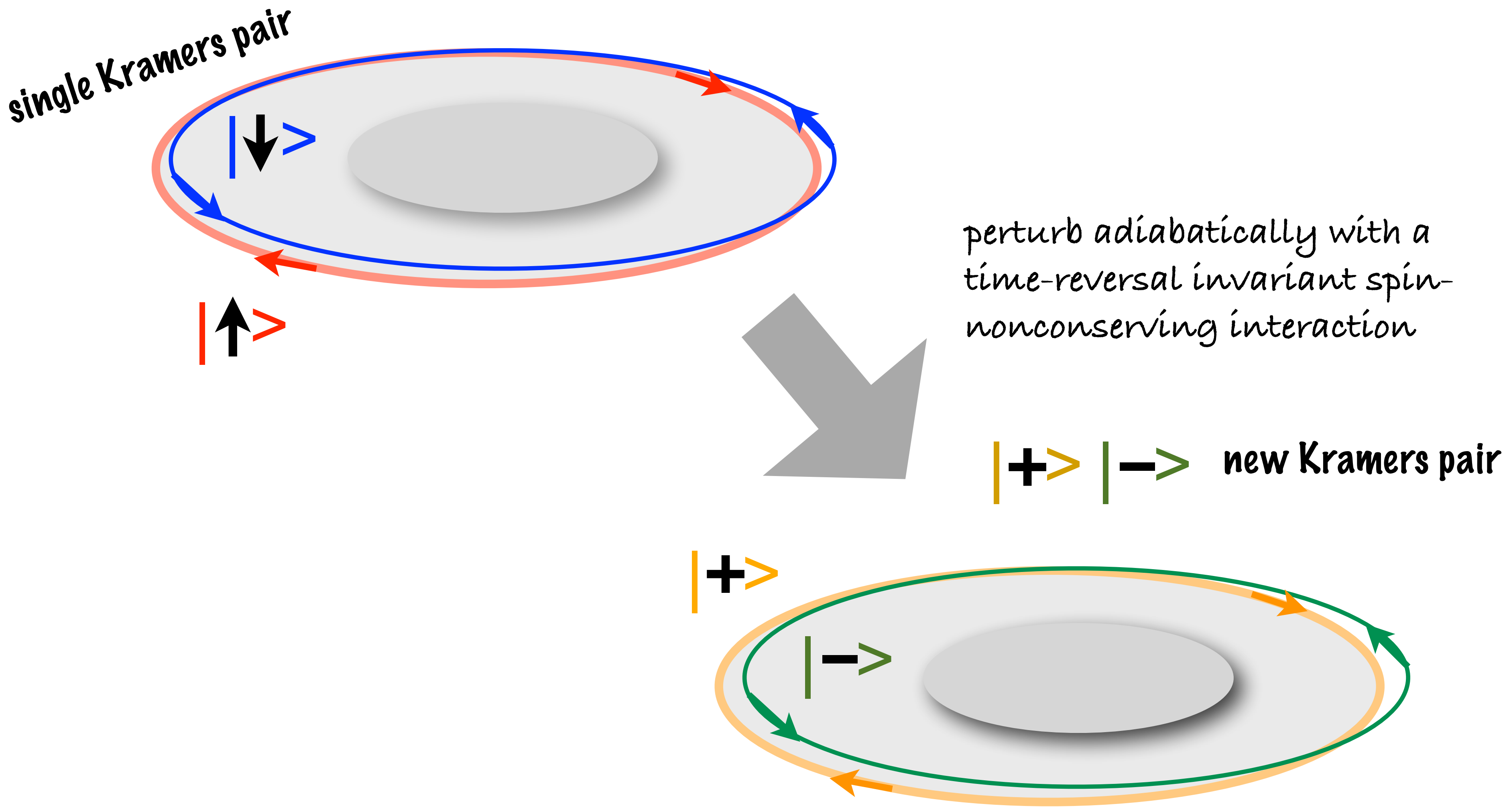

2D topological insulator

How does Nature do it?
Also by spin-orbit interactions!

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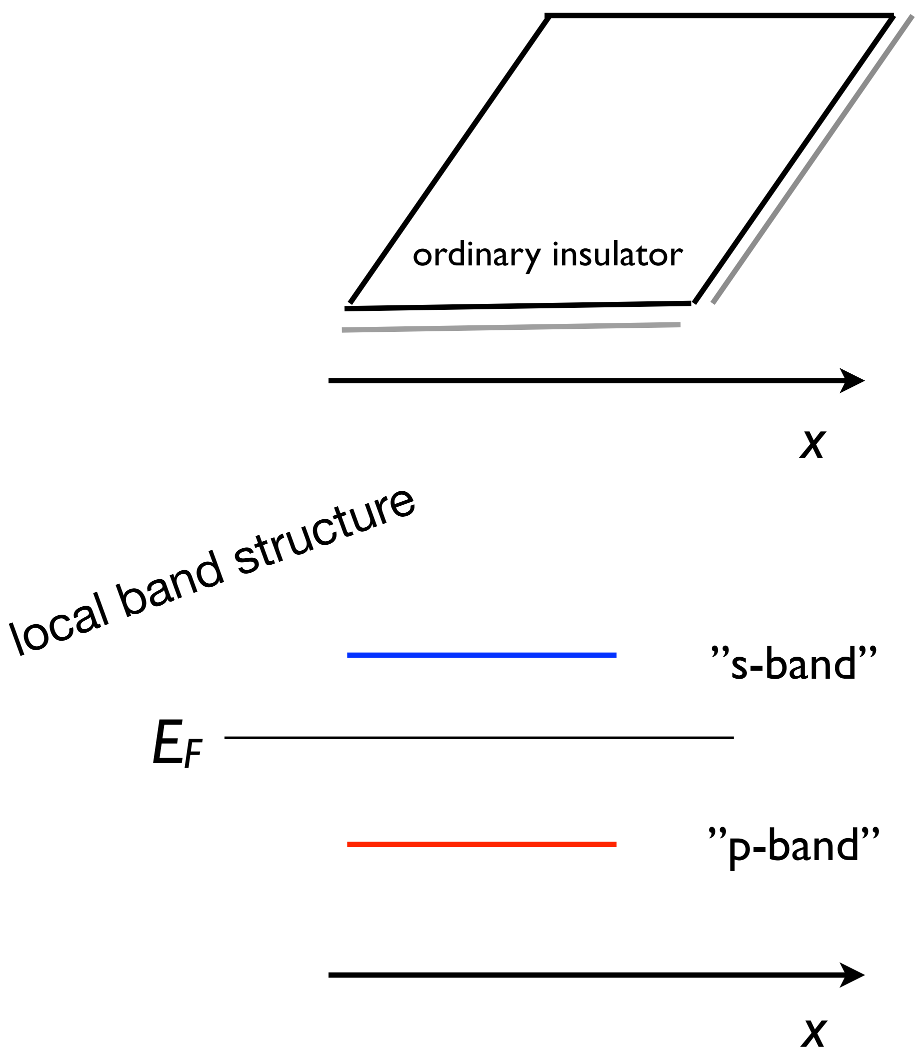
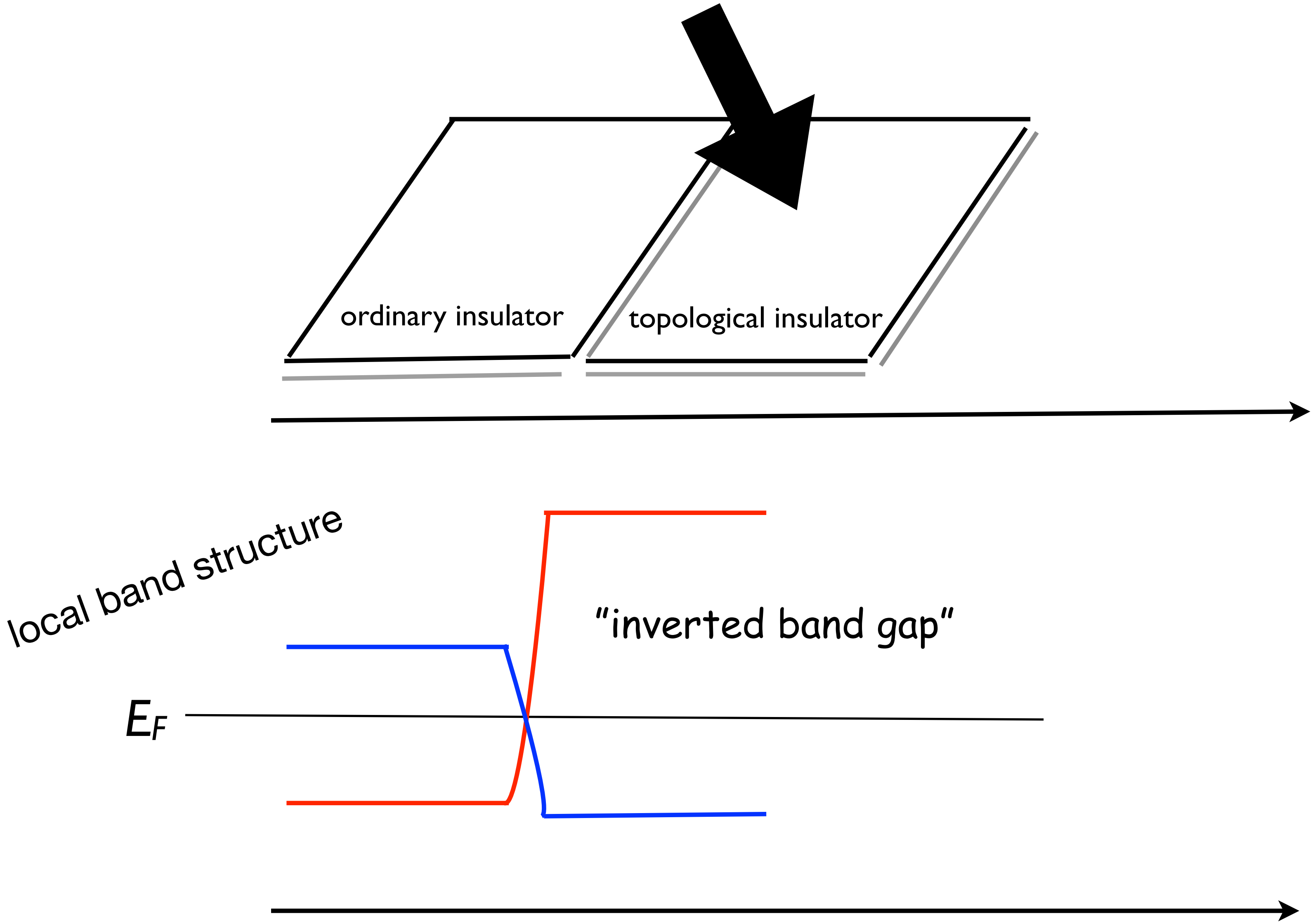


How does Nature do it?

Strong atomic spin-orbit interactions!



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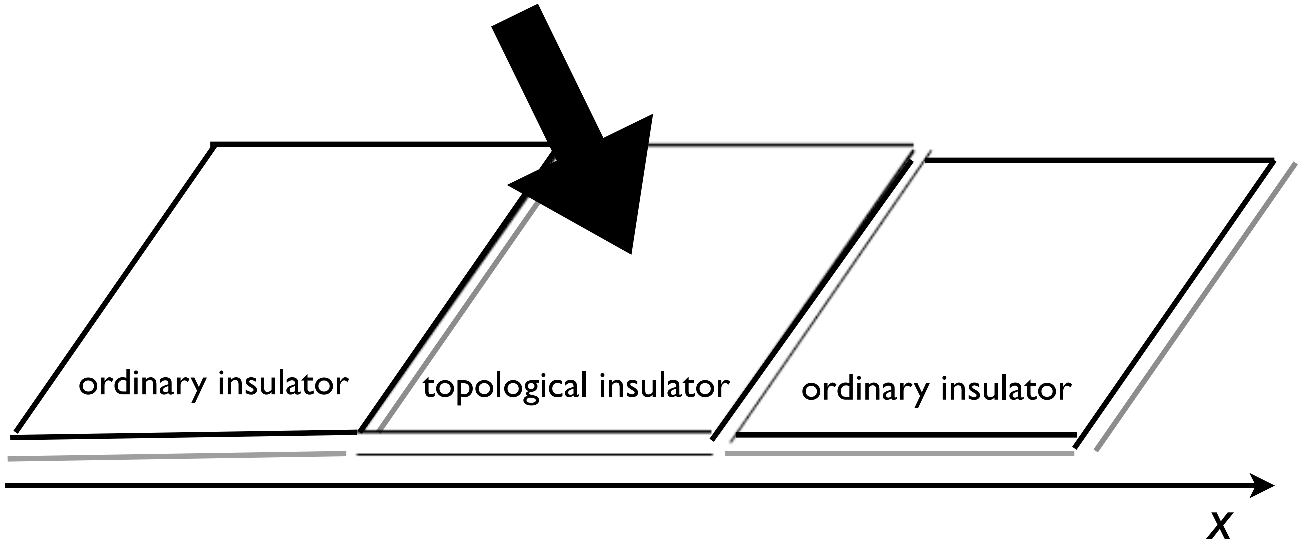

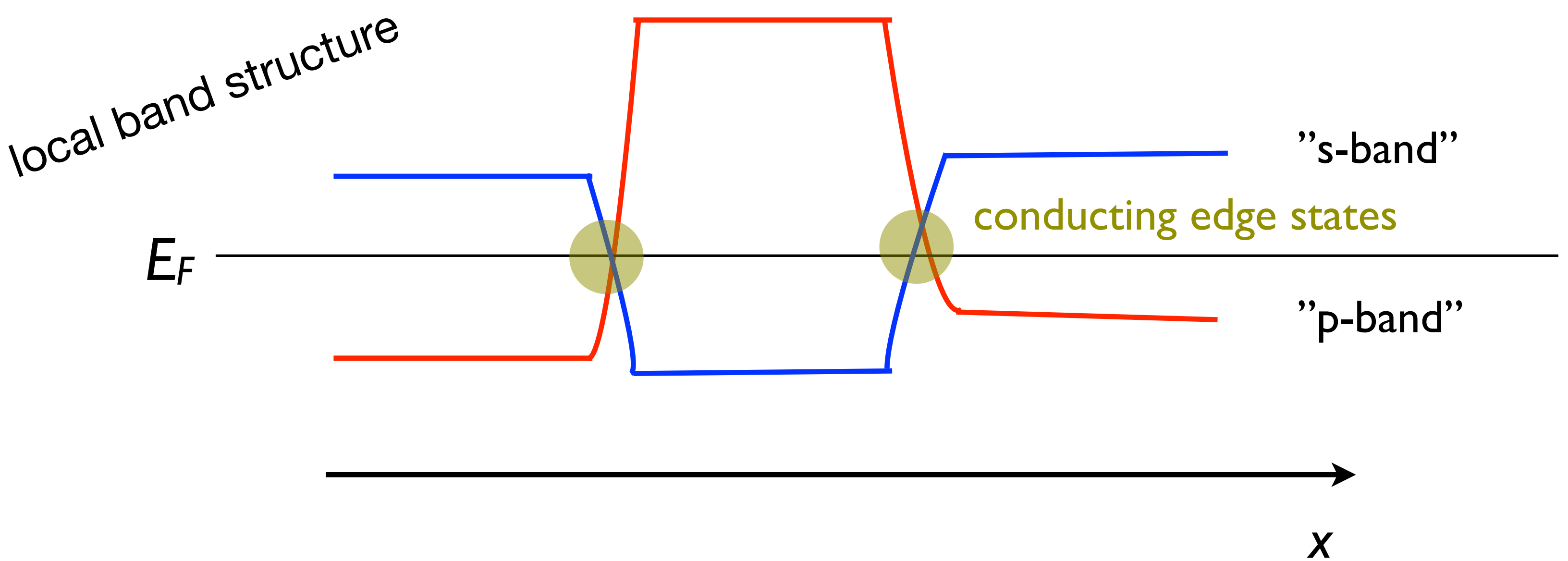

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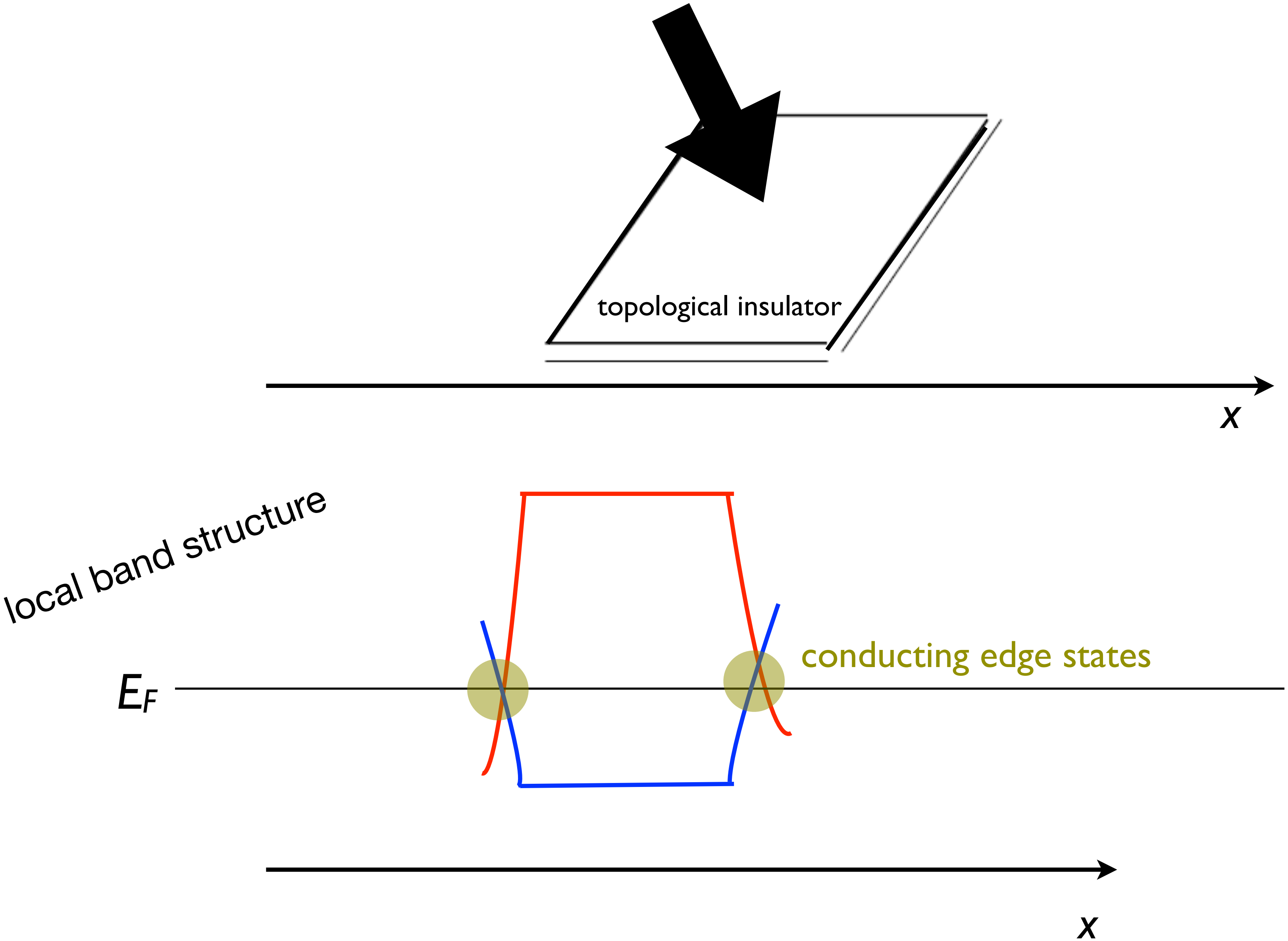
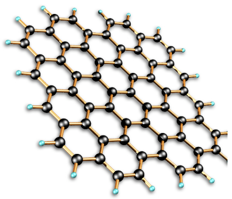
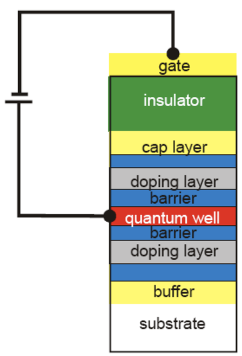


Experimental realizations...



First proposed by Kane and Mele for graphene (2005)

C. L. Kane and E. J. Mele, PRL **95**, 226801 (2005)

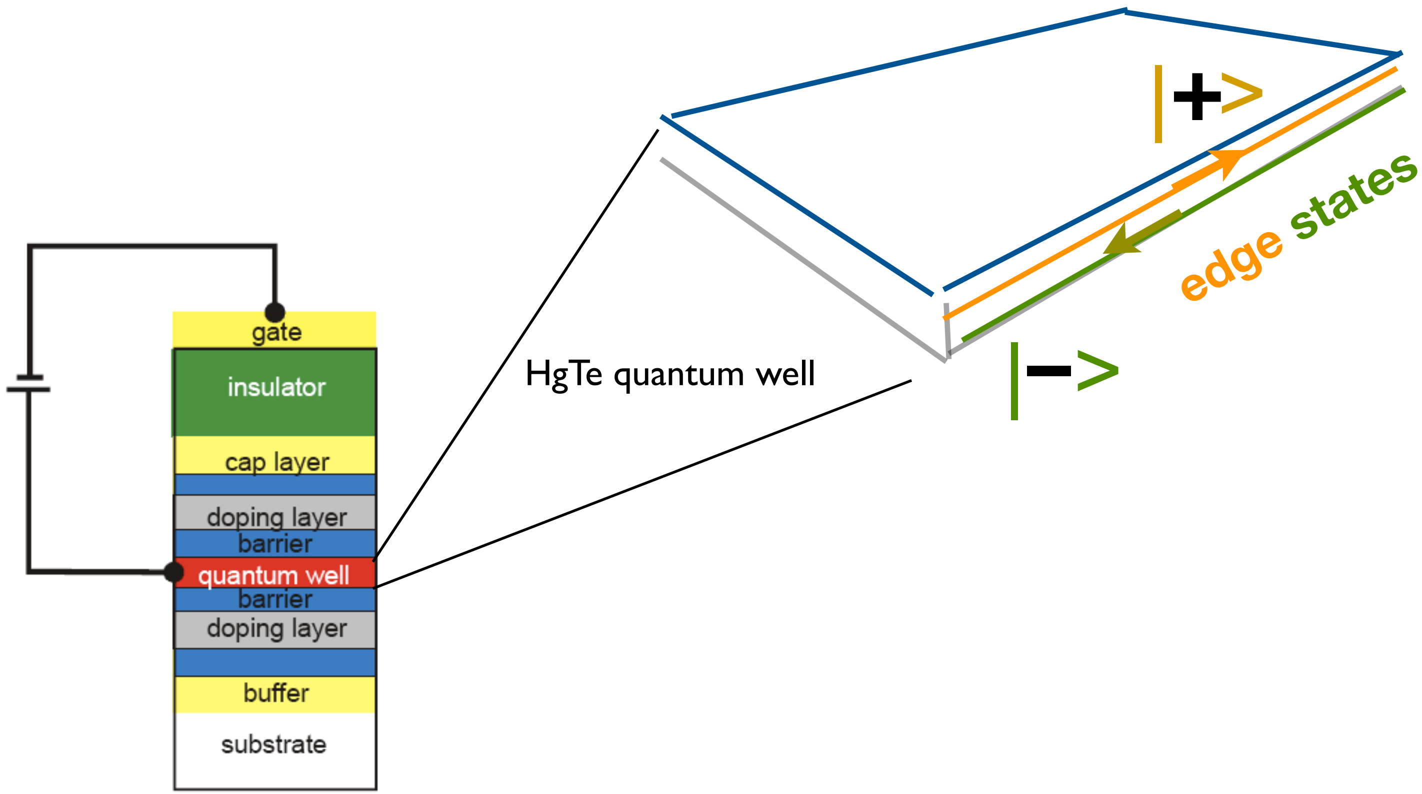


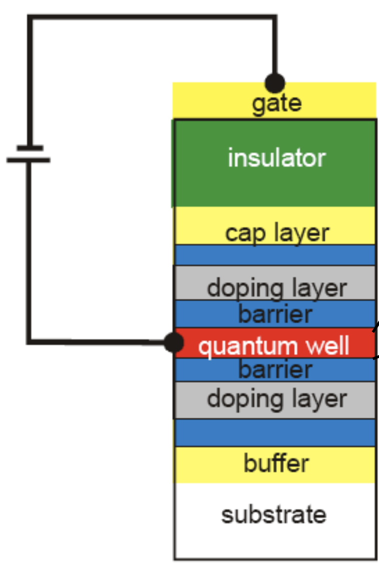
Bernevig et al. proposal for HgTe quantum wells (2006)

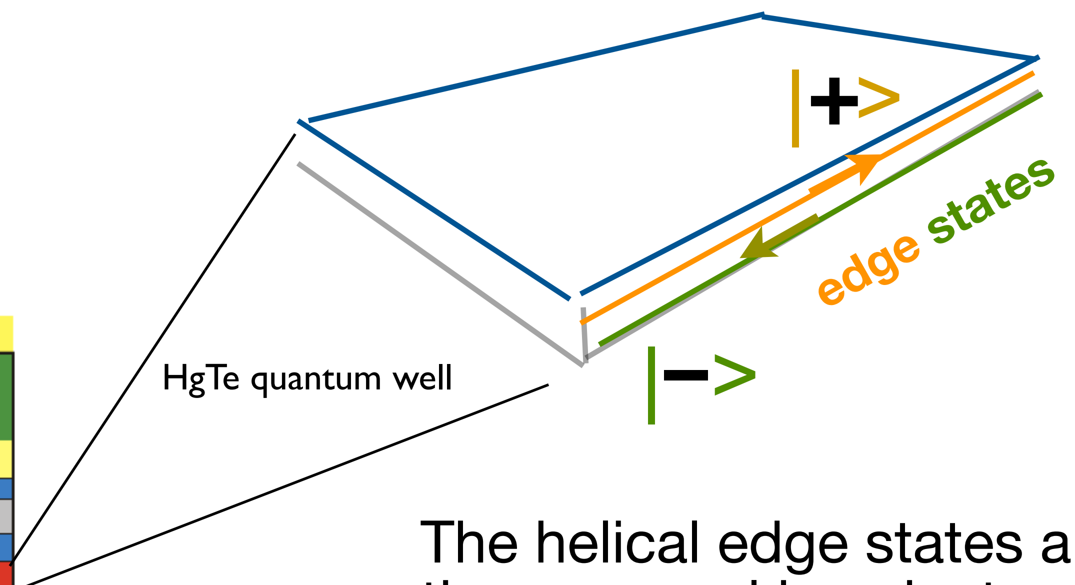
B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

Experimental observation by König *et al.* (2007)

M. König *et al.*, Science **318**, 766 (2007)


gate
insulator
cap layer
doping layer
barrier
quantum well
barrier
doping layer
buffer
substrate
HgTe quantum well
|+>
edge states
|−>

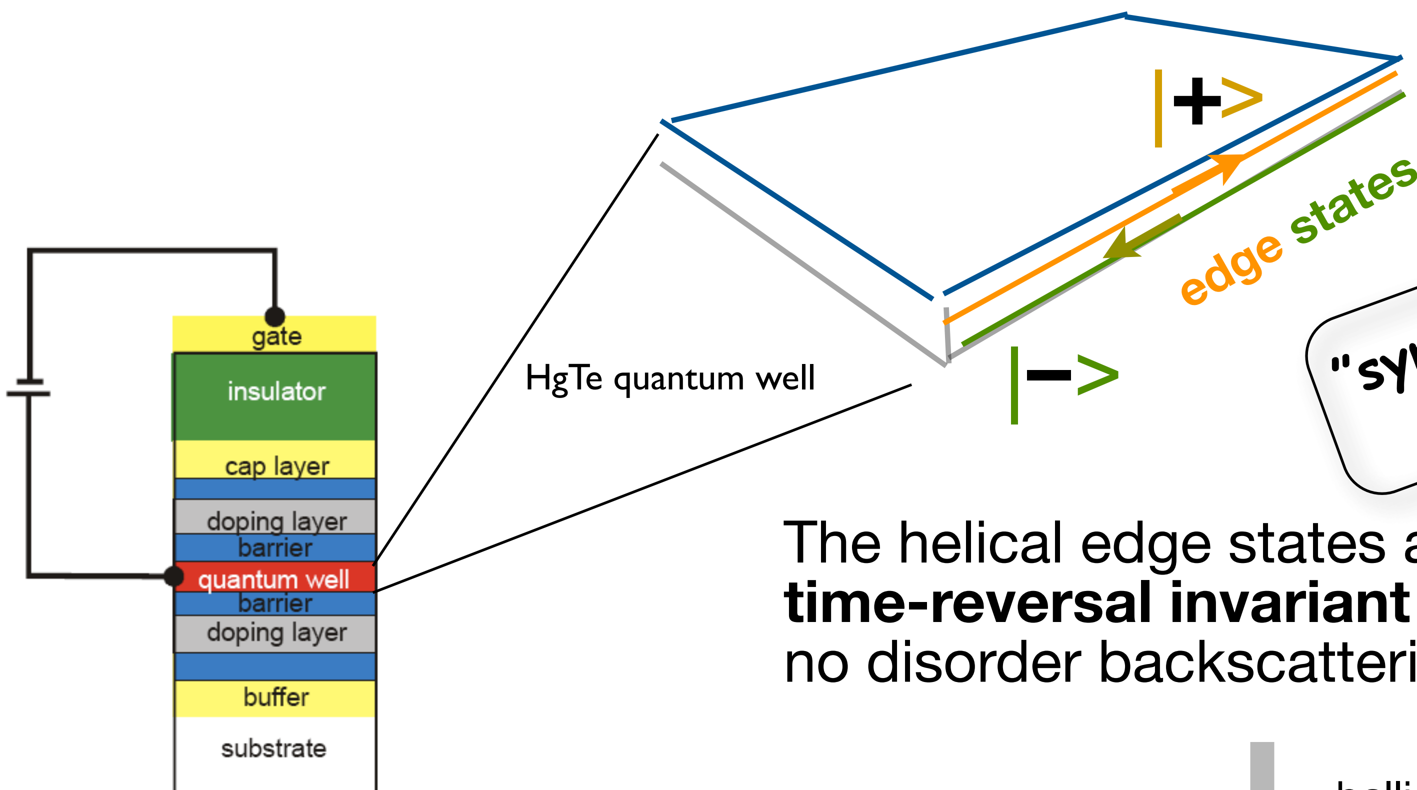




The helical edge states are stable against time-reversal invariant perturbations, no disorder backscattering

ballistic transport

$$G = \frac{2e^2}{h}$$



"symmetry-protected" topological phase

The helical edge states are stable against **time-reversal invariant perturbations**, no disorder backscattering

ballistic transport

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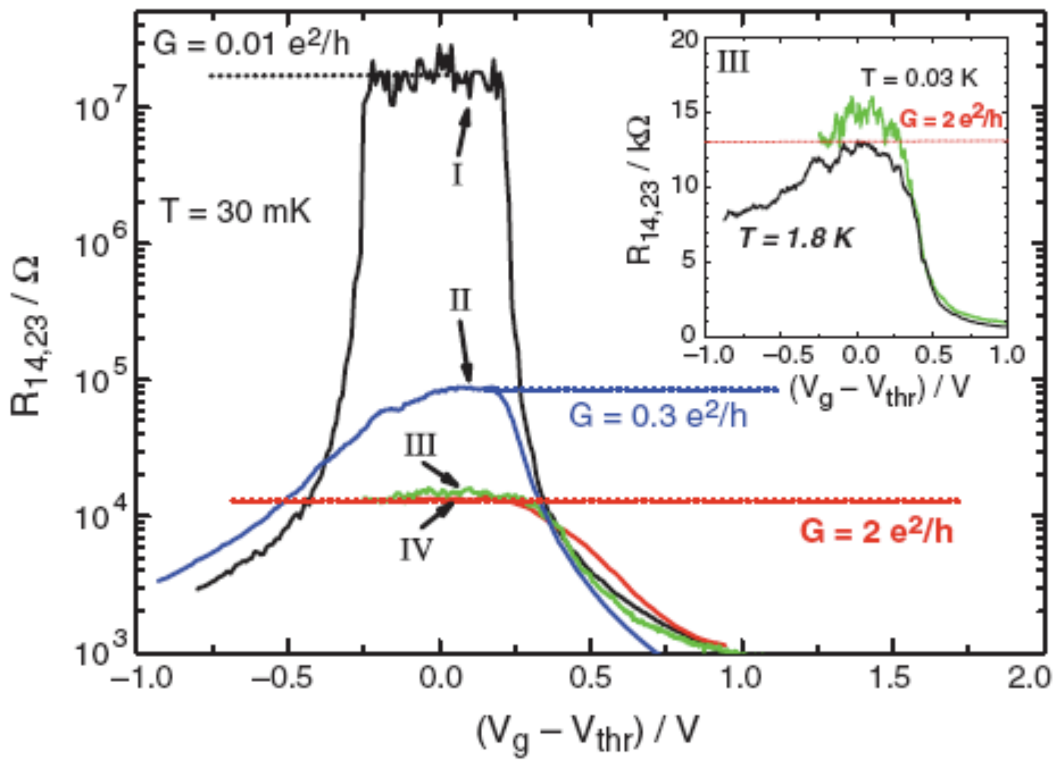
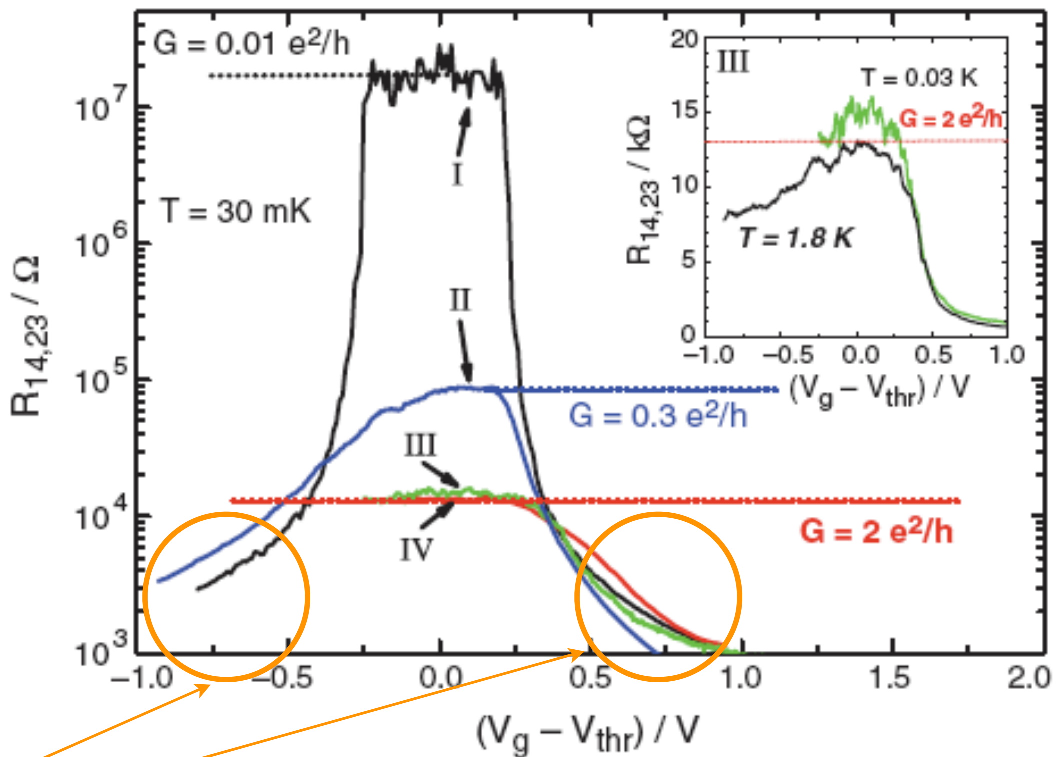


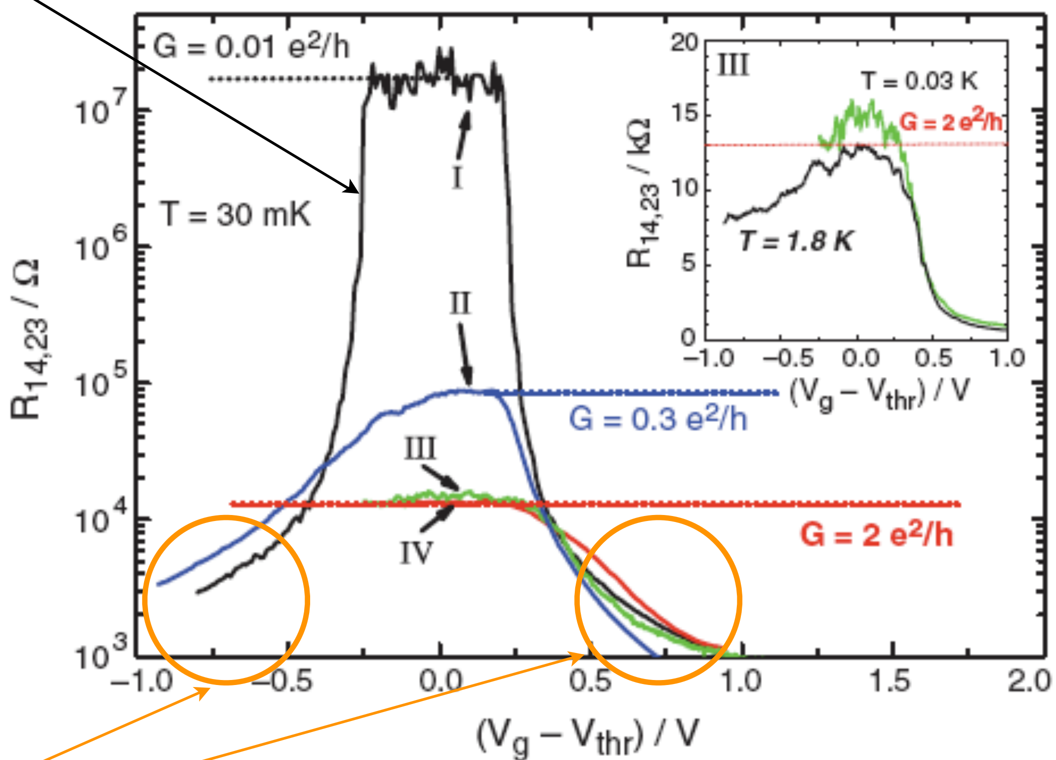
König *et al.* (2007)



Fermi level *not* inside the inverted gap

König *et al.* (2007)

normal band gap



Fermi level *not* inside the inverted gap

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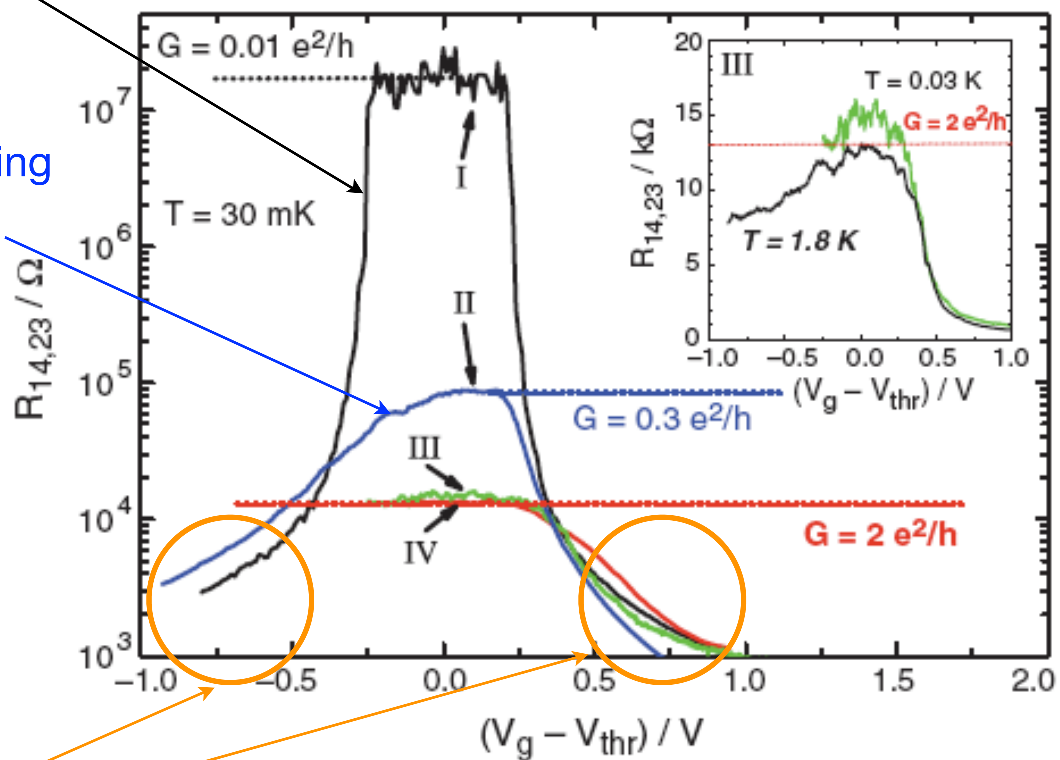
normal band gap

large samples
(inelastic scattering
from the bulk)



Fermi level *not*
inside the
inverted gap

König *et al.* (2007)

What is ”topological” about a topological insulator?

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Warm up... *Gauss-Bonnet theorem*

$$\frac{1}{2\pi}\int_{\mathcal{M}} K dS = 2(1-g)$$

Pierre Ossian Bonnet, 1819-1892

Ancien élève de l'École polytechnique, ingénieur des Ponts et Chaussées, Bonnet préféra l'enseignement et la recherche. Répétiteur puis examinateur à l'École Polytechnique,

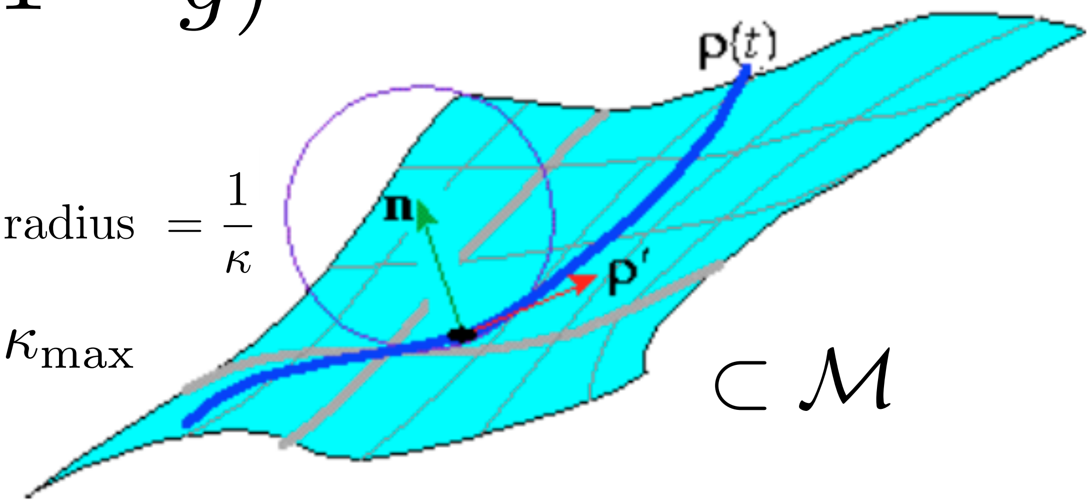
What is "topological" about a topological insulator?

Warm up... *Gauss-Bonnet theorem*

$$\frac{1}{2\pi}\int_{\mathcal{M}} K dS = 2(1-g)$$

$$\text{radius} = \frac{1}{\kappa}$$

$$K = \kappa_{\min} \cdot \kappa_{\max}$$

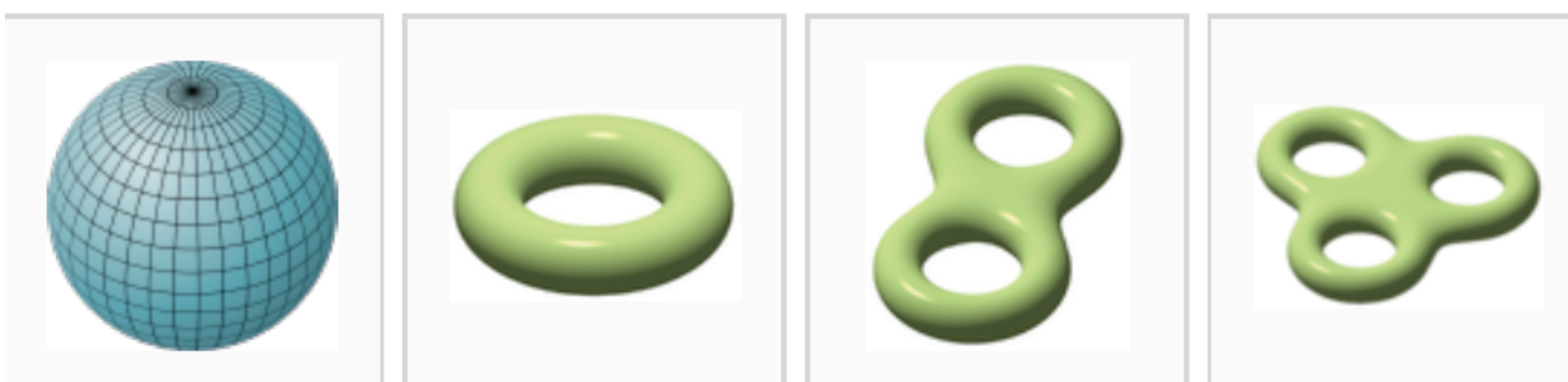


$$\subset \mathcal{M}$$

What is ”topological” about a topological insulator?

$$\frac{1}{2\pi}\int_{\mathcal{M}} K dS = 2(1-g)$$

$g = 0$ $\quad$ $g = 1$ $\quad$ $g = 2$ $\quad$ $g = 3$



What is ”topological” about a topological insulator?

Generalized *Gauss-Bonnet theorem*

$$\frac{1}{2\pi}\int_{\mathcal{M}} K dS = 2(1-g)$$

What is "topological" about a topological insulator?

"Chern theorem"

Generalized *Gauss-Bonnet theorem*

$$\frac{1}{2\pi}\int_{\mathcal{M}} \mathcal{F} \cdot d\boldsymbol{S} = C$$

Berry curvature

Chern number

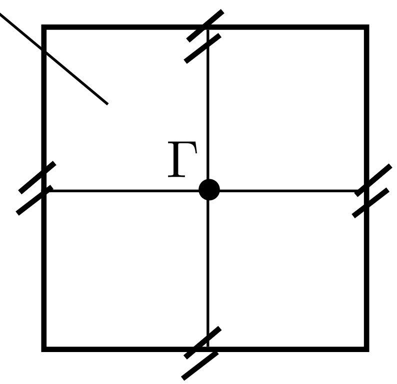
Shiing-Shen Chern, 1911-2004

$$\frac{1}{2\pi}\int_{\mathcal{M}} \mathcal{F}\cdot d\boldsymbol{S} = C$$

Electron wave function in a crystal

$$\psi_{n,\boldsymbol{k}}(\boldsymbol{r}) = u_{n,\boldsymbol{k}}(\boldsymbol{r})\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

band index



Brillouin zone (BZ)

$$\frac{1}{2\pi}\int_{\mathcal{M}} \mathcal{F} \cdot d\boldsymbol{S} = C$$

Electron wave function in a crystal

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Bloch wave function $u_{n,\boldsymbol{k}}(\boldsymbol{r}) = \langle\, \boldsymbol{r} \,|\, u_{n,\boldsymbol{k}} \,\rangle$

$$\frac{1}{2\pi}\int_{\mathcal{M}} \mathcal{F} \cdot d\boldsymbol{S} = C$$

Electron wave function in a crystal

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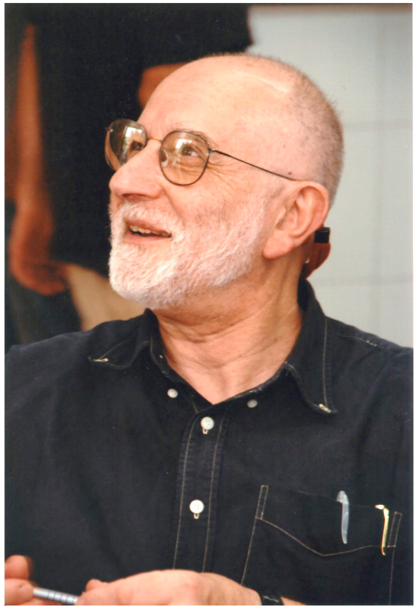
Berry curvature Berry, RSPSA (1984)

$$\boldsymbol{\mathcal{F}}_n(\boldsymbol{k}) = -i\nabla_{\boldsymbol{k}} \times \langle u_{n,\boldsymbol{k}} \mid \nabla_{\boldsymbol{k}} \mid u_{n,\boldsymbol{k}}\rangle$$



Michael Berry

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$$\frac{1}{2\pi}\sum_{n=1}^{\sharp\text{ filled bands}} \int_{\mathcal{M}=\mathrm{BZ}} \mathcal{F}_{n,z}(\boldsymbol{k})\, dk_x\, dk_y = C$$

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odd under
time reversal

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vanishing
Chern number

broken time-reversal symmetry (like in the quantum Hall effect)

Electron wave function in a crystal

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~~odd under time reversal~~

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broken time-reversal symmetry (like in the quantum Hall effect)

Electron wave function in a crystal

$$\psi_{n,\boldsymbol{k}}(\boldsymbol{r}) = u_{n,\boldsymbol{k}}(\boldsymbol{r}) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

~~odd under time reversal~~

Berry curvature Berry, RSPSA (1984)

$$\mathcal{F}_n(\boldsymbol{k}) = -i\nabla_{\boldsymbol{k}} \times \langle u_{n,\boldsymbol{k}} \mid \nabla_{\boldsymbol{k}} \mid u_{n,\boldsymbol{k}} \rangle$$

$$\frac{1}{2\pi} \sum_{n=1}^{\sharp\ \text{filled bands}} \int_{\mathcal{M}=\mathrm{BZ}} \mathcal{F}_{n,z}(\boldsymbol{k})\, dk_x\, dk_y = C$$


David Thouless

integer quantum Hall effect: ”TKNN invariant”

Thouless *et al.*, PRL (1982)

What if we identify time-reversed points in the BZ?

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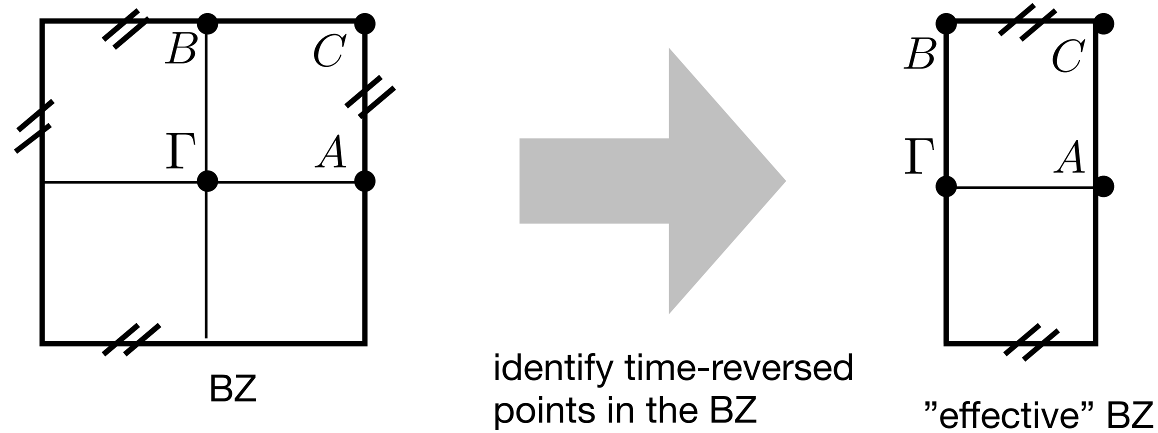
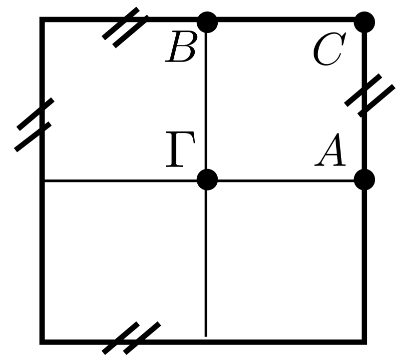
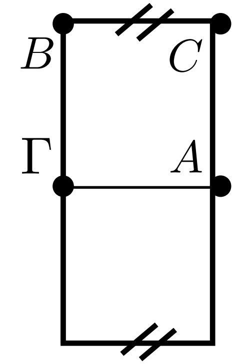


B
C
Γ
A
BZ
identify time-reversed points in the BZ
B
C
Γ
A
”effective” BZ

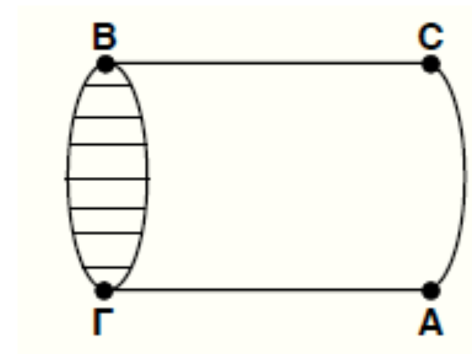


BZ

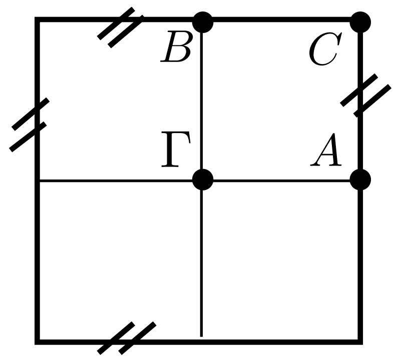
identify time-reversed
points in the BZ

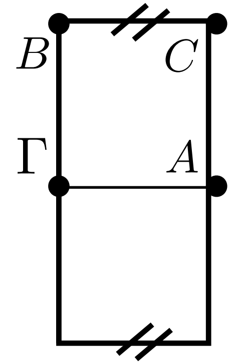


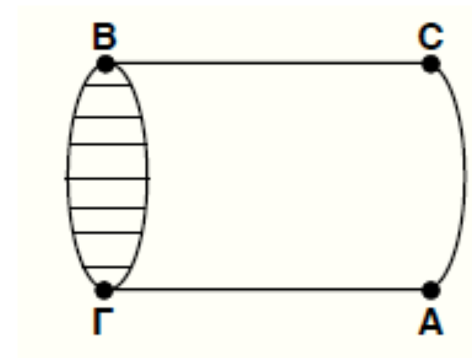
"effective" BZ



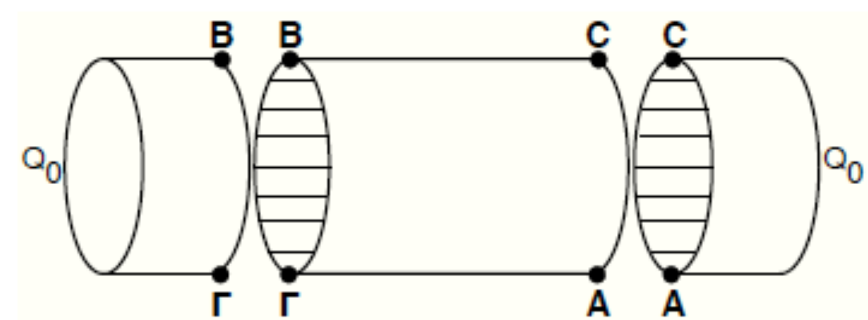
open manifold:
NO quantization
from the Berry curvature



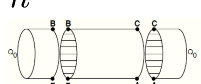




"close" the cylinder!



$$\frac{1}{2\pi}\sum_n \int \boldsymbol{\mathcal{F}}_n(\boldsymbol{k}) \cdot d\boldsymbol{k} = C$$



Doesn't give a unique Chern number!

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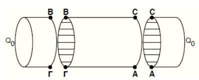

The *parity* of the Chern number is unique!

Kane & Mele, PRL (2005)

$$C = \begin{cases} 0 \bmod 2 & \text{ordinary insulator} \\ 1 \bmod 2 & \text{topological insulator} \end{cases}$$

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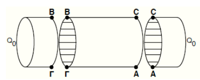

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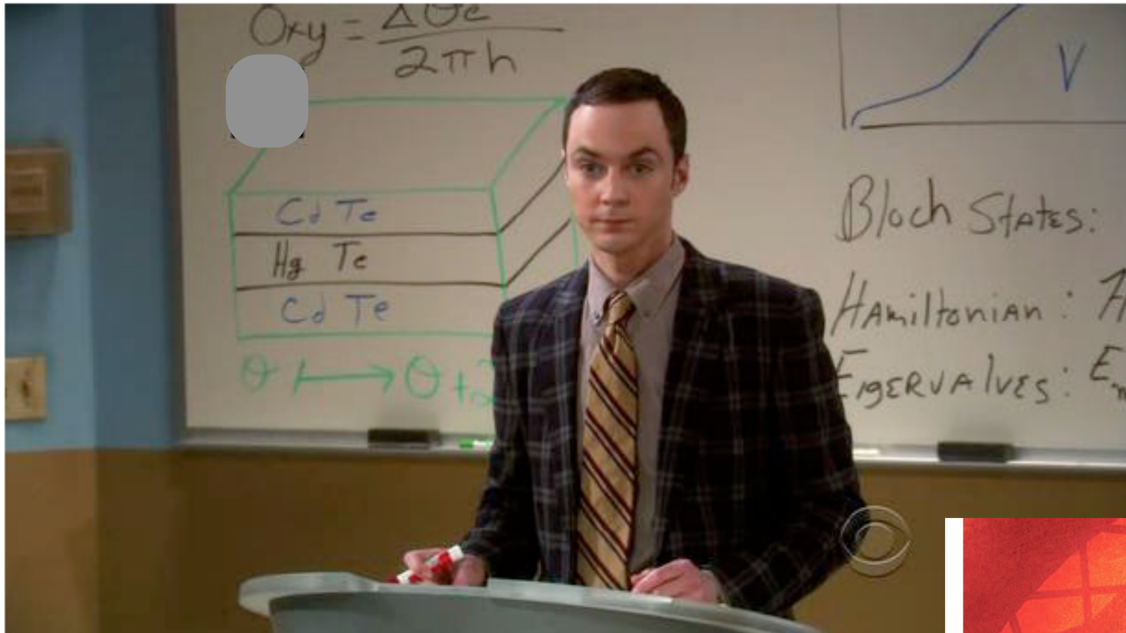
”Z_2 topological invariant”, counts the number of Kramers pairs at the edge of the topological insulator *(bulk-boundary correspondence)*

Why all the hoopla?

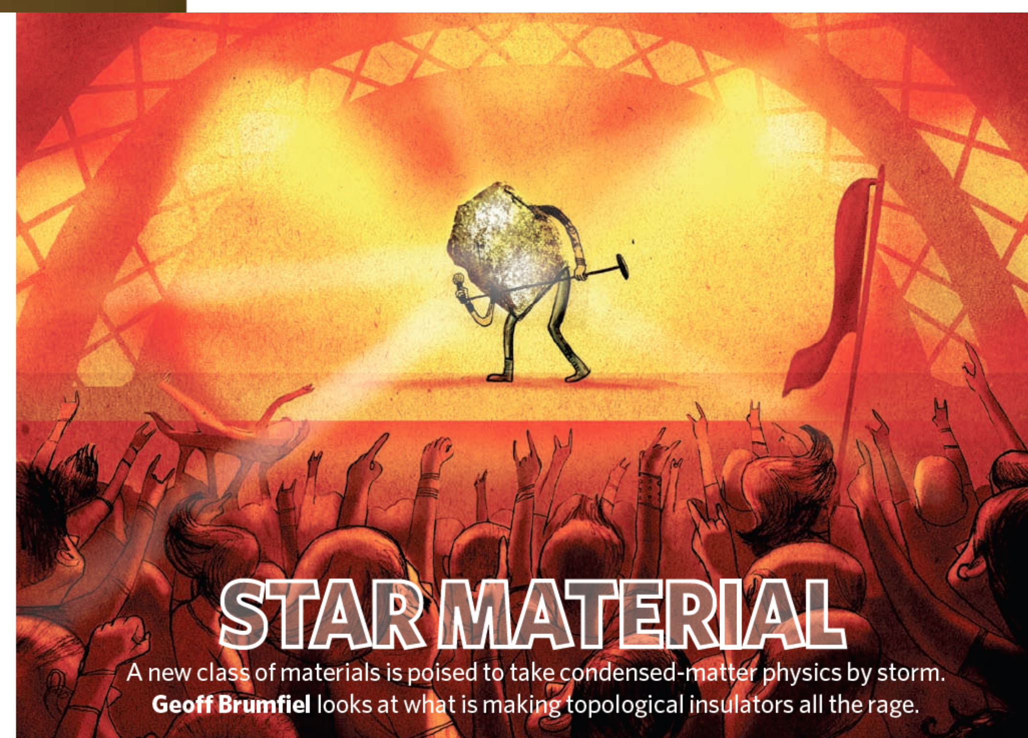
"Who here is familiar with the concept of Topological Insulators?"



TV-Series: „The Big Bang Theory"

https://www.youtube.com/watch?v=HBuLMrzgbgM

BREAKTHROUGH OF THE YEAR
ELECTRONS TAKE A NEW SPIN

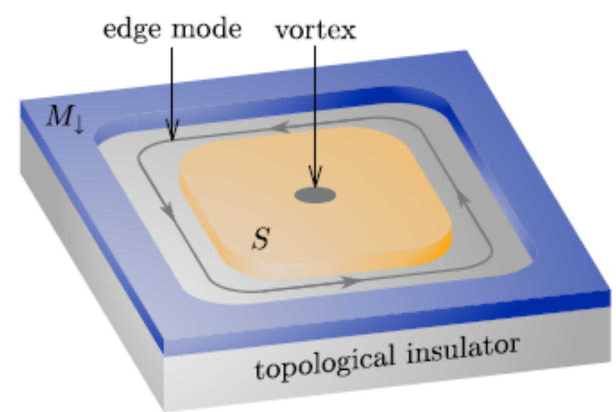


Why all the hoopla?

- Future electronics/spintronics:

Toward dissipationless spin transport in semiconductors

- New physics in hybrid structures:
  Majorana fermions, magnetic monopoles, "dyons",...



- An inroad to the general study of

TOPOLOGICAL QUANTUM MATTER

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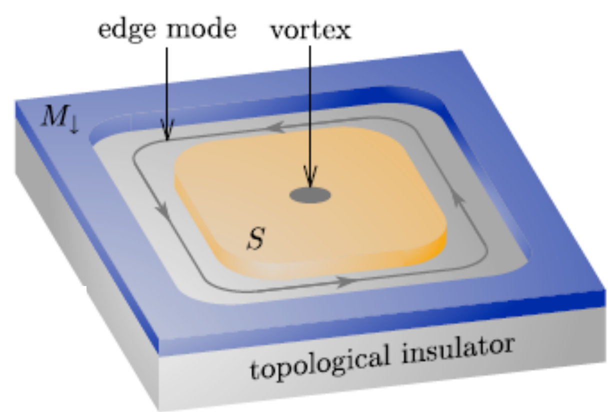


- An inroad to the general study of

symmetry-protected

TOPOLOGICAL QUANTUM MATTER

Some cool stuff exploiting the helical edge states:

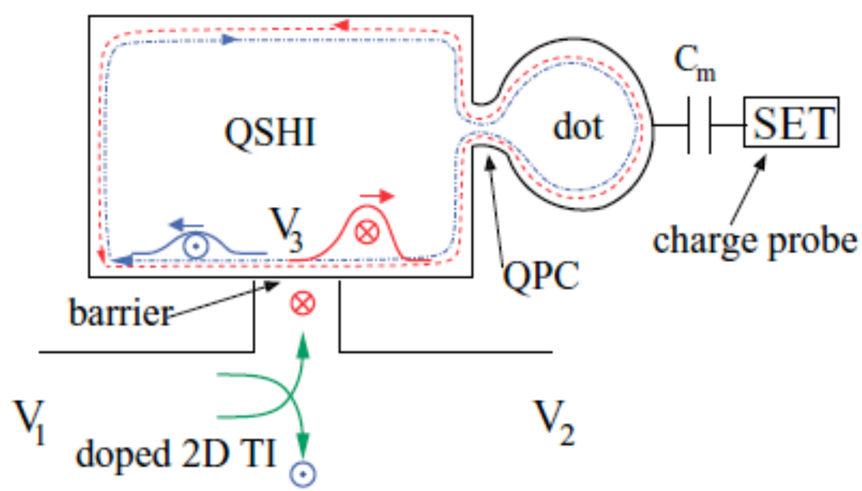
Probing charge fractionalization

PHYSICAL REVIEW B **85**, 195465 (2012)

Noninvasive probes of charge fractionalization in quantum spin Hall insulators

Ion Garate[1] and Karyn Le Hur[1,2]
[1]*Department of Physics, Yale University, New Haven, Connecticut 06520, USA*
[2]*Center for Theoretical Physics, Ecole Polytechnique, CNRS, 91128 Palaiseau Cedex, France*
(Received 19 November 2011; published 30 May 2012)

When an electron with well-defined momentum tunnels into a nonchiral Luttinger liquid, it breaks up into two separate wave packets that carry fractional charges and move in opposite directions. A direct observation of this phenomenon has proven elusive, mainly due to single-particle and plasmon backscattering caused by measurement probes. This paper theoretically introduces two topological insulator devices that are naturally suited for detecting fractional charges and their velocities directly and in a noninvasive fashion.



Some cool stuff exploiting the helical edge states:

Spin filtering

Stephan Rachel[1] and Motohiko Ezawa[2]
[1] *Institute for Theoretical Physics, TU Dresden, 01062 Dresden, Germany*
[2] *Department of Applied Physics, University of Tokyo, Hongo 7-3-1, 113-8656, Japan*

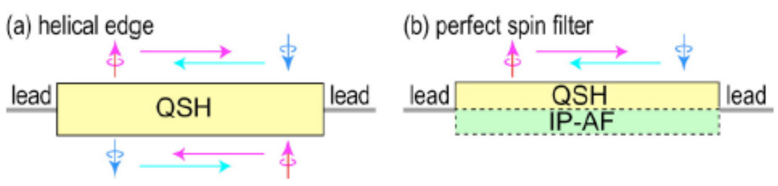


FIG. 6. (Color online) Illustration of a perfect spin filter. A solid (dotted) boundary represents an edge where metallic edge modes (do not) emerge. (a) In general, helical modes circulate around a sample. (b) By introducing the IP-AF order to the lower half of the sample, helical modes only propagate along the upper edge. In this example, only ↑ spins are transported from the left to the right lead.

the simplest case, we assume that the direction can be adjusted in the xz plane by parametrizing

B. Perfect spin filter

The second application is a perfect spin filter, which is realized when we turn on the IP-AF order only on one half of the nanoribbon [Fig. 6(b)]. Since helical edge states are present only on the other half of the nanoribbon, we have a *one-way helical edge state*. That is, by sending a spin-unpolarized current through the nanoribbon, only ↑ spins (or ↓ spins) can pass the nanoribbon, hence it is a spin filter. The spin filter is *perfect* since the spin-momentum locking is an inescapable property of the topological insulator. We note that usual helical edge modes circulate around the sample, that is, the direction of two helical edge modes are opposite on opposite sides of the nanoribbon [Fig. 6(a)]. In the latter case, there is no spin-filtering effect. Note that similar ideas about blocking a helical edge channel have been considered in Refs. [36,37].

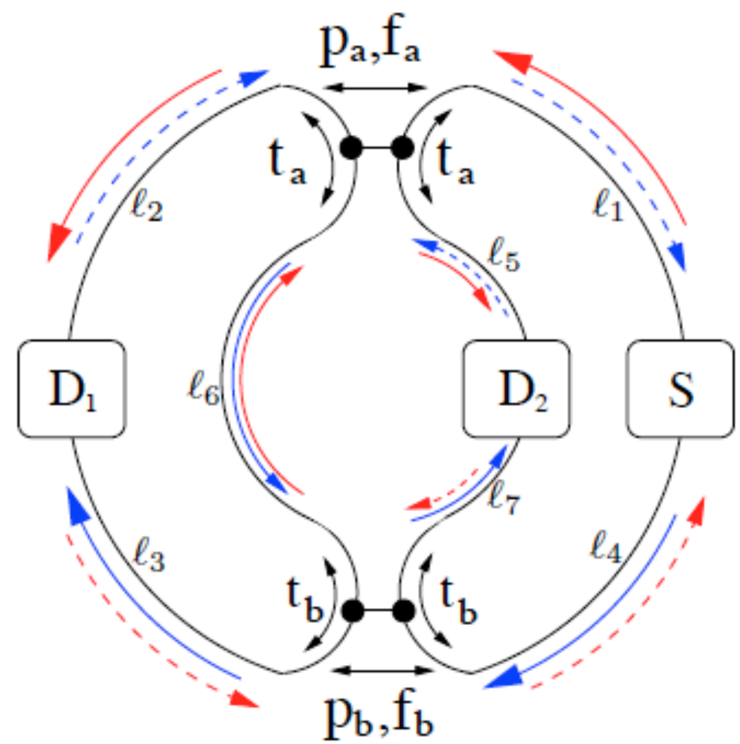
Some cool stuff exploiting the helical edge states:

”On-demand” spin entangler

Controllable spin entanglement production in a quantum spin Hall ring

Anders Ström,[1] Henrik Johannesson,[2] and Patrik Recher[1]
[1] Institute for Mathematical Physics, TU Braunschweig, 38106 Braunschweig, Germany
[2] Department of Physics, University of Gothenburg, 412 96 Gothenburg, Sweden
(Dated: June 11, 2014)

We study the entanglement production in a quantum spin Hall ring where electrons of different spins are emitted from a source and detected in two different detectors. Post-selection gives rise to entanglement in the system, measurable through correlations between the outcomes in the detectors. The production of entanglement depends on the dynamical phases picked up by the edge states as they move along the ring. The dependence of the phases on the chemical potential and Rashba interaction in the system allows for electrical control of the entanglement production in the ring.



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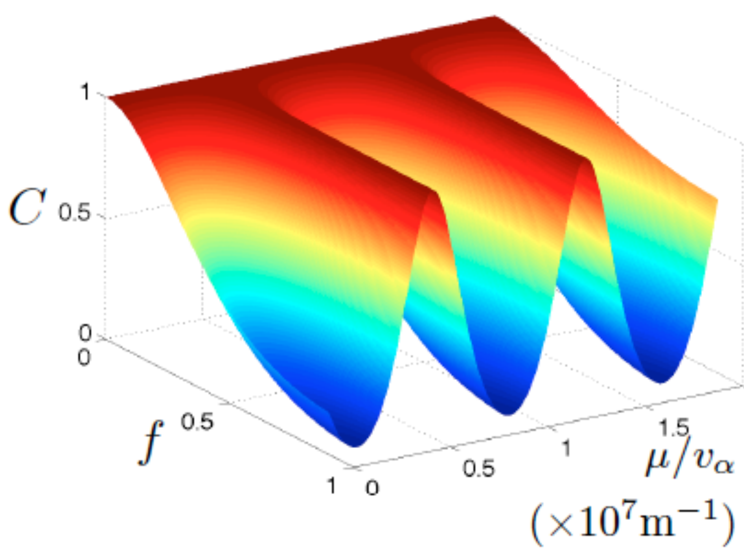

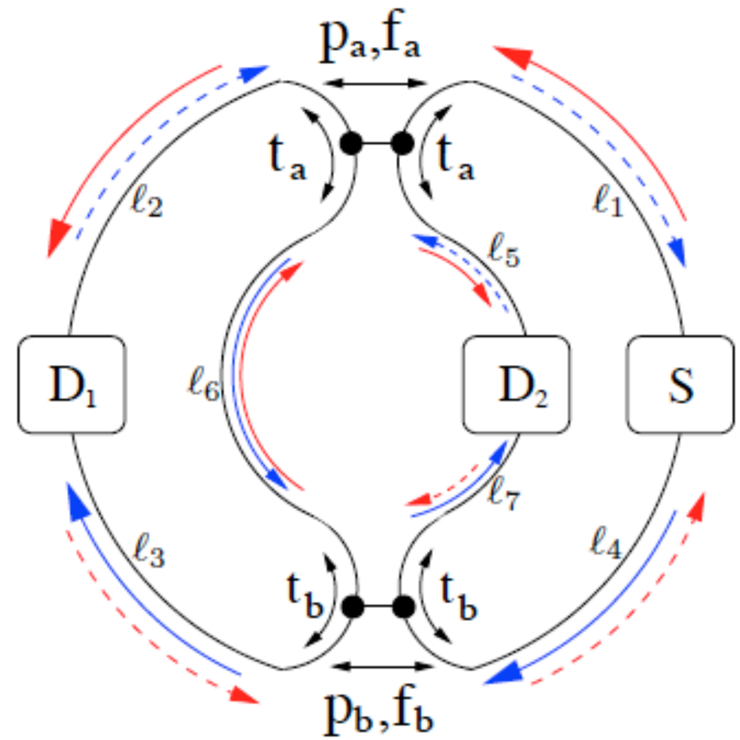

Bad news: Experimental realizations of 2D topological insulators are tricky to handle!

Since its discovery in 2006, the topological phase of the HgTe/CdTe quantum well has still only been probed experimentally in Laurens Molenkamp's lab in Würzburg.

Laurens Molenkamp

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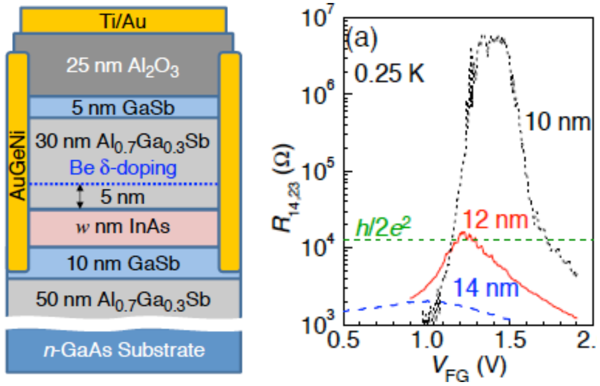
Laurens Molenkamp

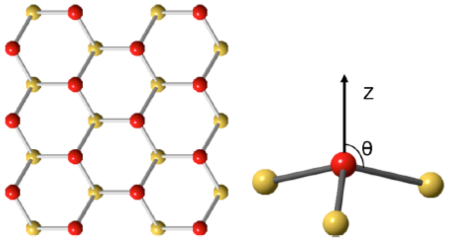
Candidate 2D topological insulators:

"Stanene" (single atomic layer of tin)
Xu *et al.,* PRL (2013)

InAs/GaSb quantum wells
Suzuki *et al.,* PRB (2013)



Silicene
C.-C. Liu *et al.,* PRL (2011)



Alternative realizations of helical electron liquids in high demand...

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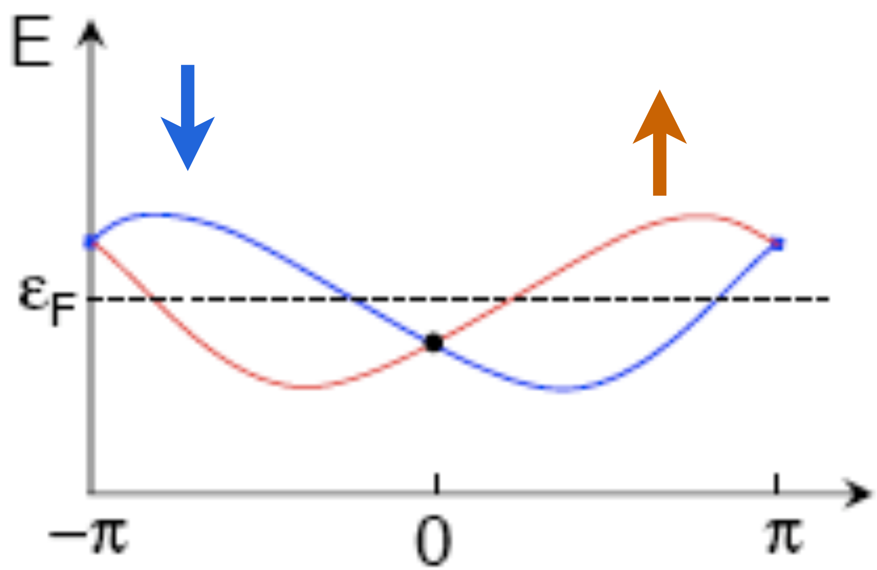
... but there is a catch!



”Fermion doubling” in 1D time-reversal invariant systems

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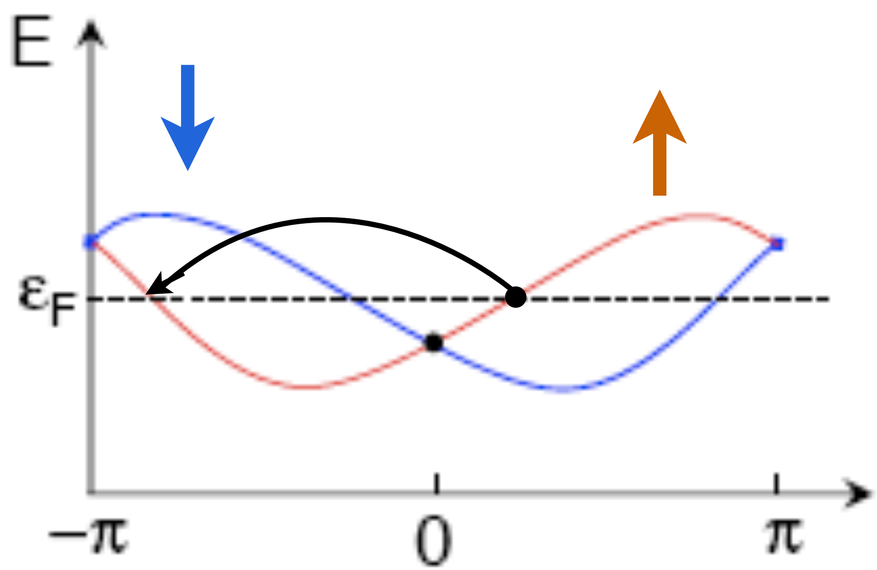
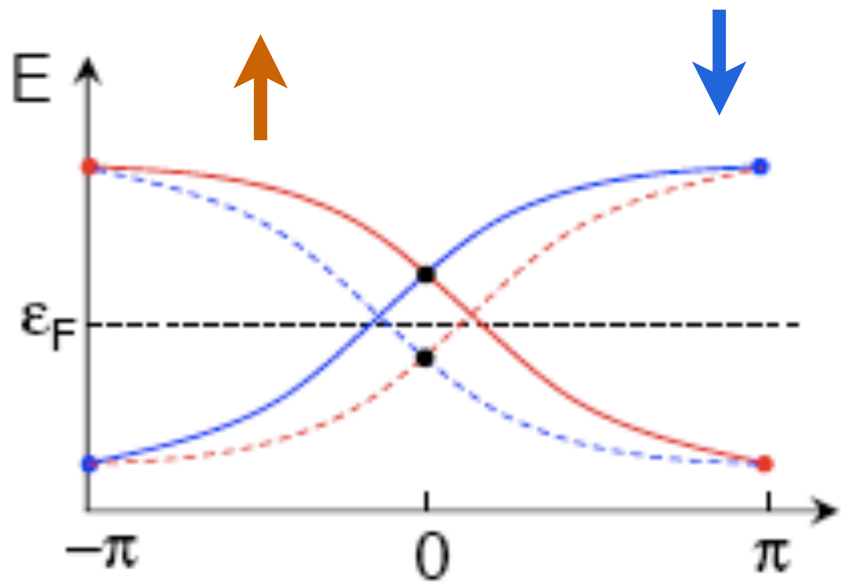


”Fermion doubling” in 1D time-reversal invariant systems

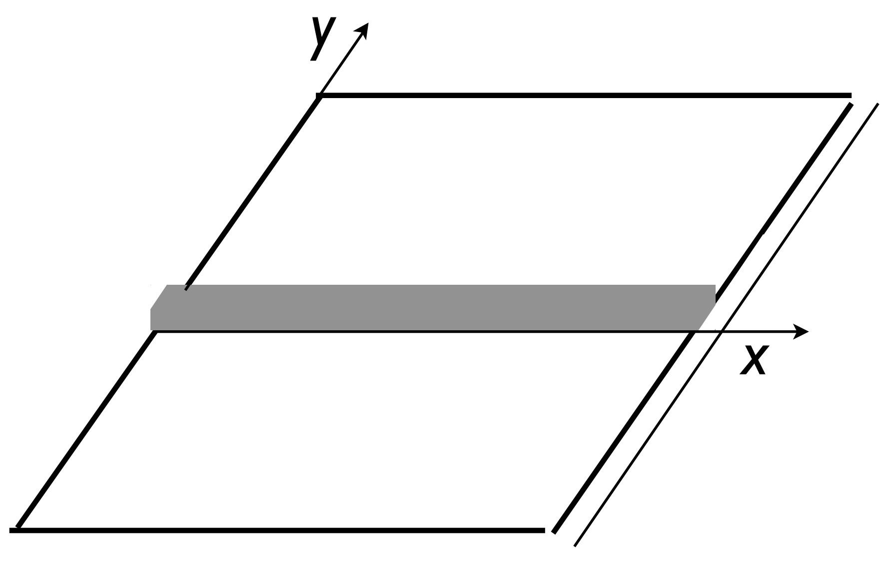
single-particle backscattering in presence of disorder

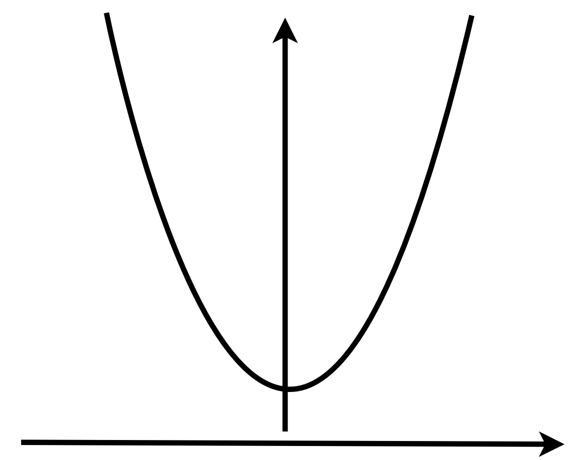
The 2D topological insulator ”solves” the fermion doubling problem in an elegant way:



P. Streda and P. Seba, PRL (2003)

Another way out: start with a quantum wire, ...





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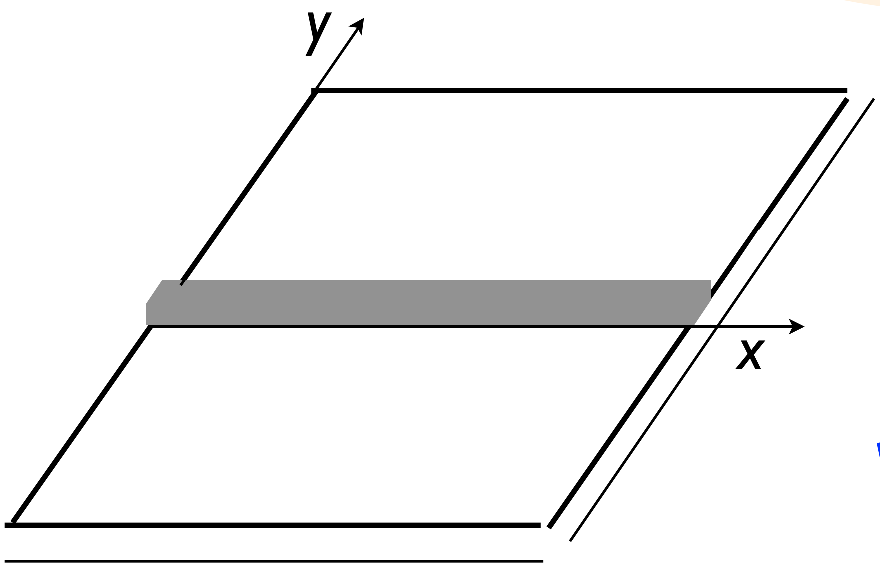
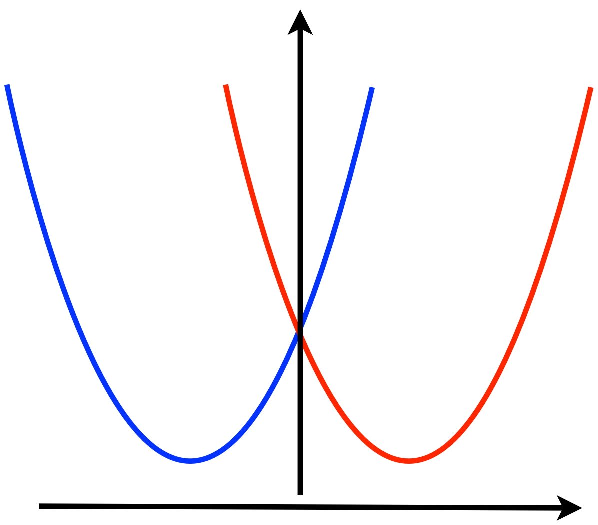
Another way out: start with a quantum wire, spin split, ...

$$H_{\mathrm{Rashba}} = \alpha k_x \sigma^y$$

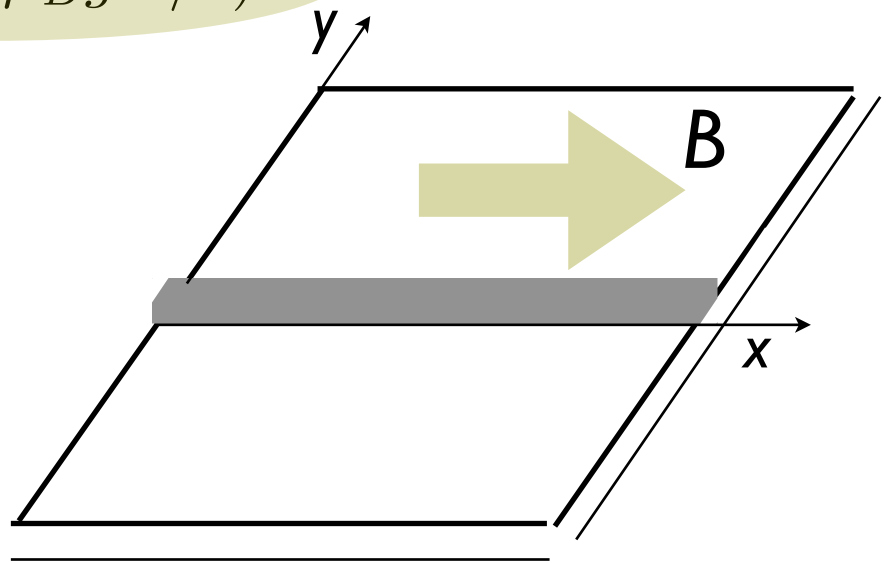


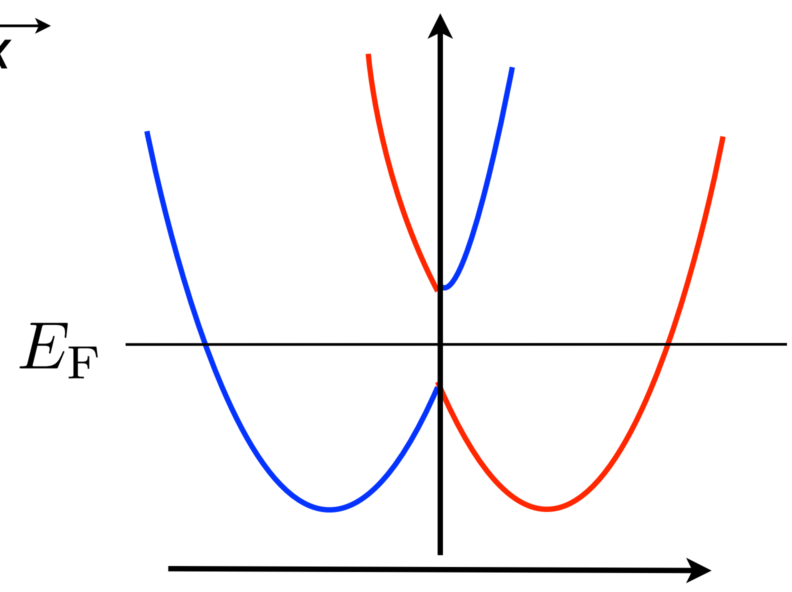


P. Streda and P. Seba, PRL (2003)

Another way out: start with a quantum wire, spin split, and break time reversal!

$$H_{\mathrm{Zeeman}} = (\mu_B g B/2)\sigma^x$$





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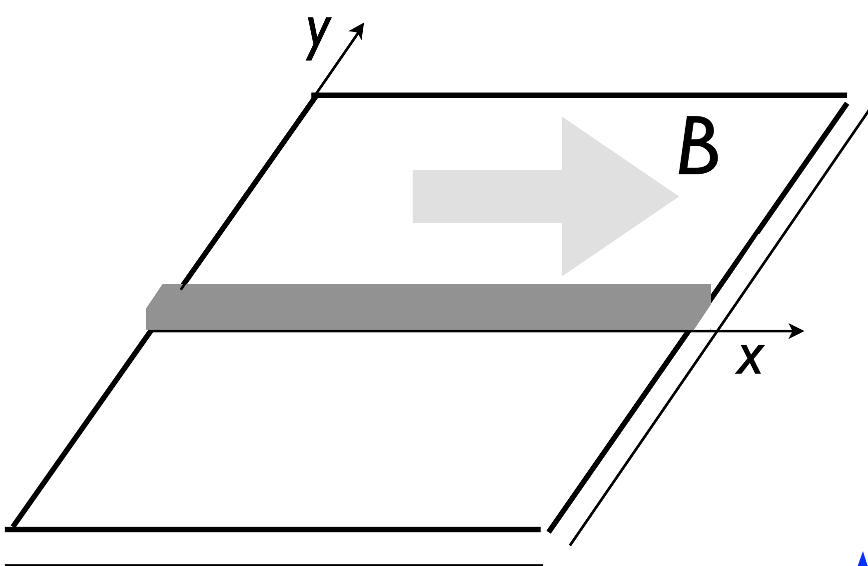

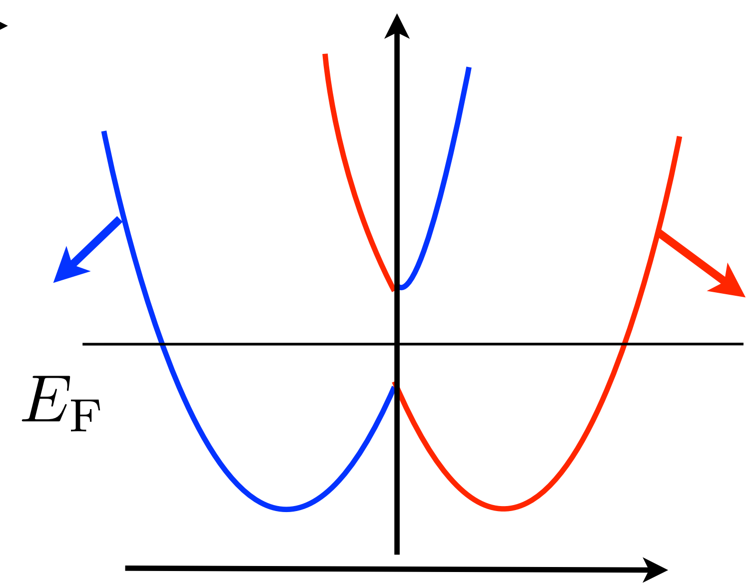

Spin overlap again enables disorder backscattering

Braunecker *et al.*, PRB (2013)

Can one do better?

PHYSICAL REVIEW B **89**, 201403(R) (2014)

Synthetic helical liquid in a quantum wire

George I. Japaridze,[1,2] Henrik Johannesson,[3] and Mariana Malard[4]
[1]*Andronikashvili Institute of Physics, Tamarashvili 6, 0177 Tbilisi, Georgia*
[2]*Ilia State University, Cholokasvili Avenue 3-5, 0162 Tbilisi, Georgia*
[3]*Department of Physics, University of Gothenburg, SE 412 96 Gothenburg, Sweden*
[4]*Faculdade UnB Planaltina, University of Brasilia, 73300-000 Planaltina-DF, Brazil*
(Received 14 November 2013; revised manuscript received 17 April 2014; published 15 May 2014)

We show that the combination of a Dresselhaus interaction and a spatially periodic Rashba interaction leads to the formation of a helical liquid in a quantum wire when the electron-electron interaction is weakly screened. The effect is sustained by a helicity-dependent effective band gap which depends on the size of the Dresselhaus and Rashba spin-orbit couplings. We propose a design for a semiconductor device in which the helical liquid can be realized and probed experimentally.

Mariana Malard
University of Brasilia

Gia Japaridze
Andronikashvili Institute

Idea: Replace the magnetic field by a spatially periodic Rashba coupling. When the *e*-*e* interaction is weakly screened, a helical liquid is generated dynamically!

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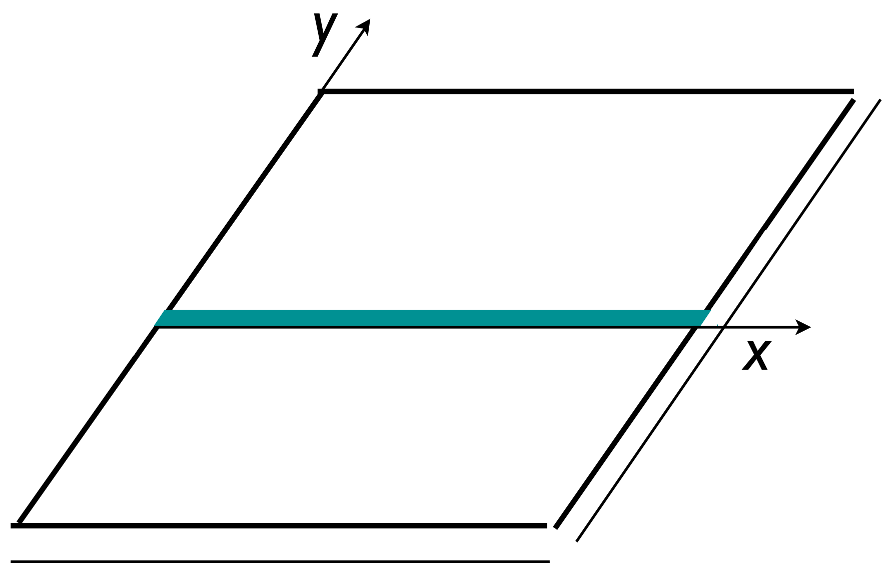
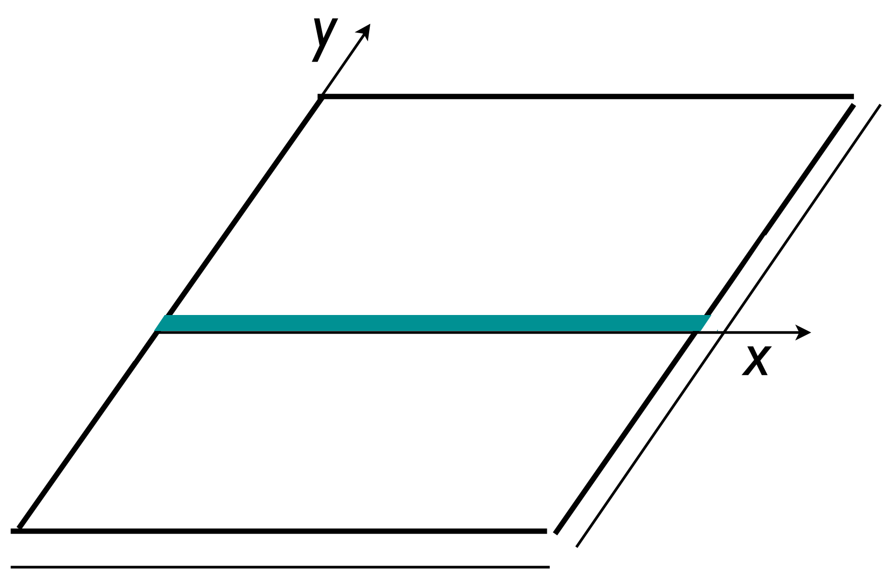
Another proposal for a synthetic helical liquid without a magnetic field:

Klinovaja et al., PRL (2011) [armchair carbon nanotube in a strong electric field]

Again: start with a quantum wire,...



Again: start with a quantum wire, but now put a periodic sequence of charged electrodes on top, ...



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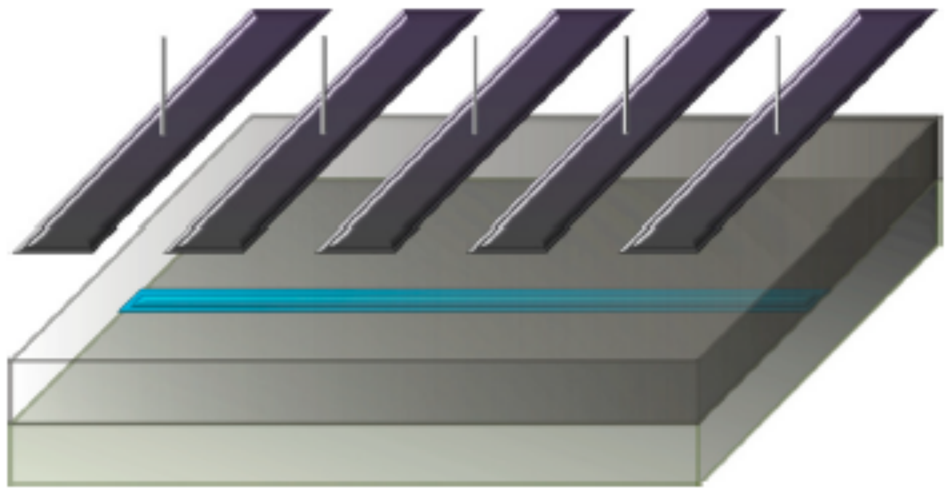

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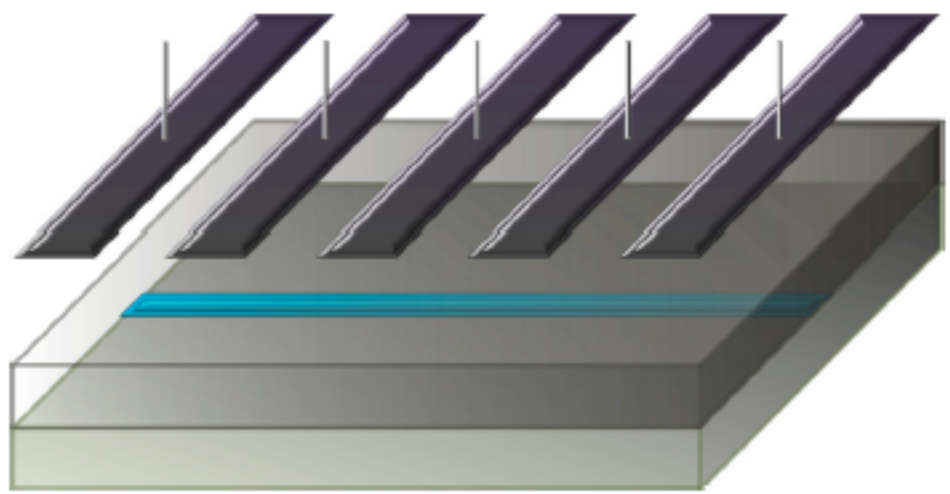
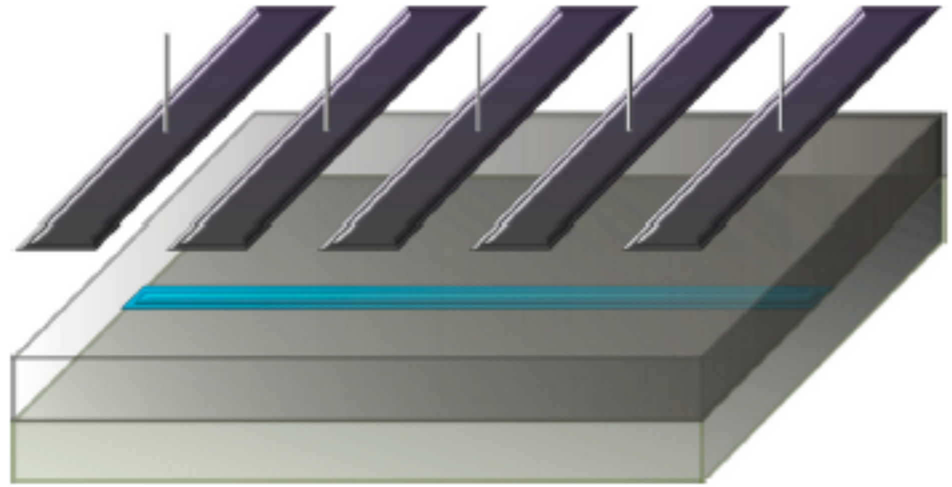


periodic modulation of chemical potential and Rashba spin-orbit interaction



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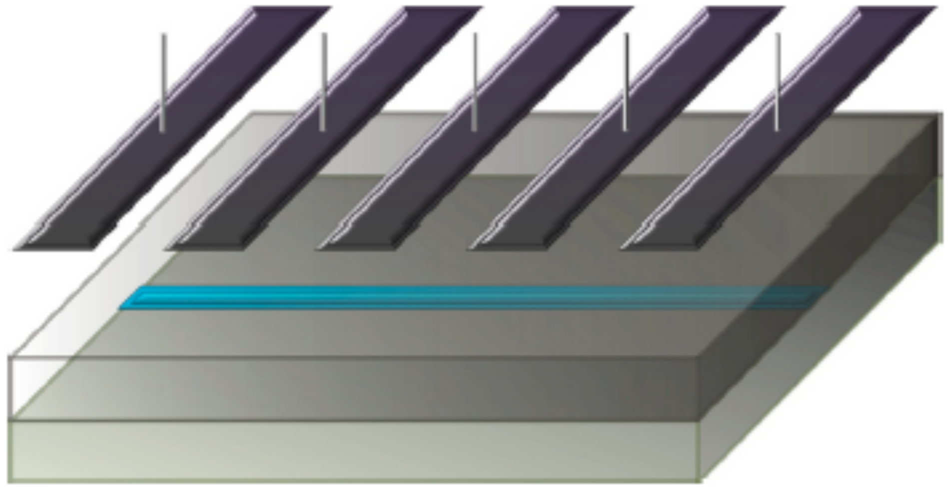
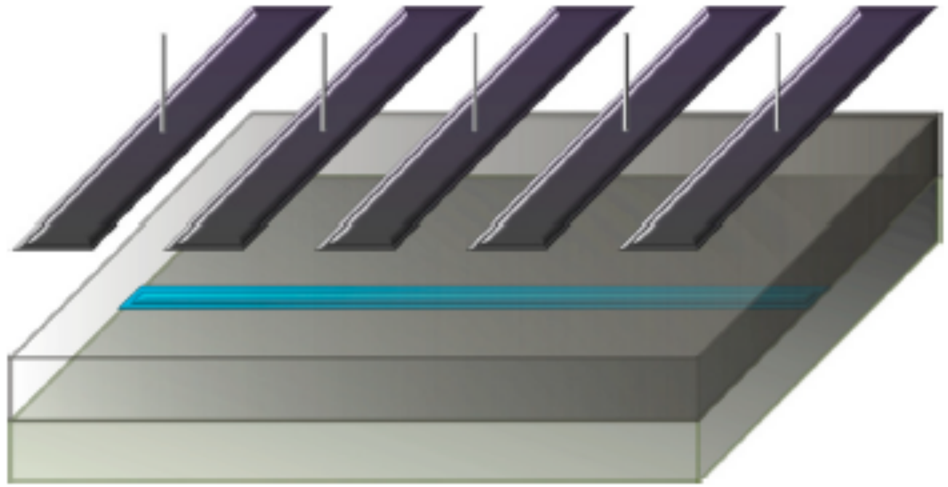


periodic modulation of chemical potential and Rashba spin-orbit interaction

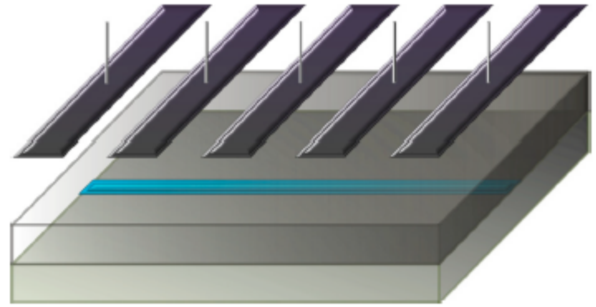
use a backgate to tune the average electron density to a value determined by the wave length of the modulation



periodic modulation of chemical potential and Rashba spin-orbit interaction

use a backgate to tune the average electron density to a value determined by the wave length of the modulation

weak screening of the *e*-*e* interaction



$$
\begin{aligned}
H &= -t\sum_{n,\alpha} c^{\dagger}_{n,\alpha} c_{n+1,\alpha} + \frac{\mu}{2}\sum_{n,\alpha} c^{\dagger}_{n,\alpha} c_{n,\alpha} - i\sum_{n,\alpha,\beta} c^{\dagger}_{n,\alpha}\left[\gamma_{\mathrm{D}}\,\sigma^{x}_{\alpha\beta} + \gamma_{\mathrm{R}}\,\sigma^{y}_{\alpha\beta}\right] c_{n+1,\beta} \\
&- i\gamma'_{\mathrm{R}}\sum_{n,\alpha,\beta} \cos(Qna)\, c^{\dagger}_{n,\alpha}\sigma^{y}_{\alpha\beta} c_{n+1,\beta} + \frac{\mu'}{2}\sum_{n,\alpha} \cos(Qna)\, c^{\dagger}_{n,\alpha} c_{n,\alpha} \\
&+ \sum_{n,n';\alpha,\beta} V(n-n')\, c^{\dagger}_{n,\alpha} c^{\dagger}_{n',\beta} c_{n',\beta} c_{n,\alpha} + \mathrm{h.\ c.}
\end{aligned}
$$

$$
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&- \ i\gamma'_{\rm R}\sum_{n,\alpha,\beta}\cos(Qna)c^{\dagger}_{n,\alpha}\sigma^{y}_{\alpha\beta}c_{n+1,\beta} + \frac{\mu'}{2}\sum_{n,\alpha}\cos(Qna)c^{\dagger}_{n,\alpha}c_{n,\alpha} \\
&+ \ \sum_{n,n';\alpha,\beta} V(n-n')c^{\dagger}_{n,\alpha}c^{\dagger}_{n',\beta}c_{n',\beta}c_{n,\alpha} + \text{h. c.}
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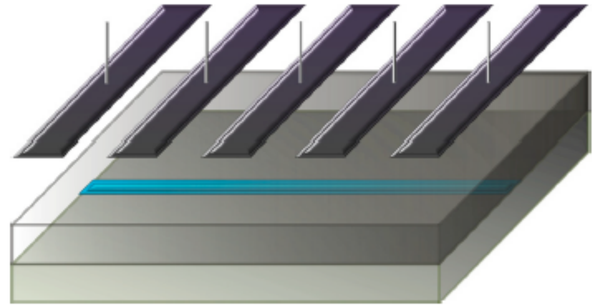
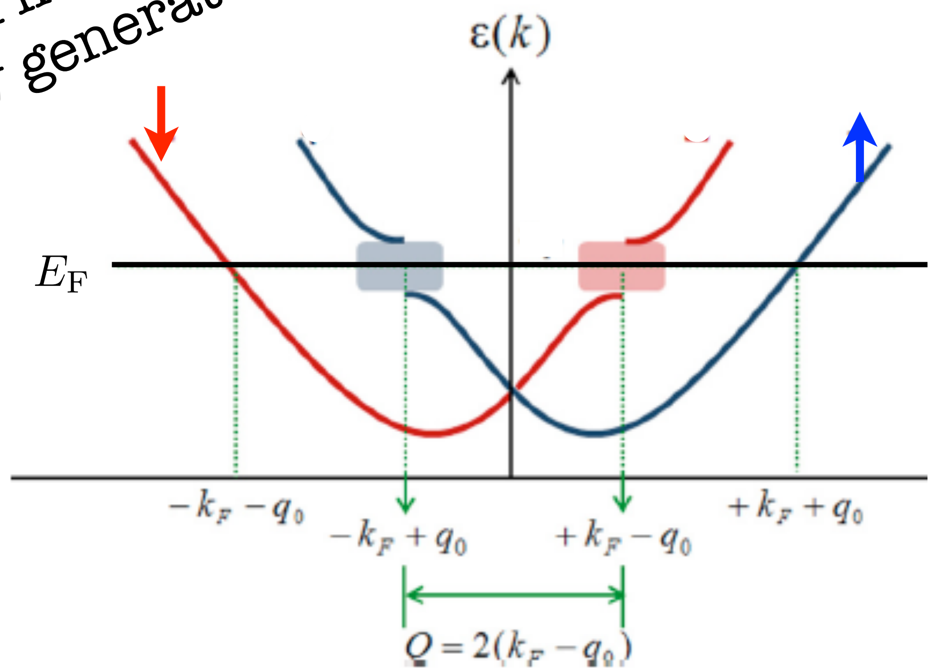
$$

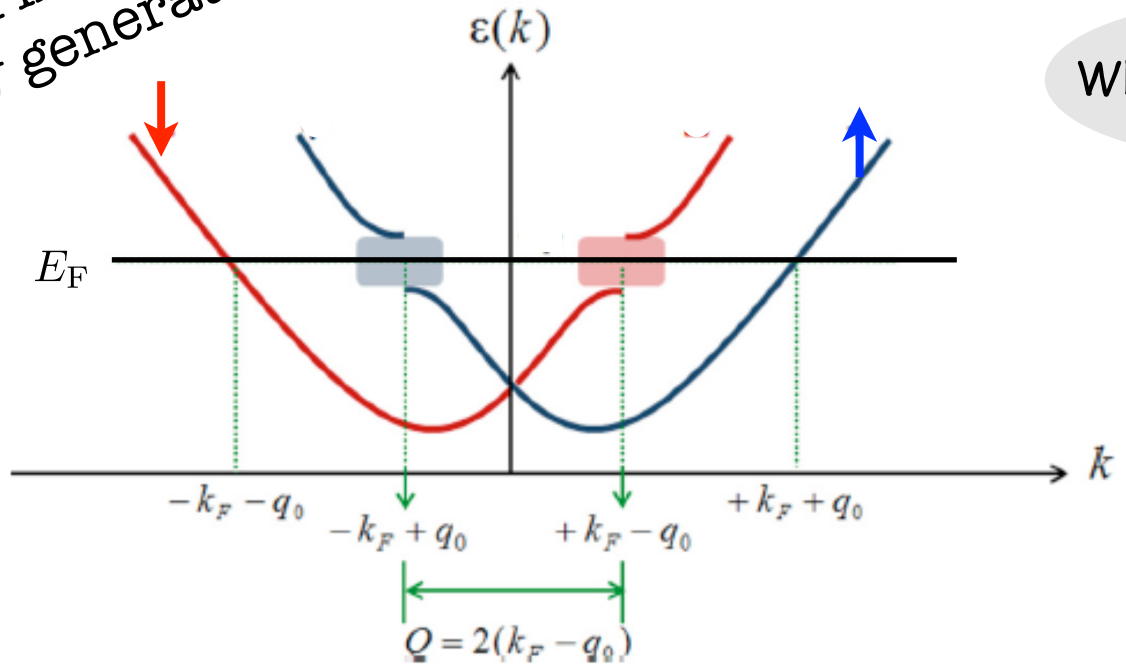
bosonization &
perturbative RG



Helical liquid inside the dynamically generated gap

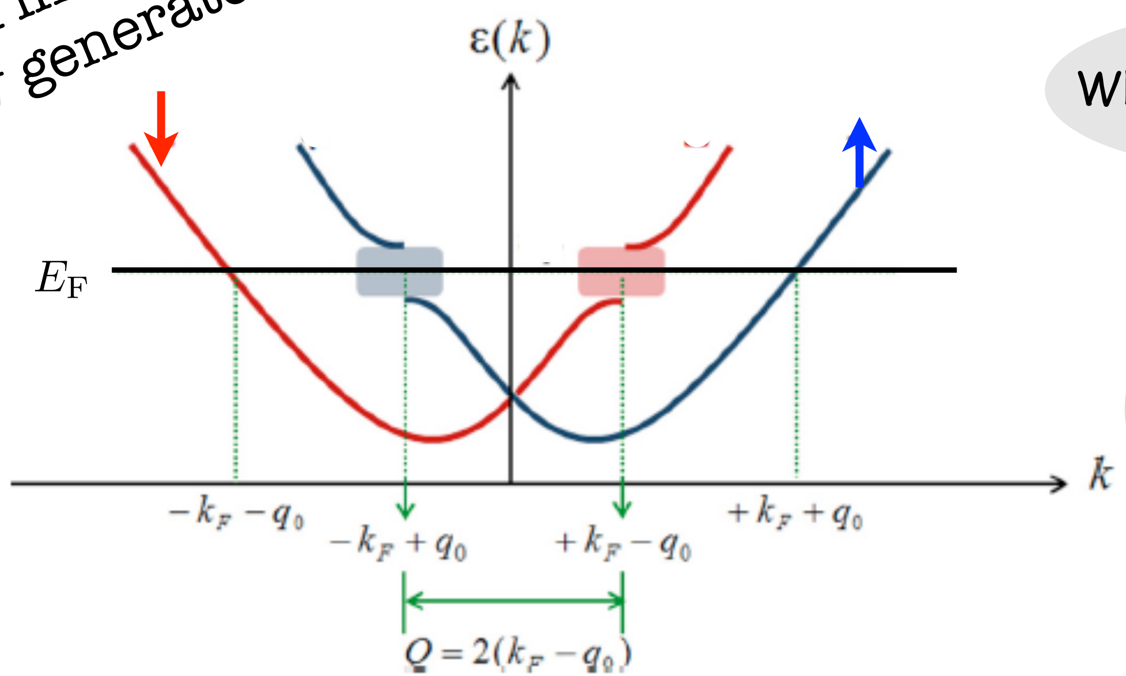


Helical liquid inside the dynamically generated gap



What's the trick?

Helical liquid inside the dynamically generated gap



What's the trick?

Spontaneous breaking of time-reversal symmetry in the gapped branch!

Will it work in the lab?

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Can the screening of the e-e interaction
be reduced sufficiently much?
(Luttinger liquid parameter K<1/2)?

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(Luttinger liquid parameter $K<1/2$)?

Can the gap be made sufficiently large to sustain the helical liquid?

when the InAs QW is separated from the top gates by a solid PEO/$LiClO_4$ electrolyte, the Rashba coupling $\hbar\alpha$ is found to change from 0.4×10^{-11} eVm to 2.8×10^{-11} eVm when tuning a top gate from 0.3 to 0.8 V.

D. Liang and X. P. A. Gao, Nano Lett. **6**, 3263 (2012)

into Eq. (15), and choosing, say, $K = 1/4$ with $c(1/4) = 1$ [31] we obtain $\Delta \approx 0.3$ meV (with smaller values of

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Is the synthetic helical liquid robust against disorder?

the localization length ξ_{rand} for an InAs wire, making the usual assumption that $\sqrt{\langle \alpha^2_{\mathrm{rand}}(x)\rangle} \approx \langle \alpha(x)\rangle$ [22], turns out to be much larger than the renormalization scale $\xi = \hbar v/\Delta$ at which the helicity gap develops [43]. Moreover, estimates of the elastic mean free path ℓ_{e} for InAs quantum wires [42] show that $\xi < \ell_{\mathrm{e}} < \xi_{\mathrm{rand}}$ when $1/5 < K < 1/2$ and $\alpha_{\mathrm{rand}}(x) < 4 \times 10^{-11}$ eVm. It follows that the synthetic HL is well protected within these parameter intervals.

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Experiments will tell!

Merci pour votre attention!